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SPRING ICE JAMS IN STREAM CHANNELS

PHYSICAL PRINCIPLES AND QUANTITATIVE ANALYSIS

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Ice jams as natural, regular formations with specific, but not identical forms, sizes and properties were examined. This permitted a description of the theoretical principles of the investigated processes of jam formation in relatively total form. The material presented includes a) the establishment of initial theoretical concepts of a model of ice movement and ice jams; b) an investigation of the causes and formation mechanism of ice jams and their special features as a function of the concrete conditions of the external environment; c) a quantitative analysis of the elements of jam formation in the form used for solving applied problems; and d) theoretical bases and practical considerations for methods of combatting ice jams and controlling them.

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Physical Principles and Quantitative Analysis

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Foreword

/3/* Ice jams, a regular and widespread phenomenon in the freezing and thawing of the many rivers of the USSR, can cause considerable loss as a consequence of the floods that they cause and the ice damage to water resources equipment, ships, etc. The losses are often aggravated as a consequence of the sharply expressed elementary nature of the phenomenon, the intense dynamics of the process of jam formation and the inconsistency from one year to the next of the size and location of these jams. In the total complex of water resources practice, the formation of jams, including artificial ones, can have a positive effect, for example, for purposes of improving the irrigation of flood plains, reducing the intensity of ice movement, and preventing ice jams in low-lying sections of channel flows.

Under the conditions of an intensive water resources organization, the problem of managing ice movement including its regulation, combatting ice obstacles, predictions, and a quantitative analysis of ice jams should be classified as an important hydraulic

*Numbers in slashes in the left-hand margin indicate the original Russian page.

engineering problem. At the present time there is a large accumulation of materials investigating the causes and conditions of the formation of spring ice jams. These investigations are to a great extent limited to a qualitative study of the phenomenon, although in recent years a number of important investigations have appeared devoted to a quantitative analysis of the individual problems of jam formation.

A cardinal problem still remains with respect to the development of a general theory of ice jams. The present study is devoted to solving some questions related to this problem. Considerable space is devoted to an analysis (based on physics with the drawing in of special fields of mechanics) of the regularity of ice jams which arise in the process of thawing and ice movement in channel flows. Investigations in this area, begun in 1948 by Iu. A. Deev, include the theoretical study of ice jams combined with field observations, experiments, and literature sources.

We examined ice jams as regular natural formations with specific, but not identical forms, sizes, and properties. The individual properties of particles of a large-unit system required using methods of investigation somewhat different from those usually used for the study of accumulations of "finely crushed ice", "granulated blocks", etc. (see Section 1.2); these were considered here in spring ice jams comprised of the accumulation of relatively large ice blocks. This permitted a description of the

/4/ theoretical principles of the investigated processes of jam formation in relatively total form. Individual questions relative to the general theory of ice movement are included in the examination. The material presented includes:

(a) The establishment of initial theoretical concepts of a model of ice movement and ice jams;

(b) An investigation of the causes and formation mechanism of ice jams and their special features as a function of the concrete conditions of the external environment;

(c) A quantitative analysis of the elements of jam formation in the form used for solving applied problems;

(d) Theoretical bases and practical considerations for methods of combatting ice jams and controlling them.

The very comprehensive set of questions and the large volume of investigations that enter into the problem of ice jams did not permit examining all sides of the investigated phenomenon with the same detail within the limits of the present study. This predetermined the necessity of future supplements, revised editions and corrected versions of some of the provisions set forth in the book. Separate aspects can be perceived, such as discussions that fully conform to the modern stage of investigation of this phenomena.

The principles of the established regularities correspond to the results of field observations and agree with experimental studies related to the quantitative aspects of the process; they can be used in solving practical and applied problems as well as in basic research.

Part of the established regularities and dependences obviously may be applied to autumn ice jams, but the special features of these latter must be taken into investigation. Included here are: jamless passage and holding back accumulations of ice blocks, losses of stability of ice blocks in collisions, the strained state of the ice accretion, the formation of jams, and the increase in ice thickness in them, etc. However, autumn ice jams as a whole require special investigations and are not examined in the present study.

It should be noted that during the time the study was conducted, its individual aspects were studied by other researchers; however, the authors introduce all of their conclusions into the book, since this helps to maintain the integrity of the discussion and points out differences in the initial positions of the analytical method itself. Some general determinations and aspects that touch upon ice movement and ice jams are included in the study in order to simplify its utilization by a broad circle of water resources specialists.

The authors express their sincere thanks to Professor B. B. Bogoslovskii, to S. N. Bulatov, and F. I. Bydin, Ph. D., Technical Sciences; R. V. Donchenko, and A. N. Chizhov, Cand. Technical Sciences; L. M. Margolin and A. Ya. Rybkina, Cand. Geographical Sciences for their valuable observations and recommendations, made by reading and criticizing the manuscript; these were taken into consideration in preparing the study for publication.

/5/

Conventional Designations

Q, $q = \frac{Q}{B}$, H, B, L, B_0 - discharge, specific discharge, depth, width, length of a flow section, "effective" flow width

u, $u_0 = ru$, u_k and u_p - flow rate: surface, average, critical, and guaranteed tightening of the ice blocks (see section 2.1)

u_v - wind velocity

r - transfer coefficient from surface velocity to the central

H_p, H_m, and H_0 - head jam level in the given line of direction (flowing), maximum level and level corresponding to the stable state of a single layer of ice block accretion

l, b, h - length, width, and thickness of the ice block

h_1, h_m - thickness of the immersed section of the ice block accretion in the given line (flow line) and the maximum thickness

v, v_k, v_1, v_2, and v_3 - velocity of ice movement: total, critical, and guaranteeing hummock formation, submerging and piling up of the ice blocks;

$v_0 = u-v$ - relative velocity of ice movement

S and S_p - ice discharge, total and probable amount (according to quantity and size of the ice blocks)

S_1 and S_0 - ice block through-put (drift) of the flow and jam (limiting) sections

γ, γ_1, ρ and ρ_1 - specific gravity and density of the water and ice blocks (and their accretion)

$\Delta\gamma = \gamma - \gamma_1$; $\Delta\rho = \rho - \rho_1$; $\rho_2 = \psi\rho_1$

ε - - porosity coefficient of the ice accretion

$\psi = 1 - \varepsilon$ - population coefficient of ice movement

Q_f, q_f, and k_f - total and specific discharge and coefficient of water filtration through the jam

$\ni$, T, $\ni_p$, and U_p - kinetic energy of the ice blocks, pressure energy of the ice accretion (including the water plus ice total), the kinetic flow energy and the potential flow energy;

A_1, A_2, A_3, A_4, A_1, A_s, A_d, A_p - critical values of the work of forces acting on the ice block, which, if surpassed, and relative to the indices, leads to a breakdown in the stability of the floating blocks, their plunging (tightening) piling up, pushing against each other, movement under the ice, the surmounting of coupling forces between the blocks, between the blocks and the stream bed, a breakdown of blocks;

P_1, P_2, P_3, P_4, P_5, P_6, P_0 - forces of hydrodynamic pressure (profile), water friction against the ice surface, the component of ice weight, wind pressure, filtration pressure, centrifugal, shore resistance, relative to a surface unit (see section 1.3). The additional index y denotes the transverse component of these forces.

σ, τ, σ_k, τ_k, σ_s (comp.), σ_i, and σ'_p (bend)	- normal and tangential stresses, their critical values, breakdown stresses in compression and bending, and the strength of the jam obstacle.
σ_2, σ_3, σ_4, σ_6	- critical values of normal stresses at which there arises a packing of the accretion, ice hummocking, jam pileup of ice, the breakdown of the jam
$F_1 = \mu_1 h_1^2$ and $F_2 = \mu_2 h_1^2$	- active and passive pressure (resistance) of the ice accretion (see section 1.3)
k_0, k_1, k_2, k_v and k_i	- resistance (pressure) coefficients of the water flow to the block movement; general, form, friction, wind pressure, and proportionality coefficient
t_0, t, t_y, and i_z	- slopes of the bottom, the water surface (longitudinal and crosswise) and in the ice accretion jam
ϕ, ϕ_1, ϕ_2, η, ζ and β	- coefficients of internal friction in the block accretion underwater, friction

against the shore and the bed, ice against ice, lateral pressure, non-prismatic form of the channel, and the "arch effect" (see sections 1.1, 2.1)

α_1 and α	- slope angles of the bottom slope of the jam and blocks in the jam (see Fig. 6)
C and n	- the Chezy coefficient and the roughness coefficient
t and t^{o} C	- time and temperature;
λ	- probability coefficient (reliability) of the given calculated value
θ	- angle of internal friction of ice accretion
C_0	- line corresponding to the stable state of a one-layer block accretion

Chapter 1

Physical Premises and Theoretical Models for Studying Ice Jams

1.1. Properties and Special Features of the Investigated Medium

Ice jams are accumulations of ice blocks which cause a crowding of the useful channel cross section and a rise in the water level associated with this; they form in the case when--for a specific quantity, strength, and pressure energy of the blocks--the throughput of the channel is insufficient for their transportion. The results of jam studies* show that their emergence, formation mechanism, distribution of ice in the jam, the value of level rise, and other characteristics are determined by many factors and their various combinations. These factors may be classified in three groups:

(1) Hydrometeorological, including the velocity and direction of the water current , the mechanical and geometric characteristics of the blocks that participate in the ice movement, the nature and sequence of the river opening and the development of high water, as well as the preceding ice regime of the river, weather conditions of the autumn-winter and spring periods, etc.;

(2) Geomorphological, which determine the nature and special features of the channel structure in longitudinal, crosswise, and planar relationships, the nature of the flood plain, trough, etc.;

(3) Factors of human activity, i.e., various water resources equipment crowding the channel, as well as measurements for control-

*Some of the published studies on this problem are given in the references. A more complete bibliography may be found in the Studies of coordinated meetings on hydrotechnology, the Studies of the GGI, and other special publications.

ling the discharge, channel, and intensity of ice movement, etc.

Some of the enumerated factors are interdependent, mutually dependent, complex (summed up) phenomena, the investigation of which is possible only after study of the regularity of action on them of separate "elementary" factors, which determine these phenomena.

/8/ Thus, ice jams are a complex, multifaceted, natural formation. In order to investigate them it is necessary to first establish the physical nature of the phenomenon and the form of material motion corresponding to it. This in turn permits determining classifications and physical laws necessary for the investigation and drawing up a diagram of the phemonenon with delineation of its main interconnections (external and internal). The basic properties of the object subject to the investigation and the method of its analysis are established on the basis of the above prerequisites.

It follows from the determination of an ice jam that this phenomenon is in its physical nature a process of retarding (building up) ice blocks entering a certain flow segment during ice movement or during the breaking open of the ice cover. Therefore, the flow section on which the process of ice block retardation arises and develops under certain conditions can be called a jam section.

Below, we will use the term ice blocks to infer monolithic polycrystalline solid bodies of finite dimensions, products of the breaking up of the ice cover, capable of travelling in the flow channel under the action of hydrodynamic and other forces, due to the absence of rigid bonds with the surrounding environment and due to their size. The breakdown of the ice cover into very large, less mobile units, "ice fields", whose longitudinal dimensions can exceed the width of the channel, usually precedes the emergence of ice blocks. Despite the usual temporary existence of these forms intermediate between the ice cover and the ice blocks, they may have a substantial influence on the opening process of the channel and are the immediate cause of ice jams.

Ice blocks, inheriting properties from their natural parent, the ice cover, acquire a number of special features, which, although they vary as a function of the concrete conditions of their formation and existence, possess, however, a specific general character, which was examined earlier in /19/.

In their physical properties, ice blocks are deformed solid bodies of nonuniform and anisotropic structure, of nonprismatic form and nonuniform size. In the movement of ice, these characteristics, including the relative density of distribution of the ice blocks in the channel are nonanalytical, discontinuous (or piecewise continuous) functions of the coordinates and time. The noted circumstance is justified both with respect to a fixed channel section as well as to the mobile system of ice-block accretion. This is due to the fact that factors which determine the emergence, characteristics, and behavior of ice blocks in the flow are themselves variables along coordinates and in time.

/9/ Various forms of ice-block movement can arise and develop next to each other in this process. In order to make an analysis, it is expedient to classify these as:

(1) The free movement of ice blocks, including periodic contact with solid bodies;

(2) The movement of ice blocks in the form of one-layer (plane or hummock) accretion;

(3) Movement of ice blocks with the formation (or break-down) of multilayer jam accretion of ice blocks.

In their movement, ice blocks are subjected to nonuniform external actions and exhibit different rheological properties of elasticitity, plasticity, brittleness, etc. Therefore, in a rapid "dynamic" application of external forces, the elastic properties of the ice step into first place, while in slow application, plastic deformations, fluidity of the ice, etc. are important.

In ice movement it is possible to determine two types of force transfer in the accretion of ice blocks examined as a discontinuous system, consisting of unbroken (continuous) deformable bodies of finite dimensions.

(1) Continual force transfer - inside the ice blocks;

(2) Discrete transfer - between the ice blocks with non-stationary (rheonomic) connections and force transfer of the contact type. In a single-layer accretion there are unidirectional connections, which create only compressive stresses (in the absence of freezing of the ice blocks), while in multilayer accretions, by remaining rheonomic, they possess in part a dual character due to the pressure of the ice blocks against each other, of regelation, and of filling the spaces between the ice blocks in the process of plastic deformation (fluidity) of the ice.

The compression resistanceof the accretion of ice blocks accumulates from those types of counteractions which a deformed system can be subject to: The internal resistance of the ice blocks to compression and breakdown (to cleavage, shearing), resistance to slipping against each other and resistance to expulsion (pressing out) of the ice blocks. From the viewpoint of mechanics, the formation of a jam is a process of particle displacements of a system from a state of less stable equilibrium to a more stable state, which can be accompanied by a change in the form and size of these particles (a breakdown of the ice blocks) as a consequence of their deformation.

By taking a general rheological stance toward the properties of the investigated medium, the evolution of a single-layer accretion of ice blocks during an increase in compressive force can be presented with a known schematization of the phenomenon as follows. When the compressive force has reached a certain critical value σ_2, sufficient to overcome the force of resistance of the ice blocks toward slipping along contact sections, a process of coming together occurs, a consolidation of the ice blocks to a more dense packing, which is accompanied by a scattering of the contacting side surfaces of the ice blocks, a shattering of fine ice chips, etc.

/10/ A further increase in the stress to σ_3 leads to the formation of slipping (displacement) areas within the ice blocks with shearing along their contacting edges (usually less strong than the inside section of the ice blocks /19/), at a certain angle to the direction of action σ_3. A portion of the pressure on these areas is transferred to a plane normal to σ_3, and if its value is high enough, a creeping and under-ramming* of the ice blocks against each other occurs (the phenomenon of ice hummocking). This process is made easier by the oblique shearing of edges in the direction of large tangential forces and by the bending of not very "rigid" ice blocks (see Section 1.3).

Fig. 1. Breakdown of the longitudinal stability (swelling) of the ice cover in a sandbank.

However, losses in the longitudinal stability with wave-form buckling, rupture of the blocks and the subsequent hummocking of their fragments can occur even earlier for large size ice blocks--with a smaller compressive force--as follows from the theory of the resistance of materials. Such a breakdown is especially characteristic for ice fields and the ice cover in the sandbanks of many rivers, where it arises under the pressure of the increasing head of the water after the formation of border zones and transverse cracks separating the ice on the sandbank from the above-situated reaches. Such a phenomenon was clearly observed in the model investigation (Fig. 1).

*or plunging--Translator

Jams can arise also directly in the collision of floating ice blocks with an obstacle (the ice field, the edge of the unbroken ice cover, etc.), omitting the intermediate stage mentioned above. This happens in the case when the kinetic energy of the ice block is sufficient for their packing or under-plunging. A similar phenomenon may take place directly in the breaking open of the ice cover under conditions of considerable compressive force.

With a very rapid growth of compressive pressure in the ice block accretions (or with a large store of kinetic energy of the blocks knocking against each other), the creeping process, which requires a known time, cannot successfully develop and after attaining the dynamic stability limit, the ice blocks will
/11/ break down. With a slow increase in the load, which does not exceed the rate of the dislocation shifts within the blocks, plastic deformation--fluidity with a bending and warping of the ice blocks will prevail. At a temperature close to zero, the examined phenomenon, as experimental investigations show /19/, will be accompanied by a thawing of the ice. Under conditions of ice movement in rivers, this process usually does not develop, but in the deformation and breakdown of ice packings in jams, it can play a significant role.

After a jam has emerged, two processes occur in it that are opposite in their action direction. On the one hand, under the effects of ice pressure and the water mass, a consolidation of the ice mass of the jam occurs due to packing, the phenomenon of regelation, and filling the spaces between the ice blocks during their plastic deformation. At temperatures below freezing, this process is supplemented by a freezing of the ice blocks which leads to an increase in the elastic connections in the ice mass. On the other hand, the piling-up strength is weakened due to the mechanical and heat action of the water filtering and flowing under the ice as well as by the insulation, the convective heat supply from the atmosphere, etc.

An increase in the longitudinal dimensions of the jam and in the head of the level in the continuing ice movement can lead to the fact that compressive stresses exceed the value of internal resistance of the ice-block accretion. In this case, the accretion thickness should increase (ice shifts in the jam), and with an insufficient quantity of ice, it breaks down by a shift of the ice blocks with respect to each other or of the entire mass of ice along the flow bed. It is obvious that the nature of breakdown depends on the ratio of the values for the coefficients of internal friction in the accretion of ice blocks and their coupling with banks, and also depends on the intensity of the growth of pressure and level head and on the nature of the obstacle to the jam itself.

The packing density of the ice-block accretion is analogous to the process of coming togther under pressure of skeletal earth particles (called compressive contraction in soil mechanics), but has a more complex character. This character is explained by deformation, nonuniform dimensions, and by the stress of the ice blocks, the phenomenon of regelation and recrystallization of the ice under pressure, as well as by the scattering and cleaving of the contacting edges. In a single-layer accretion of ice blocks, the compressive contractions correspond to plane deformation, and in multi-layer accretion, to volumetric deformation.

With an increase in compression σ to a limiting value equal to the internal reaction of the plane ice-block accretion, the porosity coefficient ε reaches a minimum value. A further increase in stress causes a loss of stability of the ice blocks, their hummmocking, and piling up with new compressive contraction, deformation, or breakdown of the blocks.

It follows from the examined properties of the investigated medium that the overall dependence $\varepsilon = f(\sigma)$, is a multifactor piecewise
/12/ continuous function and cannot be expressed in simple analytical form. The investigation of this phenomenon is insufficient to construct a functional dependence; therefore an evaluation of the limiting porosity values can now be based only on empirical data.

For an ideal free-flowing body, the coefficient of interaction with the walls is determined by a constant value for the product of thrust and friction coefficients. In the block accretion, this value is a variable dependent on the nature of the equilibrium state, the nature and speed of movement of the accretion, the channel structure, etc. Thus, according to the investigative data of the Central Scientific Research Institute of Ice Melting /57/, the interaction coefficient of the flowing material with the banks decreases according to a nonlinear law with an increase in the Froude number and with a widening of the channel, while it increases during a contraction.

A. Gan'on, R. Hausser, E. Pariset /15/ and D. F. Panfilov /58/ introduced a coupling coefficient also into the condition of interaction with the banks. However, according to the data of the Canadian investigators, it has a substantial value only in the case of a thin ice accretion. They note also that little study has been made of the internal friction coefficient of an accretion, and examine it as a variable. The results of our investigations also show that the breakdown in the stability of the blocks with relatively large longitudinal dimensions is not determined by this coefficient, but by other factors (see Section 2.4).

In addition, due to the relative mobility of the blocks within the accretion, the so-called "arch effect" can arise regularly, which is examined in the theory of free-flowing bodies /25/. It is an additional resistance to longitudinal compression and to the movement of the ice accretion. This resistance can be considered by introducing into the friction force of the shore some coefficient $\beta \geq 1$, which is also a variable depending in our investigation on the accretion structure, the relative size of the ice block, etc. (see section 2.1.).

1.2. Testing and Conditions of the Theoretical Investigation of the Investigated Object

In the study of ice jams, the object of the investigations is a regular formation consisting of interacting individualized and deformable solid bodies--ice blocks; the primary external interconnections of these formations are examined by way of a mechanical interaction with the surrounding medium. Even in the study of such processes as, for example, the external and internal thawing of the ice, the formation of "temperature cracks", the thermodynamical and physical-chemical effects in the models of calculation taken for the investigation are disregarded, since they are values of a lesser magnitude.

/13/ The construction of an overall theoretical model which takes into consideration all of the properties and effects of the object noted above, is difficult due to both mathematical complexity as well as the insufficient development of a theory of discontinuous deformable bodies and the extent of study of the object itself. In addition, an attempt at including in one diagram all of the basic properties of the investigated medium would lead to a complicated model little suited for investigation. This forces us to proceed by applying simplified diagrams designed to solve individual problems /70/.

There is already available in ice technology a new test for applying similar systems. The model of a linear elastic body with the introduction of certain additional conditions and assumptions is widely used for the "dynamic interaction" of ice blocks /39, 61/.

Models of a nonlinear elastic body, an elastic-plastic body /14/, etc. are examined for describing ice behavior beyond the elasticity limits. With a small rate of deformation, plastic deformations of ice are modelled by a viscous liquid diagram. B. V. Proskuryakov and V. P. Berdennikov /65/, for example, applied such a diagram to the investigation of temperature deformations in an ice mass.

Ice accretions form a compact mass with specific properties and regularities in their movement and evolution under the action of compressive forces. However, they can only partially be described by existing models of continuous or discrete media. Thus, a viscous liquid model is used to study the established movement and equilibrium state of an accretion of small-size particles (slush ice) /23/. With larger particles - "crushed ice", "crushed blocks" -- aspects of discontinuous body theory are applied. Here, models of an "ideal free-flowing body" /51/, a free-flowing body with interparticle coupling and variable coefficients of internal friction and interaction with the shore /7, 57/, and also taking into consideration the shore-coupling coefficient /15, 57/ were employed.

The force theory of K. Yansen (A. Gan'on, V. P. Berdennikov, etc.) and A. Kako (B. Mishel', I. Ulle) and the general aspects of the free-flowing medium theory (D. F. Panfilov, et al) were used for an evaluation of the pressure increase in an ice accretion and its equilibrium state.

The limiting value of stress which determines the jam strength is evaluated either by the limit of ice strength or by the limiting value of internal (passive) resistance to slipping which is obviously better established /15, 50, 58/. The attractive force* of water and air flow is determined by different methods:

(1) According to the usual method of streamline flow of solid bodies in a viscous liquid /39, 61/;

/14/ (2) According to the V. M. Makkaveyev diagram with the breakdown of the current along the vertical into two sections and by applying to it the Chezy formula /5, 61, 74/;

(3) According to the Strikler diagram /15/;

(4) According to a calculated structure proceeding from the A. Kako diagram where the tangential stresses from the flow and the component of ice gravity are considered simultaneously /50, 86/.

In the general dependence obtained by Berdennikov /6/ for determining the moving force of ice jam masses, the forces of flow friction on the ice and on the river bed and the component of ice weight and the water column underneath the ice are simultaneously considered.

In solving dynamic problems (nonuniform movement, change in the kinetic energy of the moving ice, etc.), ordinary Euler equations are used along with the law of kinetic energy change /36/, including taking into consideration deformation forces /19/ and the impulsive nature of ice block accretion stoppage at the obstacle /60/, etc.

A quantitative analysis of jam formation includes hydraulic calculations of the flow below the ice. The basic theoretical

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*This concept is given in more detail in Section 1.3.

foundations and principles of these calculations are established in the works of N. N. Pavlovskii, A. N. Rakhmanov, P. N. Belokon' and other researchers. In the development of these principles up to the present time, a number of working diagrams have been developed for determining these hydraulic characteristics of the sub-ice flow. The method for determining the Chezy coefficient was examined in a similar case by A. N. Marchuk /46/. Calculation diagrams for determining the coefficient of roughness have been set forth by A. A. Sabaneyev, D. F. Panfilov, V. I. Sinotin /58, 71, 78/, etc.

A less studied problem is the filtration of water through the ice accretion jam. There are still no sufficiently complete theoretical developments and calculations along this line. Closest to the given problem is the question of filtration through a stone bridge studied by S. V. Izbash /28/ et al. We took the calculation diagram calculated for this case as the basis for examination of filtration through a jam (see Section 4.1).

It must be noted that the use of a mathematical apparatus in the study of ice accretions is complicated by the fact that the internal processes occurring in them are expressed, as has been noted, by nonanalytical functions. Therefore, an analysis is usually carried out according to average characteristics and without taking into consideration the individualized properties of the particles of the system. The latter may be

considered sufficiently valid for accretions of small particles (slush, ice dams, crushed ice, etc.) for which the given models are predominantly utilized. However, for accretions of relatively large (spring) ice blocks, the consideration of some individual properties is necessary for solving single problems.
/15/ Some characteristics can be expressed here only by the law of distribution of random values. More developed models based on a continual or "ideally free-flowing" concept of the medium are insufficient for describing specific properties and features of the object. A more rigid investigation based on a discrete structure of accretion construction and based on principles of statistical mechanics is made difficult by the insufficient theoretical development.

Therefore, taking into consideration all of the properties of interest of the object within the framework of classical or statistical mechanics is practically unachievable. Here, it is expedient to apply both the continuous and discrete methods of investigations, as well as elementary methods of mechanics, inspecting and comparing the results with natural and experimental data in order to solve the various problems. In specific cases it may be necessary to apply a combined method using different rheological models, the introduction of supplementary conditions, probability characteristics, etc. A rheological diagram of jam formation is created from these considerations and a system of basic initial equations and additional relationships is established with the use of necessary "finishing" treatment of the existing models.

1.3. Rheological Diagram of the Formation of Ice Jams. A System of Initial Equations and Supplementary Relationships

A rheological diagram constructed on the basis of the regularities of ice accretion evolution examined in Section 1.1. is taken for the initial model. Here it is considered that deformation displacements in the process of ice-jam formation in the general case cause five basic forms of compressability:

(1) Elastic, viscous-plastic--within individual blocks with their destruction at $\sigma > \sigma_{i,s}$;

(2) Compressive - In ice-block accretion with packing consolidation due to a change in the blocks' mutual disposition from $\psi = \psi_0$ to $\psi = \psi_m$;

(3) "Hummocking" - In ice block accretion with consolidation by hummock formation and the loss of stability of the ice blocks;

(4) "Pilings" - In multi-layer (jam) accretion of ice blocks during formation and breakdown;

(5) Viscous-plastic - In ice-block accretion with a filling of the spaces between the blocks due to the ice fluidity.

Displacements of the compressability forms 1, 2, and 5 determine for the most part a volumetric deformation of the forms, while 3 and 4 - a form change, and forms 2-4 correspond to finite displacements of large-size particles (ice blocks).

/16/ Therefore, the application to this case of mechanics equations for a continuous medium has a formally conditioned character and is allowed mainly only to describe critical (limiting) stresses of states, with the introduction of additional parameters that take into consideration the special features of the processes examined. Taking into consideration the noted features of the behavior of a system, the degree of its study, and experience in applying simplified diagrams, a rheological model of ice movement is established for its development in stages (Fig. 2) with increasing compressive stress - from the floating of solitary ice blocks to the formation and destruction of the ice jam. Models applied in rheology and designations of internal relationships are used for this /54, 66/.

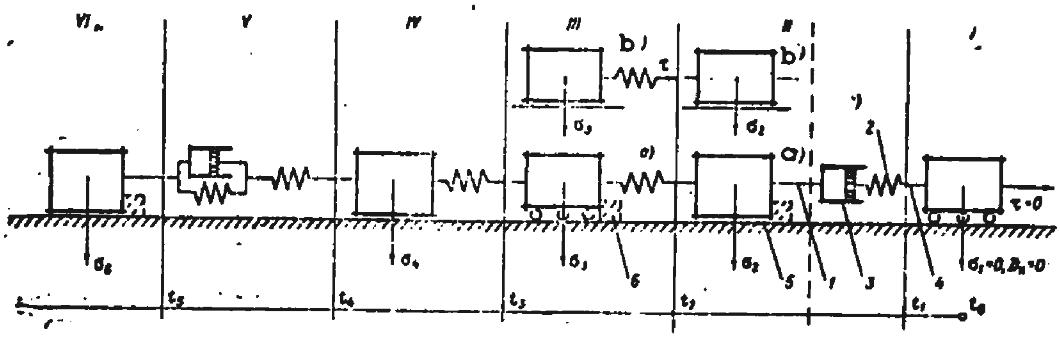

Fig. 2. Rheological model of internal connections of ice movement and ice jams.

Ice movement stages: I - floating of individual ice blocks; II - single-layer plane accretion of ice blocks with compressive contractions; IIIa and IIIb - stability losses (hummocking) of ice blocks; IV - formation of multi-layer ice-block accretion jams; V - consolidation of the jams; VI - breakdown of the jam due to stability loss; 1 - rigid connection; 2 - elastic; 3 - viscous; 4 - absence of resistance; 5 and 6 - "internal friction" resistance and brittle resistance of the ice-block accretion.

The first step is modelled with the assumption that ideal solid bodies participate in the process. In the second stage, in model IIa, ice blocks are assumed in the form of elastic-viscous relaxing bodies, as in study /59/; in Model IIb, they are assumed in the form of brittle-plastic bodies with connections corresponding to compressive contraction, while in Model IIc, where ice-block accretion/shore coupling forces are taken into consideration - in the form of brittle-plastic bodies possessing
/17/ internal friction. For the case of the loss of stability of the ice blocks, the model is made corresponding to the results of investigations (see Chapter 3) according to the diagrams: IIIa - assuming the participation of elastic-plastic brittle bodies in the process, in IIIb - elastic-plastic bodies with internal friction. The same models are used with respect to stages IV and VI of the formation and breakdown of the jam. In stage V, elastic-viscous-plastic bodies with increasing amounts of elastic couplings are taken for the ice blocks; this corresponds to the case of ice consolidation in the jam under conditions of regelation and ice-block freezing that prevail. A diagram similar to the latter, with the additional inclusion of the Newton cell was developed by T. Tabat /91/ for studying

the fluidity of ocean ice.

The corresponding rheological equations for the considered stages can be written as follows:

Step IIa - form charge:

$$D_n = k_1 \dot{D}_n = k_2 D + k_3 \dot{D}; \tag{1.1}$$

Stage IIb - volumetric deformation:

$$\dot{T}_0 = 1/\nu T_n = (1-G)\dot{T}_n; \tag{1.2}$$

Stages III, IV, and VIa

$$D_n = k_2 D; \tag{1.3}$$

Step IIIb

$$T_n = GT_0; \tag{1.4}$$

Stage V

$$D_n + k_1/k_3 \dot{D}_n = k_2 D + k_3 \dot{D}, \tag{1.5}$$

where D_n and D are stress and deformation deviators; T_0 and T_n are spherical tensors, while D, D_n, T_0, T_n are their rates; k_1, k_2, k_3, ν and G are rheological coefficients that characterize the relaxation of stresses, elastic and viscous form change, volumetric compressibility, and viscosity /4, 54, 66/.

The examined rheological diagram can be taken to solve various problems, usings its individual elements or their combinations. Thus, the free floating of ice is modelled by stage I, if the stress created by the water pressure is disregarded, whereas if the latter is considered, the diagram of combined stages I and II are used. In the investigation of the interaction of ice blocks with solid bodies, the diagram II-IIIb is taken, which in a dynamic load stage can be simplified by excluding the viscous

component of diagram II; the value σ_3 will correspond here to the limit of ice strength. In order to study the jamless passage of ice blocks and the breakdown of ice fields, diagram IIa-IIb-IIIa may be used; while taking into consideration shore
/18/ coupling, model IIa-IIb-IIIb is used or even a simpler, less precise model, without considering the viscous component in IIa or just one unit may be used directly: IIIb. The process of breaking open of the ice cover may be described by the model of units II-IIIa. Under these conditions, the critical value σ_3 will correspond to the loss of stability of the ice blocks, the limit of ice strength vs. shearing (in the passage of the ice blocks through the channel construction) and to the limit of bending strength. The model can include stages II-III-IV, etc. for the case of formation of the jam direcly in the breaking up of the ice cover.

Stages IIa, III, and IV, which are described by models that are used in the investigation of discontinuous (free-flowing) bodies /25/ are of basic significance to the present investigation. The theory of free-flowing bodies is developed more fully in the field of statistics. Problems of dynamics are solved primarily by the method of limiting equilibrium - by the fixation of the state of the system in times that correspond to the emergence of the stationary regime with specific values of the primary parameters.

General conditions for the equilibrium of a free-flowing body take the form:

$$\sum F_{x,y,z}+\sum \sigma_{x,y,z}+\sum \tau_{x,y,z}=0 \quad \sum M_{x,y,z}=0, \qquad (1.6)$$

and the stability condition:

$$D_{\Pi}=f(\sigma). \qquad (1.7)$$

For models IIa, IIIb, and IV, condition (1.7) can lead to the requirements:

(a) In contact sections between the ice blocks:

$$\tau_{\kappa}=\varphi\sigma; \qquad (1.8)$$

(b) On the surface adjacent to the water:

$$\tau_{\kappa}=\varphi_1\sigma \text{ or } \tau_{\kappa}\leqslant\varphi\sigma \text{ at } \varphi_1>\varphi; \qquad (1.9)$$

(c)

$$\tau_{\kappa}\leqslant k_0u_{\kappa}^2 \text{ or } \tau_{\kappa}\leqslant\varphi\sigma \text{ with } k_0u_{\kappa}^2>\varphi\sigma; \qquad (1.10)$$

(d) On a free surface:

$$\tau_{\kappa}\leqslant k_V u_V^2 \qquad (1.11)$$

and for models IIIa and VI - to the requirement:

$$\sigma\leqslant\sigma_{\kappa} \text{ or } \sigma\leqslant\sigma_{p}' \text{ with } \sigma_{\kappa}>\sigma_{p}', \qquad (1.12)$$

where $\sigma_{x,y,z}$ and $\sigma_{x,y,z}$ are stress components on the coordinates; σ and τ - stresses acting on the surface of the free-flowing body; and σ_k and τ_k are their critical values; $F_{x,y,z}$ and $M_{x,y,z}$ are components of volumetric forces and moments acting with respect to any pole; σ'_p is the strength limit of the obstacle which contributes the compressive forces in the ice

/19/ blocks. Here and below it will be assumed that the x axis is directed along the surface of the flow, the y axis is at right angles to the flow, and z is downward.

Taking into consideration the inter-particle coupling and the coupling between the ice and the banks (for diagram IIb) a free term is introduced into the strength equation (1.8)-(1.11) - the "coupling coefficient" which determines the value of the initial resistance to displacement. Eqs. (1.9-1.11) determine the limiting conditions, while the equality signs in Eqs. (1.7-1.12) correspond to the case of limiting equilibrium. We note here that the given conditions do not consider the isotropy of the ice-block structure and do not model their breakdown.

In order to obtain a closed equation system, which permits solving the problems set forth in the study, Eqs. (1.6-1.12) are added as further dependences that reflect the special character of the behavior of a system comprised of deformable particles of finite dimensions and moving in a water-air medium under conditions of occurrence of filtration, freezing, etc. phenomena.

1. The balanced equation that describes a change in the quantity of ice in time dt in a flow section bounded by surfaces F_i and with volume V is taken as follows:

$$dS = \int_V \frac{\partial \rho_0}{\partial t} dV = \sum_{i=1}^{4} \int_{F_i} \rho_0 v_n \, dF_i + \int_V \frac{\partial S^0}{\partial t} dv, \qquad (1.13)$$

while with finite differences for time Δt:

$$\Delta S = \sum_{i=1}^{4} S_i + \Delta S^0, \qquad (1.14)$$

where the totals in the right hand sectiors of equations (1.13) and (1.14) determine the entrance (exit) of the ice through the upper and lower boundaries of the section and through the channel of right and left tributaries; ΔS^0 is the change in the quantity of ice in the examined section due to freezing and thawing; $\vec{v}_n$ is the component of ice-block movement speed normal to the surface $F_1 - F_4$.

Equations (1.13) and (1.14) permit classifying ice movement (ice transit) according to the direction of its motion:

(1) Longitudinally - forward and backward (upstream);

(2) Crosswise - through the tributary channel;

(3) Longitudinally-transversely.

Different directional combinations yield 15 types of jams which theoretically exhaust all possible cases. The longitudinal-crosswise type movement is observed at the mouth of the tributary, and the countercurrent flow downstream of large rivers in a wind surge.

2. The dynamic equations which more fully correspond to the object's properties are equations of the quantity and moment of movement of deformable bodies /30/. Their application to the objectives examined is difficult and it is hardly expedient at the present stage; however in principle it is not promising
/20/ if we proceed from the increasing possibilities of computer application and development of the theory of ice movement itself.

3. The law regarding a change in the ice-block energy $\Delta \text{Э}$ can be used in the D. Bernoulli equation form or in another form which has been applied for this purpose by P. A. Kuznetsov /61/ and by Iu. A. Deev /20/:

$$\Delta \text{Э} = \text{Э} + \sum_{1}^{n} A, \qquad (1.15)$$

where Э is the elastic energy of the ice block; $\sum_{1}^{n} A$ is the sum of the work of all forces including those consumed in plastic deformation, local crumpling, thawing, etc.

The formation of a jam is accompanied by an increase in the specific energy (pressure head) of the flow from $Э_0$ to $Э_1$ in order to surmount the additional hydraulic resistances $_{h}$, which arise in the useful cross section in ice-block accretions. With a stationary water/ice movement regime (d/dt = 0) according to the law of conservation of energy:

$$Э_1 = Э_0 + Э_h + H_t + H_f \qquad (1.16)$$

where H_t is the change in the specific energy of turbulent scattering; H_f is the energy loss in the filtration of water through the ice block accretion.

This equation, which we examined earlier in /21/, permits obtaining additional characteristics and criteria in the study of jam formation. The particular case of Eq. (1.16) is examined by B. Mishel'/49/ in his criterion of hydrodynamic stability of the ice-cover edge.

4. The forces of interaction between the ice blocks and the surrounding medium - their composition and physical character - have been examined in detail earlier /20/. Here there is introduced interaction forces of ice blocks (and their accretion) with a water/air medium and with solid bodies contacting the ice blocks, forces caused by the earth's mass and its rotation, interactive forces between the blocks themselves, which arise when they collide, disperse, etc. The action of these forces is not uniform in its specific contribution to the jam formation process. Within the framework of the problems presented in this study, the following forces are taken into consideration.

(a) The force interaction of water or air currents (wind pressure) on a solid body can, as is known, lead to the primary vector $\vec{P} = \sqrt{\vec{P}_x^2 + \vec{P}_y^2 + \vec{P}_z^2}$ and to the primary moment. The vector $\vec{P}_x$ corresponds to the force of the so-called head pressure (resistance) of the current, which can be presented as a sum of the force $(\vec{P}_1)_x$ of the hydrodynamic pressure (profile and wave) on the edge of the ice block and the force $(\vec{P}_2)_x$ caused by the friction of the current against the ice surface.

In investigating the interaction between the current and ice blocks, the force of head pressure $\vec{P}_x$ is usually called the
/21/ "attractive force" of the current. We will use this term in the following. For a determination of forces $(\vec{P}_1)_x$ and $(\vec{P}_2)_x$ in the study, the general aerodynamic method is taken, in which these forces that refer to the action area unit are expressed by the formulas:

$$p_1 = k_1 v_0^2 \tag{1.17a}$$

and

$$p_2 = k_2 v_0^2. \tag{1.17b}$$

(For convenience we will omit writing the index "x" here and below.)

For a rectangular form of ice blocks and their accretion, as has been suggested by K. N. Korzhavin /39/, this can be written:

$$p_{1,2} = k_0 v_0^2, \tag{1.18a}$$

where

$$k_0 = k_1 + (0{,}9 k_2 h/l). \tag{1.18b}$$

If the transverse component of the current speed appears, for example in the bending of the current, the corresponding transverse force component $(p_{1,2})_y$, which is determined according to formulas analogous to (1.17) and (1.18) emerges. To solve the individual problems, it is convenient to express $(p_2)_y$ by the depth and radius of the current term:

$$p_{2y} = k' k_2 \frac{u_x^2 H^2}{R^2}. \qquad (1.19)$$

where $k' = \kappa^2 \phi(C, y)$; $\kappa \approx 1.3$; the function $\phi(C, y)$ can be determined according to A. V. Karaushev's table /33/.

The values of coefficients k_1, k_2, and k_0 given by various authors are different. In regulations for determination of ice loads (SN 76-66) these values are taken equal to $k_1 = 0.05\ t \cdot s^2/m^4$ and $k_2 = 0.005\ t \cdot s^2/m^4$. However, these data obviously relate only to the case of a horizontal disposition of the ice blocks. If they are distributed in the jam at an angle to the current surface, thus to the x axis, the calculations for the determination of the examined coefficients become much more complicated (see section 2.2).

The vector $\vec{P}_z$ in the unbalanced (or semi-submerged) state of the solid body can have positive and negative values, corresponding to the total for the three forces (Archimedes', "hydrodynamic head", "hydrodynamic inflow" - of the dip of the body into the current.

The hydrodynamic head force arises in the presence of the "attack angle", determined by the disposition of the body relative to the direction of currents flowing around it. The hydrodynamic inflow force has a complex character. In the movement of a solid body at a speed different from the speed of the current it appears and is caused by a crowding of the useful cross section, the flowing of currents under it with the formation of /22/ whirlpool zones, the subsequent loss in energy, etc. These phenomena reduce the pressure (evacuation) under the body, which expands some distance from its other boundary where the flow of the current has arisen. An evaluation of this force can be made only approximately in the solution of applied problems.

(b) The component of ice weight, which refers to a unit of surface, is expressed, depending on the problem to be solved, by the angle or average velocity of the current:

$$p_3 = \gamma_1 h l = \gamma_1 h \frac{u_0^2}{r^2 C^2 H}; \qquad (1.20)$$

$$p_{3y} = \gamma_1 h l_y = k'' \rho_1 h \frac{u_0^2}{r^2 H}, \qquad (1.21)$$

where $C = 1/n\ R^{1/6}$. Here, for accretions of spring ice blocks, which are characterized by a relatively great roughness and by a wide channel with $R = 0.5\ H$, where H is the current depth under the ice /46, 67, 73/. The value of the coefficient r of passage from the below-ice to the average speed of the current is a function of the nature of the ice and hydraulic characteristics. According to the data of some field observations /36, 39/ in the absence of slush under the ice, $r = 0.8$-0.9. The values of the coefficient $k'' = f(C)$ are given by A. V. Karaushev /33/.

(c) The force of shore resistance, which corresponds to the equilibrium level (1.9) can be expressed according to the relatively well confirmed diagram of K. Yansen /5, 15/. However, with respect to the peculiarities of the interaction between the ice-block accretion and the shore, examined in Section 1.1, we also introduced the coefficients ζ and β, which take into consideration the effect of the nonprismatic form of the shore /57/ and the emergence of the "arch effect", respectively. The force relating to the unit of side surfaces of the accretion is then determined by:

$$p_0 = 2\eta\zeta\beta\varphi_1\sigma', \tag{1.22}$$

where $\sigma' = (\sigma + \tau/\phi_1)$; τ is the coupling coefficient with the shore, which is taken into consideration if the accretion is not very thick /15/; η changes from $\mu/(1-\mu)$ to $\tan^2(\pi/4-\theta/2)$ - in the limited stress state; here μ is Poisson's ratio.

In addition to the forces examined above, the following are considered in solving individual problems: centrifugal force of inertia, which arises in a change in the direction of movement of the accretion, the force which is consumed in the deformation of the ice on impact, friction bet een the ice blocks, etc.

5. The limiting conditions of the jam strength (1.12) taking into consideration the deformability of the blocks should be supplemented by the following requirements;

(1) The nondestruction of the jam and ice blocks:

$$\sigma_s \leq \mu_s h_1 < \sigma_{i,s}; \tag{1.23}$$

/23/ (2) Nondestruction of the jam in the crushing of the ice blocks:

$$\sigma_v > \mu_s h_1 > \sigma_{i,s} \tag{1.24}$$

6. Active and passive pressure (internal resistance) of the ice accretion is determined according to the known /15/ dependences:

$$F_1 = \mu_1 h^2 l = \rho' (1 - \rho'/\rho) \tan^2 (\pi/4 - \theta/2) \frac{gh^2}{2} l; \qquad (1.25)$$

$$F_2 = \mu_2 h^2 l = \rho' (1 - \rho'/\rho) \tan^2 (\pi/4 + \theta/2) \frac{gh^2}{2} l. \qquad (1.26)$$

7. We divided ice blocks into three groups according to degree of rigidity in bending upon collision for convenience in this investigation. The idea for this classification is borrowed from the theory of beams on an elastic base /75/ with a certain change in the calculation method and substituting plate rigidity for cylindrical beam stability.

Short ice blocks in which the buckling due to bending is negligibly small in comparison to the sagging of the base. These are "rigid" ice blocks, the length of which complies to the condition:

$$(k = \text{short}) \quad l_k < 0.6 (EI/k)^{1/4}, \qquad (1.27)$$

where $I = bh^3/12$ - the smallest main moment of inertia; E - is the elasticity modulus of the ice, and $k = \gamma b$ - the coefficient of the foundation bed; at $\gamma = 0.001$ kg/cm^3:

$$l_k < 2.6 (Eh^3)^{1/4}. \qquad (1.27a)$$

For $E = 1 \cdot 10^5$ kg/cm^2 and h = 100 cm, l_k = 14.5 m, while with h = 20 cm, l_k < 4.0 m.

Long ice blocks comply to the condition $\sigma_k < \sigma_i$, where σ_k is the critical compressive stress, which causes a loss in the longitudinal stability of the block. By applying the formula for compression of a rectangular freely supported plate for determining this value, we find that the length of the ice block should comply to the condition:

$$l_d > 0.92(1+m^2)h(E/\sigma_i)^{0.5}, \qquad (1.28)$$

d = long

where m = 1/b.

With σ_i = 10 kg/cm^2, m = 0.5, and for h = 100 cm, l_d > 115 m, while at h = 20 cm, l_d > 23 m.

Ice blocks of average size should obviously satisfy the equation:

$$l_{k} < l_{av} < l_{d} \qquad (1.29)$$

As was noted, some characteristics and parameters of the investigated system and medium are nonanalytical and discontinuous functions distributed according to the law of random values. This requires introducing average values and applying probable characteristics and values of the investigated phenomena in the analysis. In this connection, a sectional averaging of hydraulic
/24/ elements is taken for morphometrically uniform segments of the current. Speeds, slopes, population of ice movement, etc. are averaged according to width and length of the segments. For a mathematical description of the form of the jam "body" the step character of piling is approximated by primary curves. A probability evaluation of the object's characteristics in view of its insufficient study is made by introducing the transfer coefficient from one confidence level to another.

1.4 Physical Modelling of Ice Jams

For the purpose of confirming and refining the mechanism established theoretically for jam formation and its individual regularities, several model investigations were made in 1968-1974. It was learned that under the conditions of multifaceted phenomena similar to what we are examining, it is possible only to approximately reproduce likenesses to simplified models designed for solving the individual problems, and also that physical modelling as applied to ice jams assumes the following indispensable conditions:

(1) Modelling by concrete models that allow the reproduction of specific aspects of the investigated objects should satisfy the initial theoretical diagram (see Section 1.2) and the results of field investigations;

(2) Only the primary active forces and deformations which determine the nature of the examined process in the given problem are considered in the modelling;

(3) The possibility of reproducing on the models formations of multi-layer accretions with a large number of ice blocks, while keeping in them the necessary individuality as particles of the system;

(4) The possibility of determining intermediate (interconnecting) steps of the character of ice jam formation under very dynamic conditions and with process instability.

The practical fulfillment of these requirements presented considerable difficulty since a modelling theory for ice jams has hardly been developed. A brief presentation of some modelling results for individual theoretical problems is published in /15, 64, 72, 82, 87/.

In the present study the available experience for model investigations in jams and the general aspects of the theory of physical modelling were utilized. The following procedures, scales, criteria, and structure of the modelling were taken in this connection and taking into account practical possibilities.

(1) Only the mechanical interaction was modelled without considering the anisotropy and the nonuniform structure of the ice blocks using an ice substitute. A rigid paraffin was used for the ice substitute which, as is noted by N. Roien, "has a
/25/ known similarity to ice, but possesses a greater viscosity" and a smaller strength. The physical-mechanical properties of the paraffin used are determined experimentally.

(2) For purposes of excluding the uncertainty of form and size of natural ice blocks, these blocks were simulated by square plates of one or several sizes, which are distributed according to the usual distribution law for random values with finite limits.

(3) For the basic model, ice blocks of an average length were taken which for rigid paraffin, in agreement with Eq. (1.29), complied with the condition $80h > l \geq 10h$. Some of the experiments were conducted with short and long blocks.

(4) The investigation of jam formation was conducted for condition (1.25) with nondestruction of the blocks. The

losses in block stability were studied both under the nondestructive condition (suberging and tightening of short and average size blocks) and under the breakdown state – long blocks model "ice fields" and the breaking open (loss of longitudinal stability) of the ice cover on sandbanks.

(5) The channel model was a hydraulic trough with a varying bottom slope (rigid channel of rectangular prismatic form) with a depth scale distortion of 5-15 times that usually observed in rivers. Various obstacles, which reduce the channel throughput were modelled in the trough (see section 3.2).

(6) The dynamic pattern of the current was modelled by two basic forces: gravity and water pressure, observing the equality and the compatability of the Froude and Euler numbers. The frictional force was considered in separate problems observing the compatability of the Reynolds' numbers.

(7) The stressed state of the blocks under a load was assumed planar described by the known equation of M. Levi /4/, while the connection between stresses and deformations was taken according to the rheological diagrams II-IV-VI (see Fig. 2).

(8) A dependence is determined from the Levi equation for converting the critical stresses σ_k, which determine the stepwise development of the jam formation (according to diagrams II-IV:

$$\sigma_{k,m} = \frac{k_m \sigma_{i,m}}{k_n \sigma_{i,n}} \left(\frac{l_m}{l_n} \right)^3 \sigma_{k,n} \qquad (1.30)$$

where l_m and l_n are the linear dimensions of the model and the field.

The critical stresses are expressed by the strength limits with respect to $\sigma_k = k_i \sigma_i$, where $k_i \leq 1$ is the proportionality coefficient.

It should be noted that an investigation of the qualitative mechanism of jam formation generally speaking did not require special conversions of the model to the field. This is touched upon and
/26/ confirmed by a number of formulas of a physical nature taken for the model and for the field assuming the constancy of the respective physical constants and parameters characterizing the material used, the hydrodynamic conditions, etc.

The model investigations of ice jams were conducted along the following basic lines.

(1) The study of the ice jam formation mechanism:

(a) On an unopened section of the river and on "blocked-up" ice fields, taking into consideration jams only of transit ice and rigid participation of local ice which forms in the breaking up of a portion of the ice cover;

(b) For $B_0 < b$ (see Section 3.2), i.e. with insufficient throughput of the current along the width of the channel (division of the river into branches, stone sections, bridge formations, etc.);

(c) For $H < h$, i.e. with an insufficient channel depth (shallow rifts, sandbars, etc.). More than 70 tests were conducted for various water flow rates, initial depths, channel slopes, etc.

(2) The study of the mechanism of loss of ice-block stability (see Section 3.2) when they collide with obstacles (dynamic form) and when the longitudinal compression increases in the ice-

block accretion to a critical value (static form). More than 60 tests were conducted under various hydraulic conditions, ice-block forms and dimensions.

(3) The study of the process of ice-cover buckling and breakdown with an increased water flow rate, keeping it in a stretch of water* (see Fig. 1); 15 tests were conducted.

(4) A study of the breakdown of ice jams of various types with water influx; 15 tests were conducted.

In the investigation of jams on a sandbank/reach channel section. the bottom stretch of the trough was raised on the sandbank and had a large slope. The models of the ice cover and ice fields were made of paraffin, while the stylized models of obstacles for $B_0 < b$ and $H < h$ were made of modelling clay or solid materials. In order to established the coupling moments of the examined processes, scale photographs were taken, which were then used in the treatment of the results.

The conducting of model investigations has a known limiting character with respect to possibilities, volume, and detail, which permits only clarifying the qualitative nature of the investigated phenomena and processes. However, their results permitted confirming the earlier formulated basic theoretical aspects /18-20/, and also defining somewhat more accurately and detailing the mechanism of the jam formation. In particular, we were successful in establishing a gradation of jams according to the extent of their development (see Section 3.1) clarifying the form of the jam body and the regularity of ice-block distribu-

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*the "reaches"--Trans.

tion (see Sections 3.2 and 3.3), specifying and generalizing the mechanism of ice-block stability loss; it was also possible to determine some quantitative dependences.

Chapter 2

Jam-Less Regime of Ice Movement and Losses of Ice-Block Stability

2.1. Channel Throughput in Jam-Less Ice Floating

The ice discharge through a channel section normal at each point to vector $\vec{v}_n$, expressed by the mass per second, i.e., the mass passing through surface F per unit of time, is expressed by the equation:

$$S=\int_F \psi h \rho_1 v_n \, dF. \qquad (2.1)$$

In the general case in the current section L, the value $S = f(x, y, z, t)$ has a large S_m with a maximum integrand for Eq. (2.1). In a stationary regime, $S_m = f(x)$ will correspond to the discharge passing through the "elementary section" line with the smallest limiting throughput S_0 in the given section L. The "elementary section" is the current section, the length of which is equal to the ice-block length, and bounded below by the investigated line. A change in the ice transit regime in this line is evidently immediately and automatically distributed upward over the entire length of the section.

By averaging $\vec{v}$ according to F and assuming h and ρ_1 constant, for the examined case we find that the floating capacity of the current section L with the boundary line S_0 is determined by:

$$S_1 - S_0 = h\rho_1 (\psi B_0 v)_m, \qquad (2.2)$$

where B_0 is the "effective width" - the current width with depth $H_0 \geq 0.9h$ for $b \leq B_0$ (in the case $b > B_0$ and $T \leq A_p$, the value $S_0 = 0$).

The index m in Eq. (2.2) indicates the maximum value of the product $\phi B_0 \vec{v}$ in the line S_0. This dependence corresponds to the jam-less movement of the ice blocks in the current section with the boundary line S_0.

/28/ The jam-less floating of ice blocks can occur in three basic ice-movement modes in the form of:

(1) A single-layer plane ice-block accretion;

(2) A single-layer hummock accretion without block breakdown;

(3) A single-layer hummock accretion with block breakdown (shearing - in contraction; rupture - on curves of decrease; etc.). Floating is also found along the free channel, including its bottlenecks, turns, etc., and in the presence of head resistance of natural or artificial origin (ice dams, bridges, embankment crests, etc.).

Ice passage through hydraulic structures is examined in a number of special investigations /16, 17, 34-39, 46-48/. The condition of jam-less advance of finely crushed ice accretion subject to the basic law of mechanics of a free-flowing medium, for the case of a wide rectangular and straight channel, was formulated by D. F. Panfilov /58/. The jam-less floating of a packed, single-layer ice-block accretion taking into account the blocks' individual properties, particularly the condition $h < l$ and the presence of transverse components of force acting on the accretion is examined below.

In this case, some laws of free-flowing medium mechanics are insufficient to describe the system behavior and Eq. (2.2) requires the consideration of some of its peculiarities, inherent to:

(1) The discontinuous (free-flowing) body: (a) for condition (1.6) and (1.9) - (1.11) or (1.12); (b) for $\tau_k \leq \tau_a$, where τ_a are tangential forces on the surface of contact with the shore, created by the "arch effect";

(2) The system of solid bodies: (a) for plane-parallel movement of the blocks ($v_z = 0$), which generally speaking is already guaranteed by Eqs. (1.9) - (1.12); (b) observing the requirement $B_0 > b$; (c) in agreement with (1.14) of the condition $\Delta S = 0$ for $S > 0$;

(3) The system of deformable bodies - ratio $A_{2,3,4,p} < \sum_1^n A_t$, where $\sum_1^n A_t$ is the total work of all forces acting on the ice blocks.

Therefore, for investigating jam-less (and in general the equilibrium) regime and the S_0 determination, we have a closed system of equations:

(1) The balance $\Delta S = 0$;

(2) The equilibrium equation (1.6);

(3) The rheological dependences (1.8)-(1.12) for diagrams IIIa and IIIb, which determine the boundary conditions, while for the S_0 calculation, Eqs. (1.8)-(1.12) will correspond to the limiting equilibrium condition;

(4) The requirements $B_0 \geq b$ and $\tau_k \leq \tau_a$.

The possibility of the emergence of the arch effect is considered by the latter condition - the transfer of forces in

/29/ the ice blocks at an angle to their movement with the force distribution in the form of an arch pushing against the shore, so that $\tau_a > 0$ can cause a retardation, while for $\tau_a > \tau_k$, there is a wedging of blocks. According to R. L. Zenkov's investigations /25/, the emergence of arches (domes) in free-flowing bodies is possible with a compressive stress that guarantees dome formation. This phenomenon is particularly characteristic of constricted segments and channel turns, but takes place also in rectangular sections of prismatic channels.

This phenomenon has not been sufficiently studied, and a possibility of considering it obviously lies within limits of the direct use of the experimental data. The braking action of unstable arches of small blocks (for $B_0 >> b$) in the estimation of v can be approximately considered by introducing the empirical correction factor $\beta = f(B_0/b) \geq 1$ into the formula for determining the shore interaction force. The appearance of stable arches can be estimated by a probability method using the results of field tests. V. V. Piotrovich /62/ examined the case of formation of stable arches of three blocks in a river turn.

The results of the investigation (see Section 2.4) show that the arch effect appears both in the ice-block movement, and in an immobile accretion found in the stressed state. This phenomenon is particularly characteristic for "narrow rivers", where the ice-block dimensions are relatively large. Thus, in modelling ice-jam formations for $b = (0.10 - 0.25)\ B_0$ in the rear section of jams, due to the formation of arches upstream, sections free of ice blocks emerge. These break down with an increase in pressure or a change in the level of discharge [of water]. It may be assumed that the ·paces free of ice observed in the Dniester River within the jam limits are associated with this phenomenon /31/.

The block accretion/shore interaction coefficient (η, ϕ_1) is a variable, as noted above, dependent on the velocity of ice movement, the nonprismatic form of the shore, etc. /57/. In the current level of study of this phenomenon, it can also be approximately considered by introducing into the shore interaction force p_0 the channel nonprismatic coefficient established empirically, $\zeta \gtrless 1.0$ (see Section 1.3).

The general expression of ice throughput through a river section was obtained earlier /20/ proceeding from the following reasons. The moving accretion of contacting ice blocks bounded by the locking lines can be examined with known approximation as a material system of mass M and all forces may be related to its center of mass. Taking Eq. (1.14) for this case and the theorem of a change in kinetic energy of a material system, we obtain:

$$S_0 = hB_0\left[\psi_1\left(\frac{2\sum_{1}^{n} A_i + C_1 v_0^2}{C_2}\right)\right]_n^{0.5}. \qquad (2.3)$$

/30/ where $\psi_1 = \psi an$; a is the portion of the river width occupied by the ice movement; n is the number of block layers; $\sum_i^n A_i$ is the work of all forces in the block movement plane relative to mass M and path L; $C_1 = C_2 = M$ applies to a rectangular river section, while $C_1 = M$ and $C_2 = I/R^2$ at a turn; I is the moment of inertia of the system; R is the turn radius; v_o is the velocity of motion of the system at the section origin.

The application of Eq. (2.3) for practical purposes presents known difficulties in the general case. However, it provides relatively complete qualitative information on the dynamic conditions and possible regimes of ice movement including conditions that determine the "jam risk" of a flow section /20/.

The dependence for the determination of ice movement velocity v in the S_0 calculation according to Eq. (2.2) for the first type of jam-less regime, more convenient for practical use, is obtained by applying the equilibrium equation (1.6) to the case which establishes the motion of a single-layer block accretion in a wide channel with a turn radius R for $B_0 > 0$ and $\tau_k > \tau_a > 0$. Here we assume that with sufficient distance from the frontal edge of the accretion and under the action of external stresses, the longitudinal stresses in the packed block accretion (rheological diagram IV in Fig. 2) reach a critical value in the boundary line and do not change further. The forces p_2, p_3, p_4, p_0, and the transverse forces resulting from these directed along the y axis: p_6, p_{2y}, p_{3y}, and $\pm p_{4y}$ act on the transverse strip taken here with a length equal to unity. The first two of these are directed to the concave shore, the third to the convex, and the fourth can have either direction.

The equilibrium condition can be written as:

$$p_2 + p_3 \pm p_4 - \varphi_1 (p_{2y} + p_{1y} - p_{3y} \pm p_{4y}) - p_0' = 0, \qquad (2.4)$$

where $p_0' = p_0\ h/B$, while in parentheses is given the difference of cross forces in absolute value.

Eq. (2.4) holds both for the free channel, and when a front obstacle is present as long as this resistance prevails with a stress not exceeding σ_k given above. By expanding the force value p_2 and expressing the values p_3, p_{2y}, and p_{3y} by the cross current velocity according to Eqs. (1.19) - (1.21) and considering the energy loss for impacting, friction, and deformation of the ice blocks, by means of the coefficient $\lambda_0 \simeq 0.90$-0.85, we find the desired ice movement velocity in the form:

$$v = u - \frac{\lambda_0}{k_3^{0.5}} (p' + \varphi_1 p_0 - Fu^2)^{0.5}, \qquad (2.5)$$

where: $F = \left[\frac{2\gamma_1 h}{C^2 H} + \varphi_1 \left(k''\rho \frac{h}{R} - k'k_2 \frac{h^2}{H^2}\right)\right]$; $p' = p_0' \pm p_4'$; $p_4' = (1 - \varphi_1)\, p_3$.

/31/ The value v is determined by trial and error. σ_k entering into p_0 may be taken according to Eq. (2.20), while for crushed ice, the product $\eta\sigma_k h$ will correspond to the active ice pressure on the shore calculated according to Eq. (1.25). In previous calculations, the β coefficient which considers the arch effect for relatively large ice blocks $B \leq (4$-$10)b$ may be taken equal to 1.2-1.3, while for small blocks - 1.0 - 1.10.

These coefficient values must be taken as orienting values subject to further precision by special investigations.

To determine the throughput $S_1 = S_0$ for a known v, it is necessary to find the smallest value (for the length of the section) of product $(B_0\vec{v})_m$ for each line for the greatest value of ice movement population ψ = 0.80 - 0.85 and to put the value obtained in Eq. (2.2). In the general case, the analytical expression $B_0\vec{v} = f(x)$ is not assigned and the probelm is solved by trial and error or by the method of control lines in characteristic river sections: constrictions, turns, a decrease in the flow velocity, etc.

Fig. 3. Jam formation at the ice-cover edge.

/32/ By putting v = 0 into Eq. (2.5), we obtain the value of the flow velocity for which there results the standstill of the block accretion:

$$u_n^2 = \frac{p_0 \pm p_i'}{k_0 + \frac{2\gamma_1 h}{C^2 H} + \varphi_1 \left(k_\rho \frac{h}{H} - k' h_n \frac{H^2}{H^3} \right)}. \quad (2.6)$$

and for a rectangular river section without considering wind pressure:

$$u_n^2 = \frac{p_0 h}{B\left(k_2 + \frac{2\gamma_1 h}{C^2 H}\right)} \qquad (2.7a)$$

or

$$\mathrm{Fr} = \frac{p_0 h}{BH\left(k_2 + \frac{2\gamma_1 h}{C^2 H}\right)}, \qquad (2.7b)$$

where Fr is the Froude number.

A series of curves Fr = f(h/H) for various values h may be constructed according to Eq. (2.7b) for B = const. Eqs. (2.6) - (2.7b) may be used for solving applied problems, for example, for the calculation of ice-retaining barriers, etc. Eqs. (2.5) -(2.7) were subjected to a certain confirmation on paraffin block models in a straight channel section and in a 90° turn for R = 3B (Fig. 3). The mean-square deviation of the calculated and experimental data was 17%. Here the reasoning resulting from an examination of the equations is confirmed by the fact that with a block movement velocity close to the current velocity, when $p_6 + p_{2y} > p_{3y}$, the accretion experiences an additional constriction (piling up) into a concave shore, while at low velocities, on the other hand, a convex shore.

2.2. General Conditions of Ice Block Stability Loss. Hummocking and Plunging of Ice Blocks in Collisions with Obstacles

Under a nonstationary regime, $S > S_0$ may appear both with an increase in ice discharge S, and with a decrease in the channel throughput S_0. Here, a portion of the block energy equal to $\Delta Э = Э - \sum_i \hat{A} > 0$ (or all energy at $S_0 = 0$), is converted to the energy of interaction between the ice blocks and both the obstacle and current. This causes at $\Delta\vartheta > A_1$ a breakdown of block stability, while for $\Delta\vartheta \leq A_1$, the joining of blocks floating one under the other.

In the latter case, as the length of the block accretion increases upstream, their pressure against each other σ will increase (see Section 1.4), so that at $\sigma > \sigma_k$, this also leads
/33/ to a loss of their stability. Thus, block stability losses may arise both in their collision with an obstacle ("dynamic diagram" of the phenomenon), as well as due to their compression in the accretion ("static diagram").

The stability breakdown phenomenon for ice blocks under field and laboratory conditions has been studied by many researchers: F. I. Bydin, Ye. I. Ioganson, L. G. Latyshenkov, G. F. Kennedy, E. Pariset, G. S. Shadrin, L. G. Shulyakovskii and others, including the authors of the present study. Analyzing and generalizing these data lead to the conclusion that a breakdown in ice-block stability upon collision with an obstacle can be manifested in the following forms:

(1) A hummocking of the ice, i.e., partial submerging of the blocks in water at an angle to the surface and locking them into this position by other blocks;

(2) A plunging of the front-edge blocks under the obstacle;

(3) A tightening of the blocks under the obstacle by the lowering of the rear edge;

(4) A packing (piling up) of the ice blocks on the obstacle and on each other;

(5) A sinking (flatwise) of the blocks in the water due to their loss of buoyancy.

The individual forms of block stability loss have already been subjected to a special analysis and theoretical study: The tightening of blocks - L. G. Latyshenkov /42/, V. K. Troinin /76/, G. F. Kennedy and M. S. Uzuner /77/; the plunging - V. I. Sinotin /72/; buoyancy losses - E. Pariset, R. Hausser /89/; the plunging and creeping of blocks - Iu. A. Deev /19/, and others. Establishing qualitative dependences of the various forms of stability loss is done on the basis of assumptions in the present study for the initial conditions and models given in Chapter 1. Analyzing the examined phenomenon permitted us to draw the following conclusions:

(1) The breakdown form of block stability in collision with an obstacle is basically a function of:

(a) The relative kinetic energy of the blocks Э, determined by the block approach speed and block dimensions;

(b) The block form;

(c) The nature of the obstacle;

(d) The state of the current.

(2) Three cases for the breakdown mechanism (form) of block stability are established:

(1) The kinetic energy of the blocks Э is directly converted to the energy of mechanical pressure on obstacle T; hum-

mocking, plunging, and piling of the blocks (Model IIIb) occur depending on the value of $\ni$ and the nature of the obstacle;

(2) The ratio between kinetic and potential energy of the current under the block changes when it stops at the obstacle, which causes a tightening of the blocks (Model IIIa);

/34/ (3) The total pressure of block accretion increases to $\sigma > \sigma_k$, upon which there occurs a conversion of the system (according to diagram IIIa) to a state of more stable equilibrium (hummocking, formation of a two-layer accretion, etc.).*

In the first case, the stability loss** is caused by the formation of torque primarily as a consequence of the central impact of the block with plane obstacles (the edge of the ice cover, a block, a barrier, etc.). The process may be described in the form of two steps;

(1) Initial - The advance of the front (impacting) edge of the block with its partial breakdown under the obstacle, (or on it) which causes the phenomenon of hummock formation;

(2) Plunging per se (creep) of the block under the obstacle (on it) with the surmounting of the friction force.

A general expression was obtained earlier for the approach velocity of the block v_1, which guarantees the first step - the hummocking of the ice, by solving a system of differential equations of motion and a change in the kinetic energy of the block /20/. Analogous results are given by a simpler method. The cal-

*The mechanism of vertical block sinking is not examined here; we refer those interested to the studies by E. Pariset /89/ and V. K. Troinin /76/.

**See Sections 2.3 and 2.4 for the second and third cases.

culation for the phenomenon, the reasoning behind it and assumptions for this method may be presented as follows.

1. When the block collides with the edge (or with another block) and there is a sufficient approach velocity, the edge of the impacting ice moves out to the border (or under it) as a function of the direction of torque, which arises due to the nonuniform thickness of the blocks /19/, the nonuniform strength and form of the edge of the blocks along the vertical, the tangential stress due to water friction against the lower surface of the ice, the drop in pressure under the rear portion of the block; the curve or decline of the movement in water under the ice-cover edge.

2. In the block's advance, its center of mass effects a forward motion, while the impacting edge effects a plane-parallel rotation in the xoz plane. The kinetic energy of the block is consumed for this work, i.e.

$$n'\text{Э} = A_p + A_v \qquad (2.8a)$$

where n' is the coefficient that considers the additional pressure of ice masses approaching from above; A_p - the work expended in the forward motion of the center of block mass; A_v - the same for rotation.

3. It is assumed that a projection of the block hits at angle α_2 against the plane face of the cover without slipping, but with the partial breakdown of the contacting sections. This process, on the one hand, is similar to the phenomenon of friction with dispersal, and on the other hand, it is analogous to

/35/ tion with dispersal, and on the other hand, it is analogous to a cleaving (shearing) of the ice, which takes place in the region of maximum tangential stress. For conditions of linear compression by longitudinal force, P can be written:

$$\tau_m = 0.5 n_2 P/\omega_1, \qquad (2.8b)$$

where n_2 is a coefficient which accounts for a weakening of the block edge; ω_1 is the area of the sheared section, normal to the direction of action of horizontal force.

Under the assumed conditions, the shearing depth is equal to h with a drop (rise) of the block edge to 0.5 h, while the block lowering (rise) angle will be presented respectively by $\alpha = 2h/l$.

4. We will express the work A_p and A_v by the average force value $p = p_s$, which corresponds to the average value ω_1:

$$A_p + A_v = p_s h + p_s r_s 2h/l, \qquad (2.8c)$$

where r_s is the average value of the force arm p_s, which can be assumed equal to h at h << 1.

The dependence ω_1 on path x takes the form:

$$\omega_1 = 2(h - x)\, x \tan \alpha_2. \qquad (2.8d)$$

By integrating Eq. (2.8d) in limits of x = 0 to x = h and dividing the result by h, we find ω_s. Substituting this value in Eq. (2.8c) we find that:

$$p_s = 0.166\, n_2 \sigma_s h^2 \tan \alpha_2.$$

By evaluating ϑ in Eq. (2.8a) and solving it jointly with Eqs. (2.8c) and (2.8d) accounting for the probable nature of the values n_2 and tan α_2, we obtain:

$$v_1 = h\left(\frac{n_2 \sigma_s}{\rho_1 l h}\right)^{0.5}. \qquad (2.9)$$

where $a_3 = n^2 \quad \lambda_1^2$; $n = 0.6n_1$; λ_1 is a coefficient which considers the probable nature of the value n and tan α_2.

In guaranteeing p < 10%, the value λ_1 = 0.5-0.6; for P = 50%, the value λ_1 = 1 (n_2 = 0.8, tan α_2 = 1), and with P > 90%, the value λ_1 = 1.5-1.6. The coefficient $n_1 = (1/n')^2$ with sparse ice movement is taken equal to 1.0, for average movement - 0.8, and for dense movement - 0.6-0.7.

The approach velocities of the block for which there occurs its plunging (v_2) or creep (v_3) are obtained from considering the values of work loss on these processes:

$$v_2 = (v_1^2 + 0{,}6h)^{0{,}5}; \qquad (2.10)$$

$$v_3 = (v_1^2 + 10h)^{0{,}5}. \qquad (2.11)$$

/36/ A confirmation of these equations under field conditions in the Prut River (5 surveys) and on models (30 tests) gave satisfactory results. The mean-square deviation of experimental values from those calculated was 12-15%.

Assuming a connection between the pressure force p and the values h, σ_a, which correspond to the strength of an elastic plate, in the form $p = 3.5 \ \sigma_a \ (h/l)^2$, the value v_1 (for ρ = 0.094 $t \cdot s^2/m^4$ and $lb = l^2$) is attained in the form:

$$v_1 = 1{,}2\lambda_1 n_1 \sqrt{h}. \qquad (2.12)$$

Eq. (2.12) is similar to the dependence of Pariset /89/ obtained by another method:

$$v_1 = 1{,}25k\sqrt{h}, \qquad (2.12a)$$

where $k = 0.7\text{-}1.2$ is the form coefficient. However, it is obvious that Eq. (2.12) has a more general character.

I. Ya. Liser /43/ obtained for Siberian rivers $v_1 = 1.2\text{-}1.5$ m/s according to Eq. (2.12a) for $k = 1.2$ and $h = 0.6 - 1.0$ m, but it is noted that this value should be smaller taking into account the pressure of other blocks. Eq. (2.12) under the same data and for $\lambda = 1.0$, $n_1 = 0.8$ gives $v_1 = 0.7\text{-}0.8$ m/s, and $v_2 = 1.0\text{-}1.1$ m/s.

2.3. Tightening of the Blocks in the Collision with an Obstacle

The second of the stability loss forms enumerated in Section 2.2 is the tightening of the blocks in the impact with a plane obstacle, such as the border of the edge cover, a block which has stopped earlier, etc. A generalizing of the results of an experimental-theoretical study of the tightening mechanism for the blocks in the case of their sudden stoppage at an obstacle (considering the results of investigations of the authors given in Section 2.2) shows that it is caused by:

(1) a decrease in the kinetic Э_p and an increase in the potential U_p energy of the current on the rear face of the block as a consequence of the braking of the jets; and (2) an increase in Э_p and a decrease in U_p under the block due to the crowding of its useful section.

The consequence of the first phenomenon is the rise (increase) in the wave pressure force. This force is introduced into the general equation of hydrodynamic pressure which, in the given case, acting on the rear edge of the block, will have a component directed downward p_{0z}. The consequence of the second phenomenon is the appearance of the hydrodynamic inflow force P_{pz}, also directed downward. This force is determined by a decrease in pressure under the block in its back section for a certain length l_1.

In addition to this, a wave pressure force causes a braking of the jets along the length of the current (next to the block) with a change in the form of free water surface in the wave form
/37/ with a corresponding decrease in $\ni_p$ and U_p. The force which arises as a consequence of a formation of whirlpools along the side surfaces of the block is related to the same category. And finally, in the case of an increase in the block thickness toward the front surface, which creates an "attack angle" (see Section 1.3), there arises a force P_{hz} of the hydrodynamic head directed upward.

If a sudden stopping (collision) of the block does not cause a complete or partial destruction of the obstacle or the edge of the block (which causes its plunging or creeping), and also a slipping of the block in the vertical plane (loss of bouyancy) then the front edge of the blocks under the pressure of the current links up with the obstacle by propping against it. In this case, the forces named above as well as the force of gravity P_{gv} and Archimedes' force P_{az} form, relative to the supports, the total for force moments acting in the vertical plane. If the total of force moments directed upward is more than the main resistance force moment, than the ice block begins to sink by its rear edge under water with increasing speed as a consequence of the increase in forces P_{p2} and P_{vz}. Here, the block is tightened under the obstacle or for $H < 1$ it is wedged between the stream bottom and the obstacle. Moments of tightening and wedging of block models in a hydraulic trough are given in the photographs (Fig. 4). In the presence of a block accretion, the tightening process can encompass two or more blocks at one time due to the emergence of a total torque for them. It should be noted that the total of force moments directed downward increases with a decrease in the current depth as a consequence of an increase in the hydrodynamic inflow force (see Eq. 2.13b).

The establishment of a critical current velocity, an increase of which gives rise to the tightening of blocks of given dimensions, is carried out for the example of a sudden stopping of a block with rectangular shape of constant thickness under conditions of a plane problem, taking into account the probable form of contact. For this case, the equilibrium equation of force moment relative to the y axis lying in the intersection of the front vertical face and the lower block surface, after excluding second-order values of smallness takes the form:

$$0{,}5lP_{gz} + 0{,}5l \tan\alpha_z P_{vz} + (l + 0{,}5l_i) P_{pz} - 0{,}5lP_{az} - 0{,}5hP_{ez} = 0, \qquad (2.13)$$

where l_i is the length of the distribution of decreased pressure under the block (in the detachment zone of the jet).

The determination of forces which enter into Eq. (2.13), excluding force P_{pz}, do not cause difficulties. The latter may be evaluated only approximately. V. K. Troinin /76/ introduces in Eq. (1.17a) for this coefficient, which considers the

/38/ vacuum gauge pressure, the coefficient k_1 which is similar in structure.

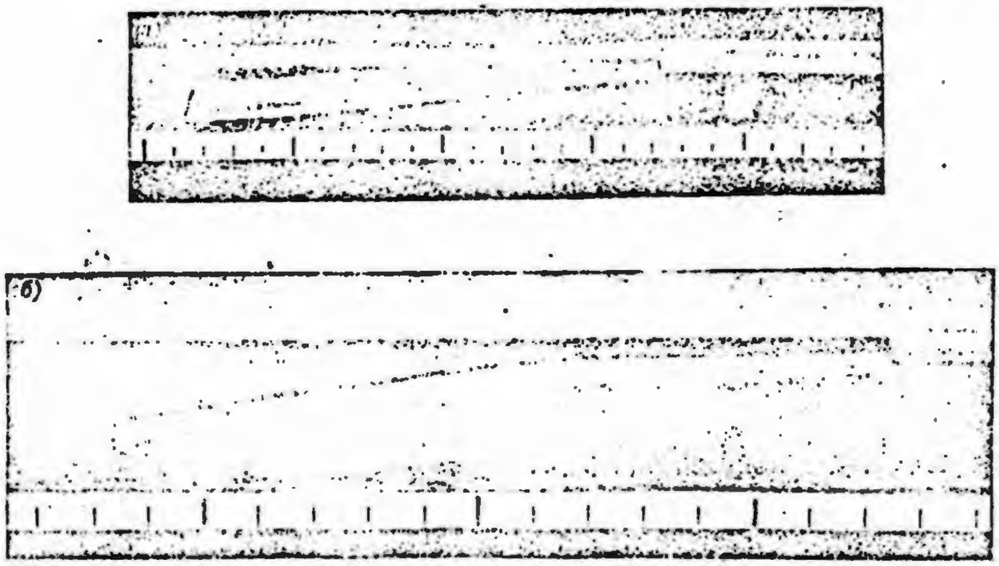

Fig. 4. Tightening of blocks upon collision with an obstacle.

a - Initial step of block tightening under the edge of the ice cover - submersion of its rear edge in water; b - Wedging of the tightened block for $l >$ h propped between the ice cover edge and the bottom of the stream.

/39/ The expression of the unknown force is presented more generally by the pressure drop under the block Δh, which considers both the current contraction with a disruption in the jet as well as local head losses, i.e.:

$$P_{p,s} = \gamma \Delta h l_1 b. \qquad (2.13a)$$

The value Δh is determined from Bernoulli's equation which consists of two sections: in front of the rear face of the block and under it:

$$\Delta h = \frac{u_0^2}{2g}\left[(\alpha_2 + \zeta_m) H_s^2 - \zeta_m H_s - \alpha\right] \qquad (2.13b)$$

where $H_s = H/(H - h)$; α is the kinetic energy correction in the Bernoulli equation; ζ_m is the coefficient of local losses in the inflow of jets under the block.

The joint solution of the given equations with respect to flow velocity for $\alpha_2 = 45^{\circ}$, $\alpha = 1.1$ and $\zeta_m = 0.5$ with consideration of the various block forms leads to the equation:

$$u_p^2 = \lambda_2 \frac{0{,}5\, \Delta\rho g h l}{A l_1 (1 - l_1/2l) - k_2 h} = \frac{a_p l}{A l_2 - k_2 h}, \qquad (2.14)$$

$$A = 0{,}55\rho r \left(1{,}5 H_c^2 - 0{,}5 H_c - 1\right);$$

where

$$a_p = 0{,}5\, \Delta\rho g \lambda_2; \qquad l_2 = l_1\left(l_1 - \frac{l_1}{2l}\right); \qquad (2.14a)$$

l_2 is the coefficient of the probable contact form which is taken with 50% probability equal to 0.9; for P = 25% the value λ_2 = 0.85-0.80 and for P = 75%, λ_2 = 1.1-1.2.

M. S. Uzuner and G. F. Kennedy /77/ in concluding a similar dependence also used the Bernoulli equation, but in another stricter form. However, the value C_m, which they introduced into the formula and which depends on ρ/ρ_1, h/l, h/H and has a considerable effect on u, was established by them only in an approximate experimental manner.

Eq. (2.14) was confirmed under laboratory conditions and partly in the field. For the natural river ice blocks, there was obtained a good agreement between the field tests and the calculated data with probability limits of 50-75%. Comparative calculations according to Kennedy's and Pariset's equation show that Eq. (2.14) provides a close similarity of results, but the Trainin formula gives higher velocities. We note that due to the introduction of the probability coefficient λ_2, Eq. (2.14) is more flexible for practical application.

The theoretical analysis and the experiment show that the form of stability loss of ice blocks upon collision with an obstacle is a function of the kinetic energy and the velocity of block approach, as well as the relative depths h/H and dimensions
/40/ h/l. With large Э and v values and a small h/H value, there is observed (or there prevails) a plunging and creep of the blocks. The tightening process here either does not arise or it develops with a retardation with respect to plunging. With small energy values, insufficient for a partial destruction (cleaving) of the edge of the blocks, and also with small relative depths, the tightening of the blocks prevails.

The phenomenon of flat dropping under the water is characteristic of ice blocks with small h/l values (according to Kennedy's data, for ice blocks with h/l $<$ 0.1). The process of plunging or tightening (see Fig. 4) can arise, as was shown by the experiment, with a relatively high approach velocity even in these cases. In general, the principle of minimum energy consumption can be followed in the calculations and the loss of stability can be taken in the form which requires the least energy consumption (smallest u).

There is an interest in establishing the critical current velocity, the exceeding of which causes slipping μ_s or rotational-forward motion $\mu_{v,p}$ of the blocks under the ice accretion. In order to derive the respective formulas, the same method is used as for obtaining Eq. (2.14). However, a more general case is examined here, when the lower surface of the accretion and consequently the ice blocks pressing downward against it are distributed at some angle Θ to the current direction. This case is characteristic in the formation of "developed" jams (see Section 3.5). Here, some values, which enter into the equations for determining the force of hydrodynamic pressure $P_{1,2}$ and of hydrodynamic inflow P_{pz}, are variables. A term is introduced into the formula for determining the hydrodynamic inflow force, which takes into consideration the additional compression of the current under the block as a consequence of its inclination. Taking this factor into consideration, the equations for determining the indicated forces take the following form:

$$P_{1,2} = k_0' u^2 l b = \left(k_1 + k_2 \frac{a_5}{a_4}\right) u^2 a_4 b = (k_1 a_4 + k_2 a_5) u^2 b = k_0 u^2 b; \qquad (2.15)$$

$$P_{pz} = (A_1 l_v \cos\Theta + A_2 a_5) b, \qquad (2.16)$$

where $a_4 = l \cos\Theta + h \sin\Theta$ and $a_5 = h \cos\Theta + l \sin\Theta$; A_1 and A_2 are determined according to Eq. (2.14a) with the substitution there for the value H_s:

$$H_{s1} = \frac{H}{H - h\cos\Theta} \text{ and } H_{s2} = \frac{H}{H - l\sin\Theta}. \qquad (2.16a)$$

Then we obtain the desired dependence for obtaining μ_s from the conditions of equilibrium of the active forces and for $\mu_{v,p}$ - from the conditions of equilibrium of the force moments with respect to the axis lying at the intersection between the front edge surface and the lower ice accretion surface:

$$u^2_{S} = \frac{\Delta\rho g h l a_1 \lambda_2}{k_0 a_2 + (A_1 l_1 \cos\Theta + A_2 a_3)\, a_2};$$

$$u^2_{\text{в. п}} = \frac{0{,}5\, \Delta\rho g h l a_4 \lambda_3}{k_4\left(\frac{h}{l} + 0{,}5 \operatorname{tg}\Theta\right) + A_1 l_1 \left(1 + \frac{l_1}{2l}\right) a_4 \cos\Theta + A_2\left(0{,}33 + \frac{h}{l}\sin\Theta\right) a_4}, \qquad (2.17)$$

(2.18)

where

$$a_1 = \sin\Theta + \varphi_2 \cos\Theta;\ a_2 = \cos\Theta - \varphi_2 \sin\Theta;\ a_3 = \sin\Theta + \varphi_2 \cos\Theta;\ a_4 = 1 + \frac{h}{l}\sin\Theta.$$

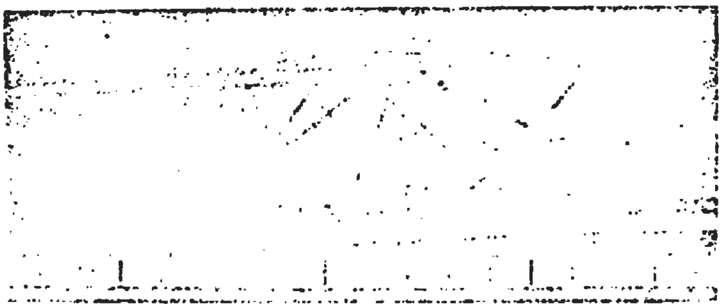

Fig. 5. Formation of a jam core.

Eqs. (2.17) and (2.18) may be simplified to the form:

$$u^2_{S} \cong \lambda_2 \frac{\Delta\rho g h l}{\frac{k_1}{\varphi_2 l} + k_2 h\left(\frac{1}{\varphi_2} + 1\right) + Al}; \qquad (2.17a)$$

$$u^2_{v,p} \cong \lambda_3 \frac{\Delta\rho g h l}{2Al_1\left(1 - \frac{l_1}{2l}\right) + 2h(k_1 + k_2)}. \qquad (2.18a)$$

For an examination of Eqs. (2.17) and (2.18), it can be seen that the form of block motion under the ice, i.e., the slip-

ping or rotating of the blocks depends primarily on the values ϕ_1, θ and on the degree of compression of the current by the block, determined by the h/H ratio and to a lesser extent on the h/l value. For $\theta = 0^{\circ}$ and $h/H < 0.1 - 0.05$, Eq. (2.18) has smaller values for the critical velocity of the current than does Eq. (2.17) for any ratio h/l only for $\phi_1 > 0.8 - 0.9$. For h/H = 0.1, the limiting value ϕ_1 is reduced to 0.7-0.8; for h/H = 0.3 to 0.6, etc. It follows from this that for $\theta = 0^{\circ}$ and small values of ϕ_1, for example, under the continuous ice cover,
/42/ the slipping of blocks will prevail, while with large values, their rotation will occur under the jam accretion of ice. The parameters h/H and h/l will effect the degree of development of this or that form. In the general case for jam formation, the values ϕ, h/H, h/l and θ are functions of spatial coordinate and time. Thus, for different sections of width and length of the current and in different phases of jam formation, this or that form of ice-block movement under the ice will arise. This regularity was confirmed qualitatively by the results of model investigations. The correlation of data from 30 experiments with calculations according to Eqs. (2.17) and (2.18) gave an average convergence on the order of 15%. The phenomena of turning and slipping of the ice blocks under the ice accretion is shown in the photographs for the model investigation and can be discerned in Fig. 5.

2.4. The Stressed State and Losses of Stability of a Single-Layer Ice-Block Accretion

The analysis of the stressed state of an immobile current-packed plane-layer accretion of ice blocks is conducted according to the diagram of K. Yansen, which has already been applied for this purpose (see Section 1.2). As in the derivation of Eq. (2.5), transverse components of attractive current force and gravity are introduced into the number of active forces, while coefficients which consider the arch effect and the non-prismatic form of the channel are included in the "side thrust" in agreement with Eq. (1.22). The equilibrium equation is set up for the elementary transverse strip of the accretion with length dx. The integration of this equation is made from x = 0 to x = L.

We will assume, as proposed by A. Gan'on et al. /15/ that for x = 0, the normal stress is equal to the hydrodynamic pressure on the rear edge of the accretion. Solving the obtained equation with respect to the longitudinal stress σ , we find:

$$\sigma = \frac{B_0 p}{p_0''} - \left(\frac{B_0 p}{p_0''} - p_1\right) \exp - \left(p_0'' \frac{L}{kB_0}\right), \tag{2.19}$$

where

$$p = p_2 + p_3 \pm p_4 - \varphi\,(p_3 - p_2)_v;\quad p_4' = (1 - \varphi_1)\,p_4;\quad p_0'' = 2\eta\beta_2''\varphi_1 h.$$

The stress in Eq. (2.19) reaches a limiting value:

$$\sigma_m = \frac{B_0 p}{p_0''} \tag{2.19a}$$

for L = aB_0, where a = 4-10 is a coefficient taken as a function of the formula used for p_0 /6, 15, 42, 88/.

It is seen from Eq. (2.19) that for $p_1 = B_0 p/o_0$ the second term is converted to 0 and the pressure σ in the given line remains constant with an increase in the length of the ice accretion upstream. For $p_1 > B_0 p/p_0$, the maximum value for pressure

/43/ will be for $L = 0$; for $L > 0$ the pressure decreases, which corresponds to the concept of the "narrow river" proposed by A. Gan'on, R. Hausser, and E. Pariset /15/. As a consequence of a more complete accounting for the forces acting on the ice-block accretion, Eq. (2.19) permits establishing not only the limiting width of a narrow river, but also the corresponding ratios between the hydraulic characteristics and the ice-block sizes. This feature is of interest in establishing conditions for possible formation of jams and predicting them, which is examined in Section 4.2.

A breakdown in the stability of a one-layer ice-block accretion, in agreement with the rheological model IIIa (see Fig. 2) and Eq. (1.12) is determined by the ratio $\sigma_m > \sigma_k$, at which there arises a shift in the particles along the slip sections. For a free-flowing body, the value σ_k corresponds to its internal (passive) resistance determined according to Eq. (1.26). As the result of published investigations show, this method is also acceptable for studying an accretion of fine crushed ice /5, 6, 15, 51, 58/.

At the same time, the investigations that we conducted of the behavior of the ice-block accretion for $l \gg h$ showed that losses in the stability of a single-layer plane accretion arise beginning approximately at $l > (3-4)\,h$ with longitudinal compression which exceeds the value σ_k, obtained according to the theory of free-flowing bodies, but is considerably less than the strength limits of the ice. The indicated phenomenon is associated with the fact that the mechanical system comprised of a packed plane ice-block accretion is more stable than the accretion of lumped particles.

The breakdown in the stability of the examined system is caused by the presence of a number of regularly occurring phenomena (factors):

(1) Off-center contraction with a nonuniform value and form of the ice blocks;

(2) The phenomenon of inertial forces during ice movement, which is caused by the crushing of ice chips, and the overthrust of the ice blocks on the shore;

(3) The formation of ice-block inclinations in the vertical plane as a consequence of these movements, etc.;

(4) The presence of oblique cuts and projections on the contacting edges of the ice blocks and a decrease in their strength /19/, which causes the appearance of vertical components of compressive force, the chipping off of the edges of the ice blocks and their slipping against each other at an angle to the water surface.

The finding of a general form for the dependence of system stability breakdown on these factors is made difficult by their complex probable manifestation and the insufficient development of the respective fields of mechanics. Therefore, the results of an experimental verification of the various mechanisms of the phenomenon are taken as a base and the following operating diagram is proposed.

/44/ For $B_0 \gg b$ and $l \gg h$, the instability of one-sided connections of the system under the action of the factors enumerated above regularly leads to the emergence of the inclination of individual ice blocks in the vertical plane with their bracing against each other and to the phenomenon of the respective moments of active forces. For $\sigma = \sigma_k$, this slope reaches a critical value, the exceeding of which causes a breakdown in ice-block stability in the system due to the progressive increase in the torque of the forces. The corresponding dependence for σ_k takes the form:

$$\sigma_к = a \Delta \gamma l_1, \tag{2.20}$$

where $a = \lambda_4 \phi_1$; $l_1 = l - \frac{p}{\varphi_1}(\eta l + h)$; λ_4 is the probability coefficient for the occurrence of the phenomenon: for P = 50%, the value of $\lambda_4 = 1.1$; for $P \leq 10\%$ the value of $\lambda_4 = 0.3 - 0.5$, and for $P \geq 90\%$, the value of $\lambda_4 = 2$-3. The values presented for the probability coefficient for $P \leq 10\%$ and $P \geq 90\%$ have an orienting nature. A confirmation of Eq. (2.20) for $\lambda_4 = 1.0$ on ice-block models with $h/l = 0.1$ gave a comparatively satisfactory correspondence for 25 experimental and calculated values of σ_k (mean-square deviation 11%).

It should be noted that in establishing the dependence for determining the critical value of compressive stress, 5 working diagrams were examined, including that of D. F. Panfilov and the diagram of the Canadian researchers A. Gan'on and B. Michel', in which the stability is determined by the internal resistance of the ice, like the resistance of a free-flowing body. This resistance depends on h_1 and ϕ , but does not depend on l. For a single-layer accretion of spring ice blocks with $l \gg h$ the basic factor determining the conditions of stability loss is the longitudinal dimension of the blocks.

/45/ Chapter 3

Emergence and Formation of Ice Jams

3.1. Causes and Focal Points of Ice Jams in Field Observations

The general conditions, causes and places of ice jam formation in channel currents have already been determined relatively completely /1, 2, 7, 8, 10-12, 24-29, 43-45, 48, 52, 56, 68-69, 77, 80-85/. Certain investigations along these lines will be recalled below. The results of a study of ice jams under field conditions leads to the following conclusions of a qualitative nature.

Ice jams may form: (1) At the beginning of the ice movement or in the breaking apart of the ice cover; (2) In the period of ice movement or as a consequence of retarding the opening of the river in individual sections. In this case they are usually formed in the rise of flood waters and in rare cases in their fall, with a stationary water flow $(dQ/dt) = 0$ or close to this.

Sections with a limited throughput (of ice transit) are focal points of jams, as has been noted, under the condition of sufficient quantities of incoming ice and the energy pressure value determined by condition Э or $T > A_{2,3}$.

In the breaking up of the ice cover, "hummocking jams" according to B. V. Proskuryakov and V. P. Berdennikov /68/ are formed in the breakdown of the ice cover as a whole, when hummocking, pushing under and piling up of ice blocks against each other and against the shore create accretions of ice masses in the channel. These same researchers classified two types of hummocking jams: (1) Jams formed when couplings with the shore were absent; and (2) In the presence of ice mass couplings with the shore, when a side thrust is included in the active forces. Hummocking jams are observed in both small and large rivers (the Severnaya Dvina, Yenisei, Lena, etc.). Jams /46/ are relatively widely distributed in holding back the opening in individual sections of the river /2, 32, 44, 68, 90/. A delay in the opening may be caused by the structure of the channel, the hydraulic flow or ice conditions - factors that are interrelated to a certain extent. Places of sharp discontinuity in the general profile (with a decrease in the inclinations and velocities of the current), channel bifurcations, etc. are characteristic in the examined situation /11, 83/. Similar jams are observed in many rivers of Siberia /2, 43, 48/, the central Volga /34/, Central Asia /24, 84/, and the Carpathians /18/. A frequent cause of jam formation is the opening delay in reaches, particularly confined to sharp turns in the river /2, 44, 68/. Such jams are characteristic of rivers flowing northward, but may also be observed in rivers of another current direction. The cause of opening delays providing risk of jams is the stronger and thicker ice in sections: (1) In autumn-winter jams, in rivers containing slush, including regions with a relatively cold winter, for example, in the Northeast region of the USSR /48/; (2) The formation of large ice layers observed in Siberian rivers /29/; (3) Incomplete winter opening with ice hummocking in regions with unstable winters, such as for example, in the Dniester River /31-32/, in the Israel River and the Wight River - USA /77/ and some Carpathian rivers (the jam on the Chernaya Tisa river of March 23, 1964, observed by Iu. A. Deev).

The later opening of lakes, reservoirs or primary rivers with respect to their inflow is also rarely the cause of jam emergence at the edge of the retained ice cover. A section of jam underwater may be the site of jam formation, when the ice cover is broken down in it by the influx of water and ice blocks accumulate downstream /1/.

Other focal points of ice jams when ice movement originates or is developed may be, as noted by L. G. Shulyakovskii /83/, a different type of obstacle to ice field passage or large transit ice flux in the ice movement. These include channel constrictions and bends, canals, islands, sandbanks, manmade structures (bridge abutments, piers, etc.). Here both the direct holding back of blocks in the case of ($B_0 < b$) as well as their wedging in a turn may occur. All of the enumerated jams are formed as a rule for $dQ/dt \geq 0$.

Ice jams during ice movement may be formed in those places where the flow throughput is insufficient for any reason for the free transit of ice blocks. This may be caused by an increase in the ice flow rate or block size as well as by a decrease in the ice transit throughput. The increase in ice flow

rate may in turn be caused by the development of ice movement on the upper section of the river or the eruption of a jam there /83/, and also by a concurrence of intense ice movement in the main river and its tributary. Such ice movement has been ob-
/47/ served, for example, by F. N. Bydin /10/ in the Sviri River and by Iu. A. Deev in the Nieman River below the Vili River junction in 1950. In similar cases, jams are formed in low-lying limited sections or in lines (see Section 2.1) confined to regions of a decrease in current inclination and velocity, increased channel resistance in sharp turns, abrupt constrictions, etc. V. S. Antonov /2/ notes that in places of sharp funnel-shaped channel constriction, ice accumulates very readily and two types of jams are formed: (1) Due to the wedging of large ice blocks; and (2) Due to the insufficient floating capacity for the passage of all the ice.

We note that the appearance of large ice blocks and their wedging is more often observed at the beginning of ice movement, but may also occur during it. The appearance of large ice blocks during ice movement may be associated with different causes, for example, with the detachment of large blocks from underneath islands - "tailings", which Deev observed on the Angara R. Jams associated with the upward thrust of very thick blocks may form in river heads, outflows from lakes and reservoirs, where the ice thickness is usually thicker than in the river. Lake ice blocks in narrow river sections ($h > H$) will be held back and can cause a jam. Such a jam was observed in 1948 in the Angara R. in a shallow sand bank at the Patron; this arose due to the inflow of Lake Baikal ice. It may be assumed that the jams on the Neva R. described by R. A. Nezhikhovskii /52/ are of this type. This is indicated by the sharp decrease in the river flow rate below the jams, which is evidently caused by the settling (landing) of Ladogian blocks at the bottom.

The decrease in ice-transit channel throughput during ice movement may be caused by a decrease in the water flow rate and a reduction in the water level or even a wind surge. In turn, the decrease in flow rate and reduction in water level are caused by a change in weather conditions or a delay of the runoff in upper-lying impound structures. Jams associated with the occurrence of negative temperatures are characteristic, for example, of northeastern rivers of the USSR /48/ where the beginning ice movement is retarded by routine cooling off. This leads to a decrease in the transport capacity of the current and the ice sinks. Such jams are evidently formed under conditions of a decrease in the value of or a change in the sign of the derivative dQ/dt to the negative. Jams which are associated with a wind surge of the ice may arise in the mouth regions of large low-land rivers, such as, for example, in the lower Ob R.

The stability and the relative risk of jams in the examined focal sites is nonuniform. In regulations for counteracting jams /47/, two types of permanent places of jam formation are classified: (1) Places with a discontinuity in the general profile

causing a decrease in the slopes and flow velocity; (2) Sharp turns of the river (greater than 110-115°). Here it is necessary to include also tapering sections of reservoir head curves, and in a number of cases places of manmade
/48/ channel constriction. Sections with a delayed opening give a greater risk of jam formation with respect to the ice conditions. Generally speaking, the other focal points of jam formation listed above are less stable and permanent. Their risk of jam formation in a number of cases is a function of the weather conditions of autumn-winter-spring and concrete morphological, hydraulic, and other special features of the river or its section.

Places with an insufficient throughput, which represent jam focal points, in agreement with Eq. (2.2) are characterized by a decrease there (up to zero) of $B_0\vec{v}$ - a multifactor function, derived from Eqs. (2.5), (2.15), and (2.16) which depends on spatial coordinates and time, i.e.:

$$B_0 \mathbf{v} = f(u, B, H, i_0, b, h, q_3).$$

The specific combination of these factors, which causes a decrease in $B_0\vec{v}$, depends on the natural features of the flow, human activities, or both together. The classification of the rivers of the USSR according to these indicators was not an objective of the present study. Similar classifications for specific purposes have been carried out in the investigations mentioned above. However, for a more complete analysis here of the cause of a reduction in $B_0\vec{v}$, it is expedient to make the following classification according to "genetic" characteristics:

(1) Ice-thermal factors which cause a reduction in B_0 and $\vec{v}$ as a consequence of the movement resistance of ice blocks of immobile ice formations (flow sections with unbroken ice cover, slow-moving sections, including blocked-up ice fields, ice layers, etc.);

(2) Morphological factors, where $B_0\vec{v}$ decreases with a decrease in the flow rate with a reduction in the channel inclination or water surface (for the river outflow from mountains, influx into reservoirs, raises in water level, and in other cases of discontinuity of the longitudinal profile);

(3) Mechanical factors which limit B_0 due to a decrease in the depth (sandbanks, shallow waters, etc.) and the flow width (constriction and branching of the channel, bridge abutment, channel dams, etc.);

(4) Aerohydrodynamic factors, which cause a change in the value or the direction of vector $\vec{v}$ with respect to the flow direction due to centrifugal forces, circulating currents, wind pressure, etc.;

(5) Combined factors, where the decrease in B_0 and $\vec{v}$ is caused by several of the causes named above.

After analyzing the literature sources, it seems that most large jams are related to the last group in the above classification; for example, jams with a simultaneous decrease in the current velocity and division of the channel into canals while retaining the ice cover there, etc.

/49/ 3.2. General Conditions of Jam Formation. Jam Classifications.

According to the causes enumerated in 3.1, a reduction in $B_0\vec{v}$ for various types of ice transit can be classified into more than 50 types of jams. Most of these were found in rivers of the USSR, USA, Canada and other countries. Disregarding the many factors involved in the many types of ice jams, their formation is subject to certain general conditions. An analysis of these conditions permits establishing general regularities and concrete features of the mechanism of jam formation and the classification of their basic form.

Jams arise in segments with an insufficient ice transit throughput (see Sections 1.2-1.4, 3.1) in the case of a loss of block stability there upon collision (dynamic formation form), and with an increase in the longitudinal compression in the accretion of blocks to a critical value σ_k (static form).

The possibility of jam formation on a flow section with ice movement S_p with a certain probability (with respect to quantity and dimensions of ice blocks) is determined by the general criterion for jam formation:

$$k = 1 - S_0/S_p. \qquad (3.1)$$

For $k < 0$, the jam cannot form. The relationship $0 < k < 1$ corresponds to a "jam-risk" section (line). The smallest value $k = 1$ for $S_p > 0$ will be in the case where $S_0 = 0$ and the jam may form only directly at line S_0. Criterion

(3.1) determines only one condition of jam formation - the insufficient throughput of a flow section.

The necessary and sufficient conditions of jam formation are determined from Eqs. (1.8), (1.12), (2.16), (2.19), (2.20) and (3.1) and result in the requirements:

$$0 < k \leq 1 \text{ with } S_p > 0; \quad (3.2a)$$

$$A_{s-4} \leq \vartheta \text{ (or } T) \leq A_p; \quad (3.2b)$$

$$\sigma_s \leq \sigma_p'. \quad (3.2c)$$

Requirement (3.2b) corresponds to the conditions $v > v_{2,3}$ $(u > u_p)$ and $\sigma = f(L) \geq \sigma$. It is necessary that the condition $\partial S/\partial L < 0$ be fulfilled for the extending of the jam formation of a certain section downward from a line with $S_p < S_0$. The dimensions and magnitude of the jam increase with $S_0/S_p \rightarrow 0$* $(S_p > 0)$; $\partial k/\partial L \geq 0$, and $T \rightarrow T_m < A_p$. By varying the dependences k, $\vec{v}$, and σ on factors controlling their parameters as a function of x and t, it is possible to establish the possibility and place of jam formation. This question is examined in Section 5.1.

By Eq. (3.2) we predict the entrance of the ice into the forming jam through some initial line C_0 in the quantity S_{0n}, equal to the throughput of this line, which applies to
/50/ $S_p \geq S_{0n} > 0$. The value S_p is a piecewise continuous function of time and bounded by the throughput of the flow section above the jam. The value S_{0n} is determined by the value (and, generally speaking, the sign) of the resulting work of all the forces acting

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*sic

on the ice blocks.

In the general case, forces act on the ice blocks in the formed jam that have a geometrical sum equal to:

$$iG + i_y G + P_1 + P_2 + F_1 + F_2 + N_1 + N_2 + P_m + P_a + I = 0, \qquad (3.3)$$

where iG and $i_y G$ are components of ice weight, directed along inclines i and i_y, respectively; $\vec{P}_1$ and $\vec{P}_2$ are forces of hydrodynamic and aerodynamic pressure (resistance) on the blocks; F_1 and F_2 are internal forces of elastic and non-elastic deformation of the ice; N_1 and N_2 are the side and vertical components of elastic compression and the weight of the ice caused by ice friction against ice, the bed, and the shore;* $\vec{P}_m$ is the force expended on thermodynamic and physical chemical processes; $\vec{P}_a$ is Archimedes' force; I is inertia.

The resultant of these forces can be indicated by the difference in the energy of pressure acting in the direction of ice-block movement in the forming jam, and the work of forces opposing this movement, i.e.: $T - A_{2-4} = f(x, y, z, t)$. The value and sign of this difference are determined by parameters that characterize the hydraulic and ice movement regime of the current, which has been examined in detail earlier /20/. Then, bearing in mind that $T - A_{2-4} > 0$ and $S_p > 0$, the value $S_1 \geq 0$, with the given inclination i_0, the following can be written:

$$\frac{dS_0 n}{dt} = f(q, H_H, S_p, n, H_n, k_f, R, u_v), \qquad (3.4)$$

where H_n is the value of the variable head, which arises due to the inflows of tributaries or of impounds and jams situated below,

*Including the movement resistant force from transverse obstacles (bridge supports, wedged ice fields, etc.).

etc; k_f is the filtration coefficient through the jam; R is the radius of curvature for the turn (bend) of the river.

The analysis of Eq. (3.4) facilitates the investigation of the conditions and peculiarities of jam formation for different water/ice movement, types of ice-block transit, and the structure of the flow channel (see Chapter 4).

Two basic mechanisms of jam formation are found as a function of the direction of the resulting forces enumerated in Eq. (3.3) with respect to the flow direction. (1) With the input of ice into the jam section in forward and forward longitudinal-side block transport; and (2) For reverse transit. Jams of the first type will be examined below.

A theoretical and experimental study shows that in the general case there are five stages of jam formation /18/: (1) Formation of the base; (2) Formation of the core; (3) Forma-
/51/ tion of the rear section - the jam body with ice packing in the jam and the formation of the head; (4) The formation of the tail section of the jam; (5) The slow consolidation of the jam mass (or its development state) during which its breakdown begins.

While there are general regularities in jam formation, there are considerable differences caused by concrete conditions in which the jams are formed. These relate to: (1) The type of obstacle which creates the ratio k > 0; (2) The hydrodynamic and ice conditions (nature and regime of ice movement, physical-mechanical characteristics of the ice block, etc.);

(3) The structure of the channel and flood plain on jam sections of the river and adjacent places. The resulting action of these factors determines a number of special features in the emergence and mechanism of jam formation, as well as in their form, dimensions, filtration properties, and certain other characteristics.

The experimental-theoretical investigations conducted by the authors combined with the results of field investigations published in the literature for ice jams permit the following classification of jam types.

1. The cause of jam formation may be subdivided into those that form: (a) according to a dynamic diagram due to loss of ice-block stability upon collision under the conditions $\ni > A_{2-4}$, i.e. for $v > v_{2,3}$ or $u \geq u_p$; (b) according to a static diagram, as a consequence of block stability loss upon collision with an increase in the longitudinal compressive pressure $T \geq A_{2-4}$, i.e. for $\sigma_m > \sigma_k$.

2. According to the formation mechanism and body shape, jams are classified as: (a) undeveloped, where the jam body formation proceeds under the conditions $\ni$ and $T < A_1$, i.e. for $\partial\sigma/\partial L \leq 0$ and $\partial h_1/\partial L = 0$; (b) developed with the formation of the body for $\ni$ and $T > A_{2,4}$, $\partial\sigma/\partial L > 0$ and $\partial h_l/\partial L > 0$; (c) the transition type, which is formed for $A_{3,4} > (\ni \text{ and } T) > A_{1,2}$. The formation mechanism of these jams is examined in Sections 3.3-3.5, and the criteria

for establishing the type of jam are given in Section 5.1. The differences in the structure and form of the body of the given jams, and also the distribution of the ice blocks in them may be examined in Fig. 6.

3. According to the nature of stress in the blocks, jams that emerge under the prevailing conditions are classified as: (a) tangential stress (type $\tau > \sigma$), which occurs in sections with $H < h$ (sandbanks, shallow water, division of the channel into small canals) and for a closed channel (ice cover, wedged ice fields, etc.); (b) normal compressive stresses ($\sigma > \tau$) under conditions of $B_0 < b$ as a consequence of retarding the blocks at transverse obstacles (constrictions and divisions of the channel (bridge supports, etc.); (c) tangential and normal stresses (τ, σ) for $B_0 < b$ and $H < h$ or in a closed channel (bridge supports with unbroken ice cover between them, etc.).

4. According to the filtration properties and special features of the formation of a maximum head level, jams are classified
/52/ as: (a) freely filtering jams with ice accretion porosity, which assures an average filtration velocity that is sufficient for passage of the entire water flow even with a complete clogging of the channel by ice blocks; (b) slightly filtering; and (c) nonfiltering jams. Filtering jams may be formed when ice-block accretions consist primarily of large, strong blocks, while nonfiltering jams form with accretions of predominantly

small blocks of varying size (crushed or ground ice) or blocks with a small strength $A_{2-4} \leq A_p$.

5. According to head section form, jams are classified as those: (a) with bottom slope (see Fig. 6a, b, c); (b) without bottom slope (see Fig. 6d), which are formed under conditions of b $< B_0$ at transverse obstacles in sharp constrictions of the channel, etc.

6. According to conditions of jam head shift (floating) along the vertical, which depends on the nature of the obstacle, jams are divided into: (a) those with free shifting; (b) with a "pinched" base and with stopping at the obstacle, which corresponds to the condition $\partial v_z/\partial z = 0$ (jams at the edge of a strong ice cover, at transverse obstacles (see Fig. 6d).

7. According to the morphological features of the jam section, the strength of the obstacle, and the ice probability, jams are examined: (a) with a formation not limited to the given causes; (b) with a formation limited to these causes, which is examined in more detail in Section 4.4.

8. According to the water discharge regime and ice movement in the formation of the jam, jams are classified as forming: (a) under stationary conditions, i.e. at $dQ/dt = 0$ and $dS/dt = 0$; (b) under non-stationary conditions with accelerated, retarded, and continuous development, examined in Section 4.4.

In addition, jams should be mentioned that form in the absence of shore binding and in the presence of this bond /64/. Jams of transit (coming to the top) blocks, of local blocks which are formed in the breaking apart of the ice cover, and from blocks of miscellaneous origin may also be distinguished.

The characteristics (forms) of jams which are given under 1-3, 5 and 6 above are the results of investigations conducted in the present study, while those given in 4, 7, and 8 represent a clarification and generalization of those data on jams known from the literature. The jam formation mechanism and its peculiarities, the form of the jam body, the equilibrium state, and their other characteristics will be examined below.

3.3. Formation of Jams Under Stationary Conditions. Emergence and Consolidation of Jams

The formation of jams under stationary water/ice movement conditions and in the absence of growth limits both in flow
/54/ length, as well as in height, i.e. for $L_3 > L_1$, $L_v > L_0 + L_2$ and $H_b > H_p$, where L_3 and L_v are the lengths of the jam and rectangular top bordering flow sections (see L_0, L_1, and L_2 in Fig. 6). The developed jam with a "dynamic form" of formation is taken as the basis of analysis. The formation mechanism of this type of jam has been examined earlier /18/, and is further refined with the following experimental-theoretical study. The features of jams of the second type are established according to a stepwise analysis of their formation mechanism.

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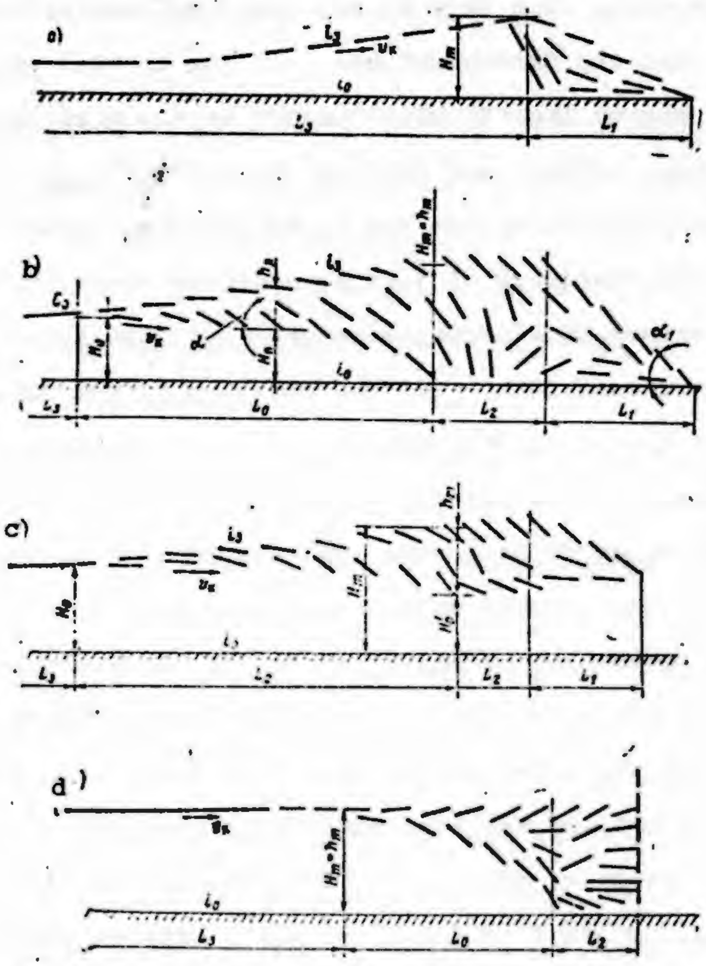

Fig. 6. Schematic longitudinal jam section. (a) Undeveloped freely-filtering jam with a bottom slope; (b) A developed freely-filtering jam with a bottom slope; (c) A developed non-filtering jam with a bottom slope; (d) A developed freely filtering jam without a bottom slope (at a transverse obstacle with b less than B_0; L_1 and L_2 - head; L_0 - rear ("body") and L_3 - tail (single-layer) part of the jam; i_0 - channel slope; α - angle of block inclination in the jam body; α_1 - angle of inclination of the bottom slope of the jam.

The emergence of jams as well as the formation of their core along with general regularities also have differences that are determined by: (1) The initial value of the block energy (Э and T) with respect to the work necessary for plunging, piling, and pushing the blocks (A_{2-4}); (2) The nature and features of the jam obstacle in the channel. We examined the emergence of jams on sections where the condition $k < 0$ arises as a consequence of: (a) Insufficient depth; (b) Insufficient width; (c) The presence of the unbroken ice cover (ice fields) in the path of movement of the transit blocks; (d) Non-simultaneous opening of the specified flow section; (e) The presence of obstacles opposing the formation of ice movement in the opening of the river.

The emergence and consolidation of the jam under conditions of $Э > A_{3,4}$ in the section with $H < h$ begin with the formation of the lower subjacent block layer - the jam base. Approaching the jam section, the blocks push deep into it under the action of energy Э and pressure T of returning blocks (for Э or $T > A_4 < A_3$). With an increase in the length of the pushed-through layer of blocks, the work A_4 increases and at $T < A_4$ the formation of the base is terminated and the core formation begins.

With an initial energy value of less than A_4, the blocks remain immediately in front of the jam section, where the core begins to form. An analogous phenomenon takes place in the presence of transverse obstacles for l and $b > B$, including also the wedging of large blocks (jams without the bottom slope, see Fig. 6).

The emergence of a jam on a section with an unbroken ice cover, in the approach of the blocks to its edge, requires the fulfillment of the additional condition:

$$S_p > S_1 + S_2, \qquad (3.5)$$

where S_1 and S_2 are the quantities of ice carried off under the ice cover and along its surface.

The blocks can move under the ice cover at a flow rate of $u > u_s$ or u_v, as established in Section 2.3, and the ice flow rate can be approximated according to the equation:

$$S_1 = \psi h B_0 (u_s - u_v). \qquad (3.6)$$

The model investigation showed that for $\ni > A_2$, the block stability losses at the edge of the ice cover caused

/55/ both their plunging and their tightening with the wedging of blocks for $l > H$ braced between the bottom and the edge (see Figs. 4a, b). These blocks are used as a springboard for the creeping of other blocks to the edge and at the same time create an obstacle to the plunging of new blocks under the edge. The respective decrease in S_1 causes the accretion of blocks at the edge and a raise in the water level with its discharge onto the ice cover, so that blocks striking there can be carried away along it in quantity S_2 which depends on the values H_p, B_0, h and the ice roughness (hummocking).

In fulfilling Eq. (3.5), the first layer of ice blocks on the ice cover forms the jam base. In the general case, Eq. (3.5) can result from an increase in S_p, as well as in a decrease in S_1 or S_2 (for example, according to the clogging of the current under the ice cover by blocks, etc.). Further, the formation mechanism of both types of jams (with H < h) is similar also on sections with the ice cover.

In the delayed opening of a relatively large section of the river, for example on a sandbank, while keeping intact the ice cover in the lower-lying reaches, large blocks may form the jam base, and these blocks are formed in the breaking apart of the ice cover and their penetration downward under the edge of the ice of the unopened section.

Model investigations permitted the rather clear tracing of the entire process of a similar non-simultaneous opening and emergence of the jam on a conventional model of the geomorphological pair sandbank / reaches (at the channel turn) with a thicker ice cover on the reaches. The process is developed in the following manner. With an increase in the water flow rate, the ice cover in the section with the smallest strength and the weakest external couplings (sandbank) lost its coupling to the shore, rose up, protruded, lost its longitudinal stability and then broke apart into large units thrust over the edge of the stronger ice cover (reaches). Depending on the current velocity and the increase in the water flow rate, two mechanisms of the examined phenomenon are discerned.

With a high current velocity and an intensive rise in the water level, the deformation and breakdown of the ice cover corresponds to the behavior of elastic freely supported plates, examined in the theory of materials' resistance. In our tests, under such conditions, the ice cover buckled, forming bulging waves and then broke into large units (according to the number of pathways of the bulging).

With a relatively low current velocity and a slow rise in the water level, the ice cover at the sandbank (which is not connected with the upper reaches) floats up, forming at its place of reinforcement with the lower-lying immobile cover only one bend. With a rise in the water level to a value 2-4 times the initial depth there is observed a breakdown and shifting of the ice to the reaches with its subsequent breakdown as a con-
/56/ sequence of collision with the edge of the strong ice in the reaches (Fig. 7).

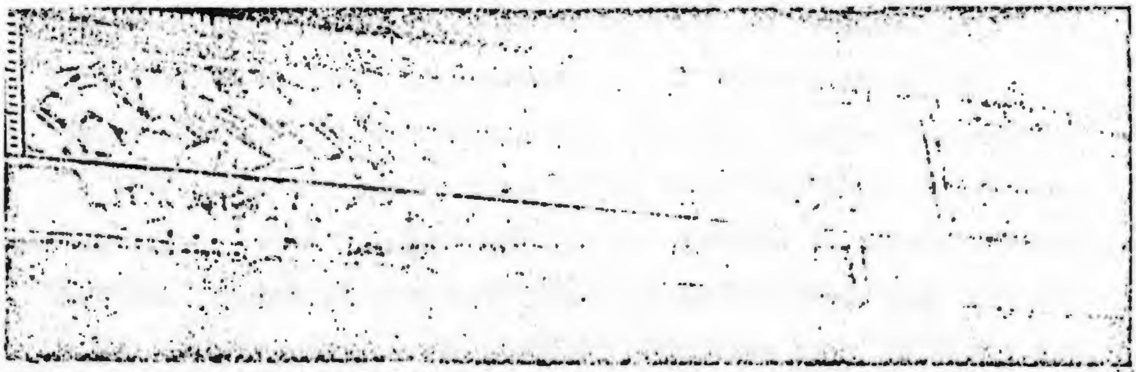

Fig. 7. A developed filtering jam on a rapid-laden sandbank.

Ice blocks formed in this way partially creeping or tightening under the edge were wedged together rather than creating the jam base. In individual cases, the wedging of particularly large blocks with the moving out of their front edge under the edge of the ice cover created a clogging up of a single-branch channel and a raising of the water level like in a type of spillway dam with a wall inclined along the current. In front of such "dams" there are observed curves of fall and an increase in the current velocity, so that the subsequent smaller (transit) blocks freely pass through the jam crest and the further development of the jam is terminated. This phenomenon can obviously be used in some cases for preventing the formation of large jams.

Therefore, in the examined type of jam, the base and lower portion of the core are comprised of large blocks of "local origin". The upper section is formed of small "transit" blocks. A similar distribution of the ice in a jam has been established also, for example, on the Lena R. by A. S. Rudnev /68/.

The consolidation of the "hummocking jams" is analogous in
/57/ its general nature, with only the difference that in this case, jams are confined not only to the edge of the ice cover, but also to places of reduced channel throughput. After consolidating the jam (the formation of its base) the core is formed, and then the jam body. The mechanism of these forces, according to the results of experimental-theoretical and field tests, are exam-

ined in Section 3.4 and 3.5.

3.4. Formation of the Jam Core

After the jam is consolidated, its core is formed, which is a relatively compact piling up of ice directly on that section where the blocks are stopped or their movement is slowed down as a consequence of insufficient throughput (Fig. 5, 8a, 9a).

With a large value for the initial pressure energy of the blocks ($\ni$ and $T > A_{3,4}$), which is characteristic for developed jams, the core formation proceeds as a consequence of the regularly alternating processes of accumulation (piling up), submerging, tightening and pushing through of the blocks into the depths of the jam section. This alternation is caused by a change in the values of $\ni$, T, A_{2-4} and the ratio between them and the growth of the water level rise and dimensions of the core.

A certain level head caused by the formation of the base reduces the work value for the accumulation of blocks A_3 and at $\ni > A_3 < A_4$, the blocks that have not pushed through earlier, upon stopping, pile against them. This piling in turn increases the crowding of the channel and the rise in the water level, decreasing the frictional force between the blocks (and consequently also work A_4) and again guaranteeing the process of the pushing forward deep into the section of the blocks piled up on the base, which occurs at $T > A_4 < A_3$. Thus a second layer of blocks forms for the specific length of which there again arises the ratio $\text{Э} > A_3 < A_4$ and the pushing through alternates with the piling up of a third layer. Therefore, the formation of the core in this stage of its development proceeds, as it were, by the subsequent accumulation of block layers. However, due to the nonuniform size of the blocks, and also the values v, u, and H_p, this process occurs irregularly, accompanied by hummocking and upsetting of blocks.

A further increase in the water level with an increase in the size of the core reduces the flow velocity and consequently the block pressure energy. Consequently, the length L_1 of each ice block subsequent in the height of the layer decreases and the bottom inclination of the core acquires a stepwise acute angled form (with respect to i_0). The upper (upstream) frontal slope of the core during this time has an angle close to a straight line (see Figs. 5, 8a and 9a), which corresponds to the essence of the examined process.

With a further decrease in the flow rate, only large blocks still possess energy of $\text{Э} > A_3$. At the same time, condi-
/60/ tions of block tightening arise at the core and are facilitated due to an increase in the depth and the emergence of the ratio $A_{3,4} \gg A_2 < \text{Э}$. The tightened ice blocks take root flatwise against the front side of the core (Figs. 9a and 10a), as a consequence of which the filtration coefficient decreases and consequently so does the filtration flow rate, while the water

level increases. Because of this, the value of work A_4 is reduced according to the pushing in of the floating blocks along the core surface.

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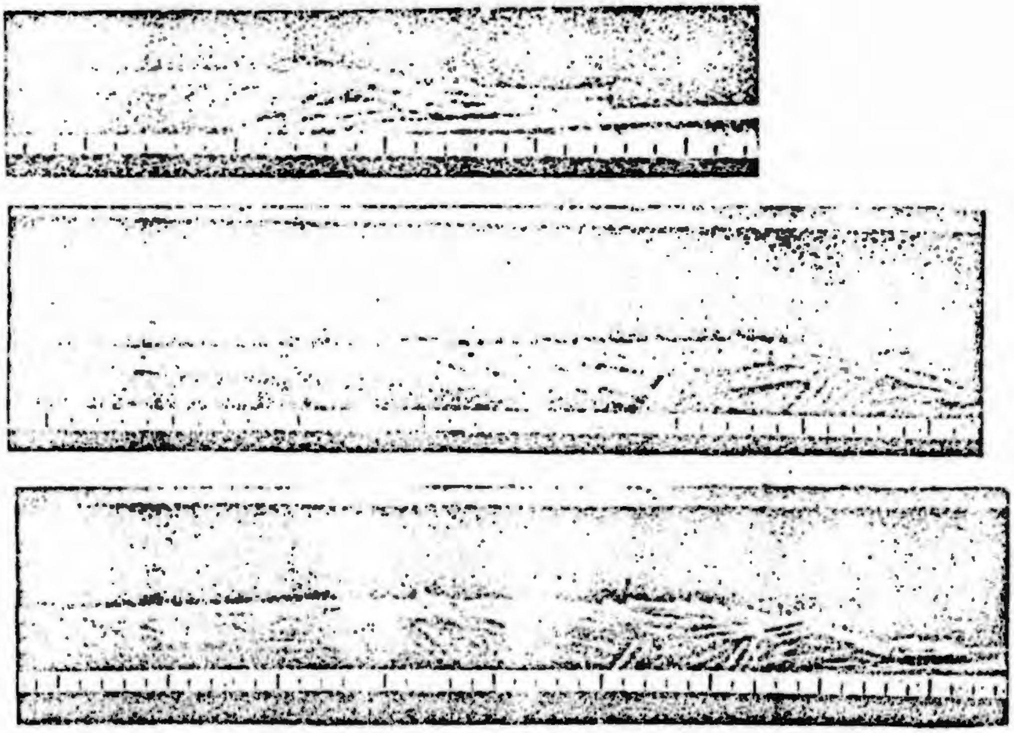

Fig. 8. The formation of the developed filtering jam at the edge of the cover.

(a) The core formation due to creeping and tightening of the blocks; (b) The formation of the jam body; (c) The formed jam after shifting.

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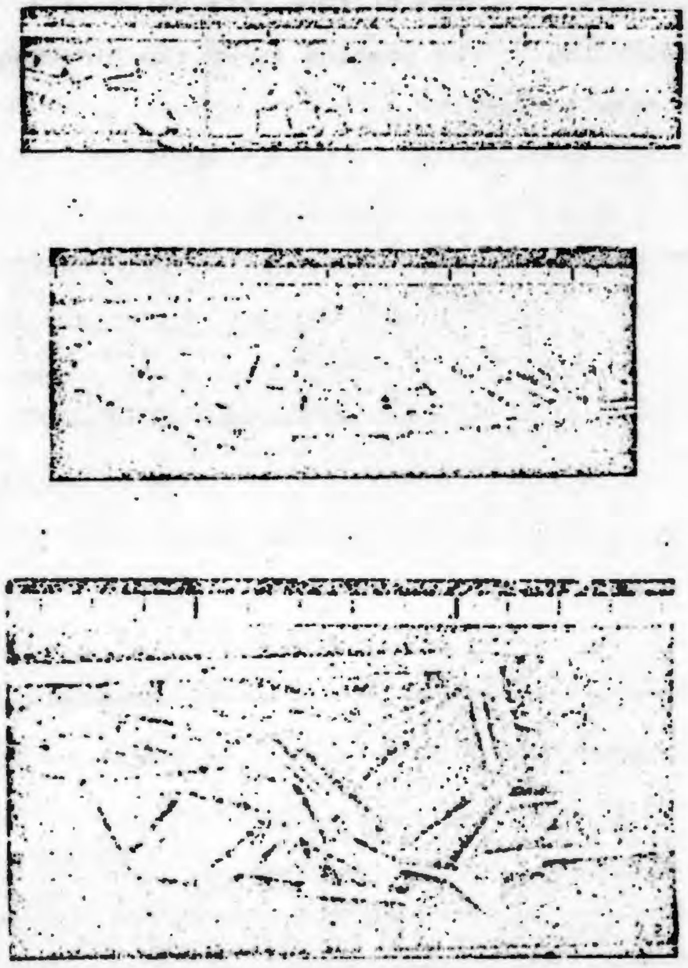

Fig. 9. The formation of the developed filtering jam at the edge of the ice cover for a turbulent flow state.

(a) Moment of core formation due to block tightening; (b) Forming of the head section and jam body; (c) The formed jam after shifting.

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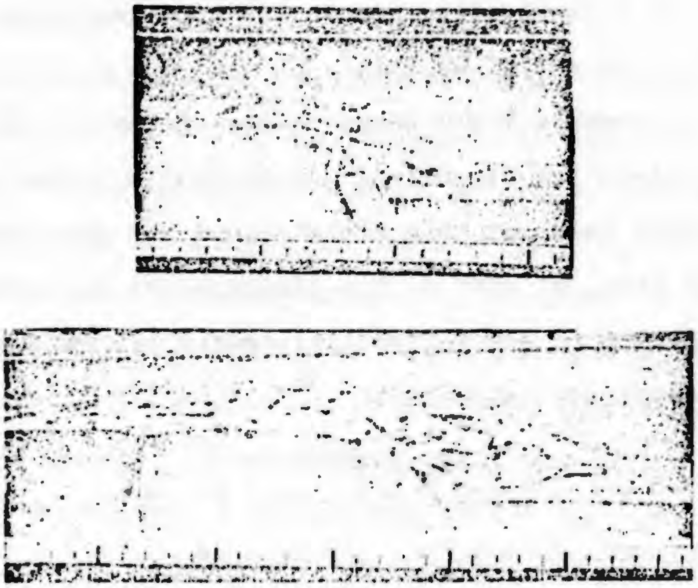


Fig. 10. Formation of an undeveloped jam at the edge of the ice cover.
(a) Core formation; (b) The formed jam.

At this second stage of core forming, the process of tightening and packing of the blocks in the front part of the core, which alternates with the pushing through of the blocks along the surface of its lower bank acquires, along with a growth in the water level, a very large specific gravity, and at the end of core formation (for $A_3 \gg \ni \geq A_2$) it becomes the only process. The plunging blocks tightening against the core are at first distributed vertically, and then as the core grows, they acquire an inverse slope (see Fig. 9a). Thus, in the formed core, the ice blocks are distributed more or less fan-shaped: in its lower section - horizontally, in its middle section - vertically, and in its front end - at an obtuse angle to i_0 (see Fig. 5, 9b).

/61/ The pressure T may cause shifts of the ice in the core, which increase its dimensions and the angle of the bottom slope The averaged value of this angle (along the line which envelopes the slope profile) lies within limits which are determined from the conditions of block stability toward slipping and shifting along the current bed, i.e.:

$$\varphi \geq \underset{\tan}{\mathrm{tg}}\,\alpha \leq \varphi', \qquad (3.7)$$

where $\varphi' \approx m_1\varphi_1 + 0{,}7\beta\eta\zeta\varphi_1 H_p - m_1 i_0$; $m_1 = 0.4 - 0{,}7$ - is the coefficient which considers the activity of the filtration current through the jam. For $\phi < \phi'$ in Eq. (3.7) instead of $\phi_{1(?)}$, we use ϕ.

With a further decrease in the flow velocity due to the rise in the water level to a value of $v < v_2$ in front of the core, only a hummocking of the blocks will occur, and with $u = u_k$ (i.e., for $u < u_p$) and $v < v_1$, their joining together, and in this way the core formation is terminated.

The water level head at the formed, freely filtering jam and the angle of the water surface, which reaches a minimum value at this time, is determined from the equations:

$$H_1 = \frac{q}{f u_k}; \qquad (3.8)$$

$$i_1 = \frac{n^2 q^2}{0{,}4 r^2 H_1^{4/3}}. \qquad (3.9)$$

In non-filtering jams, the level head will be greater than H_1 by the value $\Delta H = H_1 - H'_0$, where H'_0 is determined according Eq. (4.19b). An explanation of the cause of this phenomenon is given in Section 4.3.

The core formation for a turbulent current condition with an initial depth of $H < H_k$ (where H_k is the critical depth) is characterized by the emergence of an hydraulic "jump" above the core, after the level head reaches the value $H_p > H_k$ with an increase in the ground current velocity and turbulent flux. As a model investigation has shown, this leads to a more intense packing of the blocks in the submerged portion of the front core inclination. Consequently, q_f is reduced, H_p increases, and so does pressure T on the core, which is experiencing in this case greater shifts and the packing of the ice with an increase in tan α up to a limiting value of ϕ (see Fig. 9b).

The core formation under the initial condition $A_{3,4} >$ Э $> A_2$ (which is characteristic of undeveloped jams) begins with the stopping (or hummocking) of the first blocks in front of the jam section. The subsequent blocks push under them, creating a rise in the head and decreasing the work value $A_{3,4}$. For $T > A_4$, the upper layer of blocks pushes into the jam section, forming a core base. Then the base forms by the repeating pattern: submerging (tightening), head [water level]
/62/ increase, and pushing of the blocks along the core surface into the depth of the jam section. In this case the core dimensions and the bottom slope angle have smaller values (Fig. 10a and 11), than in developed jams. The core is formed analogously in jams which form according to the static diagram with $\sigma_m > \sigma_k$.

Fig. 11. Transition-type jam.

The examined core formation mechanism relates fully to the case of freely filtering ice-jam accretions, where there is a more or less complete piling up by the ice of the useful cross section. In slightly filtering and non-filtering jams, there remains a space under the ice accretion sufficient for passage of the water flow under conditions of a closed channel under the ice (see Fig. 6c). A number of investigators have studied the parameters of such a flow in the established regime /5, 15, 50, 58/. A theoretical dependence between hydraulic characteristics of open and closed flows of continuous ice accretions established by Berdennikov showed that the hydraulic angle of the sub-ice flow increases considerably.

The basic feature of slightly filtering and non-filtering jams under the condition of a possible free shifting of the blocks in the vertical section (floating) is the formation of the core almost completely (excluding the initial stage) as a consequence

of the blocks pushing under those that have stopped earlier. Jams with a pinched base and supported against an obstacle form primarily due to the piling up of blocks one on top of the other. The pushing under of blocks here can develop subsequently if the upper layers of the block pilings are not choked. In the contrary case, a continuously increasing water level head will cause the rise of the core, which creates condi-
/63/ tions for the piling up of new blocks and their movements against the obstacle. The formation of the head section will proceed continuously until the ice movement terminates.

3.5. Formation and Packing of the Jam Body

In the formed core, the slope, as noted, has a minimum value corresponding to the head curve type a_1, while for $H < H_k$, it corresponds to type a_2. In front of the jam core, with $u = u_k$, there is formed a single-layer accretion of blocks, which in developed jams and transitional-type jams is further transformed into a multi-layer accretion - the jam body.

It follows from the determination of the developed jam that these are formed in flows with relatively large channel slopes. Under these conditions, the level head decreases relatively rapidly with distance upstream from the core, and the current speed increases, and also at a certain line C_1 with $u > u_k$, the tightening and submerging of the blocks increases with their movement toward the core under the upper layer of the immobile ice. This movement is assured for $\ni > A_1$, which is made possible by a decrease in the hydrostatic pressure for each subsequent block layer due to an increase in H_s and the alternation of block slipping with their flatwise rotation, and also an increase in the head and consequently of the slope of the water surface (in the section, the line c_1 is the core, due to the cracking of ice blocks there). The head curve with minimum slope in this case pushes away from the core to line c_1, as a consequence of which, the current velocity decreases there and the block submerging terminates, replaced by their abutting at $u = u_k$.

The block movement toward the core under the ice accretion still continues during this time, since the current speed is greater there due to the ice crowding of the useful section. At first blocks that have reached the nucleus come to a stop and press against it along the entire plane or at a certain angle to the direction of vector $\vec{u}$ [?] (Fig. 9b), since here the work A_1 increases, and part of the block energy is consumed in packing. The growth of the ice accretion at the core again decreases filtration through the core, increasing the head level and decreasing slope i, which slows down the movement of the other blocks in the section: line C_1-core. The work of pushing the blocks under the ice accretion also increases due to the increase in its thickness. The stopping of the block movement over the entire section (and consequently an increase in the level head and a decrease in slope) begins after the ratio $\ni \leq A_1$ is established under the ice, which corresponds to the equilibrium condition (1.10).

In the final analysis in the line C_1-core section, the blocks are established obliquely with a decrease in the thickness h_1 of the accretion as a function of distance from the core (see Figs. 8b and 9b). Upstream from line C_1 at a certain line C_2,
/64/ as a consequence in the decrease of the level head, the process is repeated, and then arises in a subsequent upper line C_i, etc.

Thus the increase in the rear section of the jam consists of a subsequent regressive - upstream - formation (at $\ni > A_1$) and an automatic braking (at $\ni \leq A_1$) of the processes of block plunging and piling up in the channel (see Figs. 8b and 9c). As the line C_i moves away from the core, the thickness of the block accretion h_1 and the quantity of ice penetrating the nucleus decreases as a consequence of the general level increase, the lengthening of the block movement path, and the increase in h_i in the direction toward the core. The degree of subsequent piling up of blocks at the core will be proportional to i_0 and q; for high values, the increase in the rear section upstream will continue until the ice movement is terminated or until a new - upstream - jam forms, or until the present one is broken down.

As the jam increases in dimensions, the slope of its surface i_3 and the pressure of ice and water on the core increase. With a pressure value of $T > A_{s, d}$, i.e. when it exceeds the internal resistance of the accretion F_2 (passive pressure), there arises a shift of the entire ice mass to the core side, increasing the thickness and density of the accretion there.

The increased and deformed core forms a head section of the jam (see Figs. 8b and 9b, c) with a corresponding increase in the level head there and a decrease in the filtration discharge u_f and angle i_3. The nucleus packing in freely filtering jams in turn causes the additional rise of the head, necessary for increasing the hydraulic slope, which guarantees the ratio $q_f = q$, in maintaining jam stability. The increase in the level head and the decrease in the slope reduce pressure T on the nucleus and the packing process upstream, creating an overall increase in i_3 over the entire jam section. This process again can cause a lengthening (for $u > u_p$) and a thickening (for $\ni > A_1$) of the rear section of the jam with a subsequent increase in pressure up to $T > A_{s,\ d}$, and a new ice movement at a higher head level with a subsequent new increase in the jam head.

During these shifts, the block slope at the water surface acquires in the base section of the jam body an angle close to the angle of internal friction, which corresponds to the medium conditions in which the block accretion is found (for the paraffin models, this angle was 30-40°).

The stopping of the growth of the head level and block accretion thickness below the specific line (including the jam head)

corresponds to the simultaneous fulfilling (emergence) of the following conditions: (1) the current pressure on the blocks of $T \leq A_{s,d}$; (2) the water current pressure under the block accretion $\tau \leq \tau_k$; (3) the wind pressure on the free surface of $\tau_v < \tau_{vk}$; (4) the specific water discharge of $q = q_f + q_1$, where q_1 is the discharge under the block accretion. According to the rheological diagram IV, the first
/65/ three conditions result in the fulfilling of requirements (1.8) and (1.11). The fourth condition is determined by the filtration processes of the jam block accretion, which is examined in more detail in Chapter 4.

As the jam is formed, line C_i moves away upstream with the simultaneous increase in h_1. At some line C_0, where condition (1.12) is fulfilled in agreement with diagram IIIa, the multi-layer accretion of blocks is converted to a single layer. Here the level head will be determined by the value:

$$H_0 = \frac{q}{r u_n} + \varepsilon h + h_{min}, \qquad (3.10)$$

where $\varepsilon \approx 0.15 - 0.20$; h_{min} is the smallest block thickness in the given ice movement.

The two last terms in Eq. (3.10) present a "correction" for the head, created by the single-layer block accretion above line C_0 and the additional raising of the water from smaller size blocks, which will advance by pushing under the blocks of calculated dimensions which have stopped earlier. The value of the second correction is determined from a diagram which allows relating the phenomenon to the determination of the sum of two infinitely decreasing geometrical progressions.

Eq. (3.10) corresponds to the stability condition of both the separate blocks that are pushing under, as well as to their single-layer accretion. Thus, for the calculated value of u_k in Eq. (3.10) the smallest value obtained from Eqs. (2.9) and (2.14) should be taken. A checking of Eq. (3.10) on model and field tests (the Dniester and Prut rivers) gave relatively satisfactory results. The mean-square deviation of the calculated values from the experimental was 16%.

Below line C_0 toward the jam head on section L_0 (see Fig. 6b), the block accretion thickness h_1 increases according to a nonlinear law. The lower envelope of the longitudinal profile of the block accretion in a prismatic-formed channel approximated by a flat curve is described by an exponential-type curve, as an analysis has shown (see Section 4.1) and as experiments have confirmed (see Figs. 7, 8b, 9c).

Therefore, the developed jam which has formed with a bottom slope has three characteristic regions in the longitudinal direction (see Fig. 6b, c): (1) the head regions, where the ice fills all or most of the channel with two sections: the lower section - triangular form with a bottom slope of length L_1 and the top section - a rectangular section of length L_2; (2) the rear (jam body) region of length L_0 with decreasing thickness of the accretion h_1 upstream; (3) the tail region consisting of a single-layer accretion of blocks of length L_3.

Local changes in h_1, which deviate from the exponential law may arise in natural channels in certain sections from changes of /66/ B, H, i_0 and n in the jam body. The tail or rear section of one jam may come into direct contact with the head of another, upper jam. It should be noted that if the pressure of a single-layer block accretion does not permit establishing equilibrium as determined according to Eq. (3.10) then the jam will continue to form due to the upstream movement of section L_0 and consequently there is increase in the length of section L_2 (for a further explanation, see section 4.2).

The maximum level head H_m in the formed jam is recorded at the upper boundary of section L_2. Downstream, the hydraulic resistance creates a depression curve, while upstream, the water surface in the filtering jam is found under conditions of water rise with slope i_3. In line C_0, this slope corresponds to the value i_1 for head dimensions determined according to Eq. (3.10). The slope of the water surface $i_3 = f$ (L) in section L_0 in the general case has a nonlinear character.

The body of developed non-filtering jams forms almost like that of the filtering jams. The basic difference consists of the fact that under the ice accretion of the non-filtering jam, a space remains sufficient for the passage of the incoming water discharge. The minimum sub-ice depth H'_0 is determined at the place of the smallest ice accretion thickness, i.e., at the boundary of sections L_0 and L_2 (see Fig. 6c). The average value of this depth in a prismatic-form channel is determined by Eq. (4.19b).

Slightly filtering developed jams represent a transitional type between freely filtering and non-filtering jams. With respect to their filtration properties, which determine the discharge q_f, there remains a free space under the ice accretion that is sufficient for the discharge passage $q_1 = q - q_f$. Substituting the value q_1 for q in Eq. (4.19b) it is possible to determine the respective averaged depth H_0'.

Depending on the formation conditions, undeveloped jams can be classified into two types:

(1) Undeveloped jams, which consist of the head L_1 (or $L_2 = 0$) and the tail section L_3 adjacent to it - a single-layer block accretion. The rear section (body) is absent in these jams, i.e. $L_0 = 0$ (see Figs. 6a, 10b);

(2) Jams of a transitional type, where $L_2 = 0$ also, but which have a jam body, comprised of several layers of blocks with a predominantly horizontal disposition and decreasing upstream of value h_1 (see Fig. 11).

Jams of the first type are formed in the case when the minimum pressure of block accretion on the formed nucleus does not exceed a critical value, determined according to Eq. (2.20), i.e., $\sigma_m \leq \sigma_k$ and $u \leq u_k \approx u_p$.

Jams of the second type form when the formation of the jam body occurs under conditions of $\sigma_m \geq \sigma_k$, but $T \leq A_{sd}$. In

such a case, the formation of the jam body proceeds without shifting and packing of the ice. More complete criteria for
/67/ the determination of undeveloped jams and transitional-type jams are given in Chapter 4.

The question of a limiting value of ice filling of the useful cross section in jams has not been fully clarified as yet: certain theoretical treatments /43, 50, 73/, confirmed by a number of field tests /43, 44,74/ lead to the conclusion that a complete clogging of the channel with ice is impossible, but cases of channel clogging of rivers down to the very bottom have been noted, for example in the Lena River /2/, the Israel R. in the USA /18/, etc.

It should be noted that in this connection, first of all, the measurement of the ice thickness of jams in the field (particularly at their head section where it is smallest) is coupled with a number of difficulties and often the thickness is given as an approximate value according to indirect indicators. Secondly, the theoretical treatments mentioned here relate to the formation of non-filtering (or slightly filtering) jams of crushed ice and are fully justified for such a case.

However, in jams of large, strong blocks, which assure sufficient filtration through the jam head ($Q_f = Q$) a complete ice clogging of the useful cross section can occur in the head section of the jam, which is confirmed by the model investigation (see Figs. 7-9). A clogging of the channel up to the very bottom is also possible with crushed ice in the formation of a jam under the ice cover or under a choked ice field. Such a phenomenon is observed in the settling to the bottom of the already formed jam accretions in the decline of high water and in the retaining of jam masses in shallow water after jam eruption in the presence in both cases of an outlet path for the water to go into "channels", along reaches or rifts. With free filtration of water through the jam, the flow can be directed along the outlet path (the branch, reaches, etc.) or it can break through "pressure tunnels" in the ice accretion. The filling up of the channel with ice down to the bottom also takes place in jams at large ice layers /2/.

Therefore, the extent of filling of a single-branch (without outlet path) channel with ice at the jam head may be different and depends primarily on the filtration properties of the ice accretion determined by the dimensions and strength of ice blocks and the degree of uniformity of these characteristics, and also on the type of jam, its vigor, the channel peculiarities, the flow hydraulics, etc. We will not continue further with non-filtering jams, which have already been subjected to a detailed investigation elsewhere, but we will note briefly some characteristics of filtering jams.

In freely filtering jams, which arise with $H > h$, as the thickness of the ice accretion grows at the jam head, it settles and comes into contact with more elevated bed sections. The flow of water under the ice decreases in size and may be divided into separate pressure currents passing along the depressed bottom sections. Having a high hydrodynamic pressure, these flows /68/ may change their direction, breaking open a path in the direction of least resistance to the movement, including bursting open of the jam surface. Ye. I. Ioganson described such a phenomenon on the Volkhov R. /29/.

Deev observed pressure currents within a pile-up of ice and slush covered at the top by a continuous ice cover in the Angara R. in 1947-48. Their direction was clearly followed by the strong noise of the blocks carried along by the water and striking against each other. The breakthrough of the current onto the surface found here had an explosive nature; blocks with an area of several square meters and a thickness of more than 0.5 m were heaved up and thrown several tens of meters from the place of breakthrough. According to calculations, the maximum current velocity in the breakthrough was on the order of 10-12 m/s. With a further growth of the jam "head", continuous sub-ice flows, shrinking and dividing, can be converted to filtration flows. However, an increase in water discharge with an intensity higher than the critical intensity, i.e. at $q > q_f$, which causes a jam rise, may lead to a partial floating of the ice and formation of new sub-ice currents.

Sub-ice flows are not characteristic of freely filtering jams which arise for $H < h$, i.e. due to the mechanical contact between the blocks and the bottom. They are formed only in longitudinal recesses of the bed, and also in the rise and breakthrough of water in some section of the core ("head") of the jam as a consequence of its insufficient strength, i.e., under the condition $T > A_{s,d}$, which corresponds to the breakdown of Eq. (1.12). With an increase in the jam head size, the possibility of forming such flows decreases. An increase in the size of the jam head during its formation leads to a decrease in the concentrated sub-ice flows due to the gradual filling of the channel depressions with ice. Therefore, for large freely-filtering jams, the filling of the entire channel with ice in the jam head section is characteristic. The larger dimensions and density of the ice packing can lead here in individual cases to a considerable decrease in filtration through the jam and to a reduction in the water discharge and level in the river below the jam, i. e., a freely-filtering jam is converted into a slightly filtering one. Such a pattern evidently occurred in the jam formation in the single-branched channel of the Suhon R. at the Opock D. /74/.

/69/ Chapter 4

Equilibrium Jam State

4.1. Regularities of Ice Distribution and the Equilibrium State of the Jam Body

The static equilibrium of non-filtering jam accretions of constant thickness of crushed ice was examined by a number of authors /5, 15, 43, 50, 51, 60,73/. Applying the concepts of the theory of a free-flowing medium for this case, they establish dependences which associate the hydraulic characteristics of the current with the thickness of the accretion, the level head, etc. Essentially it is not jams that have been considered in these investigations, but the equilibrium state of masses of constant thickness of crushed ice, packed by a current. The finite dimensions, the special features of the structure and the form of the jams and regular natural formations taking into consideration the individual properties of the blocks that compose the jam are not considered nor investigated. From this point of view, an analysis is given below which allows establishing general conditions of equilibrium and certain special features of various types of jams.

The developed filtering jam is taken as the general case with consideration of the fact that other types of jams (section 3.2) may be derived from its particular cases.* The body of the developed jam is characterized by an increase in section L_0 of the accretion thickness h_1 of the ice from h in its tail section to

*Such classification of jams into general and special cases is taken only for the convenience of mathematical analysis and should not be confused with the concept of relative frequency of jams of different types observed in nature.

h_m in its head section according to a nonlinear law.

To establish the type of dependence $h_1 = f(x)$ on the jam section L_0, a crosswise strip of length dx is taken and an equilibrium equation (1.6) is established for it under the following assumptions and conditions:

(1) For a wide rectangular channel with a constant bottom slope and with straight current sections L_{pr} greater than

/70/ the jam head $L_{pr} \geq L_0 + L_2$, the jam ice accretion is found in a bounded stressed state. In agreement with form IV of the rheological model (see Fig. 2), it corresponds to some critical value of stress under the action of which a separate compartment is found. Under these conditions, the external force should be equal to the maximum internal reactance of the accretion: the passive resistance in the longitudinal direction and the active resistance in the crosswise direction, i.e., on the shore;

(2) A gradual increase in the ice accretion thickness along the length of section L_0 is approximated by a flat curve, wherein the accretion thickness increases to a value dh_1 along the length of the elementary segment dx;

(3) In the general case, the accretion consists of blocks with h < 1, inclined towards the direction of current at an angle equal to the angle of internal friction (rupture) of the accretion under the water (see Fig. 6b);

(4) Under equilibrium conditions of the jam body, the current velocity under the ice accretion is equal to a critical value u_k, at which there is still guaranteed the immobility (sta-

bility) of the blocks in the jam body. In section L_0, this velocity $u_k = u_{s,v}$ is determined by Eqs. (2.17) and (2.18), and at the upper boundary of the section for $L_0 = 0$, where the multi-layer accretion is transformed into a single layer $u_k = u_p$, is calculated according to Eq. (2.14). The average depth H_0 is established by the latter condition on the upper boundary, as it follows from Eq. (3.10);

(5) Forces are considered: p_1, p_2, p_3, p_4, p_5, p_0, which act in the direction of the x axis, and the resultant of the crosswise forces p_{2y}, p_{3y}, p_{4y}, which create an additional resistance to friction against the shore. The force p_0, directed to the negative side of the x axis is taken according to Eq. (1.22) without considering coupling, while forces p_1 and p_2 under the ice accretion are taken according to Eq. (2.15). In addition to this, the relatively small value of the slopes permits taking $\tan \alpha \approx \sin \alpha$.

Taking into consideration the above assumptions and conditions, the equilibrium equation for the strip dx takes the form:

$$-\sigma_k\, dh_1 + p_3\, dx + p\, dx - \varphi_1 (p_3 - p_{1,2} \pm p_4)_y\, dx - p_0 \frac{h_1}{B}\, dx = 0 \qquad (4.1)$$

or

$$\frac{dh}{dx} = \frac{M}{B\sigma_k} (A - h_1), \qquad (4.1a)$$

where $M = p_0 - p_3' B = 2\varphi_1 \beta \zeta \sigma_k' - \gamma_1 B (i - \varphi_1 i_y)$; $A = Bp/M$; $p = p_{1,2} + p_5 \pm p_4'$;

σ'_k is the transverse force, determined by the active pressure of the accretion on the bank; $p'_4 = (1 - \phi_1)\, p_4$; p_5 is the filtration pressure approximately determined according to the dependence $p_5 \cong \varepsilon k_0 u_f^2$; u_f is the average filtration velocity through the jam body.

/71/ By integrating Eq. (4.1a) from $x = 0$ (for $h_1 = h$) to $x = L$, we obtain:

$$h_1 = A - (A - h) \exp - \left(\frac{ML}{\sigma_\kappa B} \right). \qquad (4.2)$$

Eq. (4.2) also describes the regularity of the growth of thickness of the submerged section of the block accretion along the length L_0 of the jam body.

By analyzing Eqs. (4.1a) and (4.2) for $M \gtrless 0$ and $A \gtrless h$, it is possible to determine the following.

1. For $M > 0$ and $A > 0$, the derivative of (4.1a) is positive and consequently, the accretion thickness increases with an increase in L from the smallest value $h_1 = h$ at the upper boundary of the section L_0 (i.e. at $L_0 = 0$) to the maximum $h_1 = h_m$ at the lower boundary. With an increase in the value L, the exponential function exp- $(ML/\sigma_k B)$ in Eq. (4.2) tends toward 0, while $h_m \rightarrow A$. Such a limit is practically achieved in a flow section L_0 relatively limited in length. With the condition that the expontneial function is equal to zero, the length of this section is found by the expression:

$$L_0 = \frac{m_3 B \sigma_\kappa}{2\eta \rho_0 \sigma_\kappa' - \gamma_1 H (i - \eta_1 i_p)}. \qquad (4.3)$$

where the coefficient m_3 may be taken equal to 5-6.

Eq. (4.3) obviously determines the limiting value of the length of the body of the developed jam with a maximum thickness of the ice accretion:

$$h_m = Bp/M. \qquad (4.4)$$

Two cases result under the conditions that $M > 0$ and $A > h$:

(1) $h_m > H_1$ and $i_3 < i_1$, where H_1 and i_1 are the level head and slope for the formed jam core, which are determined according to Eqs. (3.8) and (3.9). In this case, the jam body is formed with an increase in the thickness of the ice accretion in the core and a continuous decrease of slope therein, which takes place due to shifts, ramming, and packing of the ice mass in the body and head of the jam. In this case, the jam section L_2 is formed (see Figs. 6b, 7, 8c);

(2) $h_m \leq H_1$. The formation of the jam takes place smoothly without packing and shifting of the ice.

The first case, in agreement with the classification used in this study corresponds to the formation of the developed jam, while the second is of the transitional type.

Therefore section L_2 is formed in the developed jam after the thickness of the ice accretion in front of the core exceeds its height and brings about a packing of the ice under the jam body, a compressing of the ice mass, and shifting to the core side. The respective increase in the level head at the core for some value Δh in turn creates a supplemental volume of ice accretion within the limits of the jam body: $W_1 \approx L_0 \Delta hB$.

/72/ Since the thickness of the ice accretion h_1 decreases upstream, only a portion of this volume goes into the formation of the jam body, while the remaining portion in the volume $W_2 \approx L_2 h_m B$, obtained on the basis of very simple geometrical considerations, goes into the formation of the jam section L_2. $W_2 = kW_1$, where $k < 1$ may be written, and then:

$$L_2 = kL_0(1 - H_1/h_m) \qquad (4.5)$$

According to the results of some experimental investigations, the coefficient k is on the order of 0.5.

2. For $M > 0$ and $A \leq h$, the derivative $dh_1/dx \leq 0$, so that the highest value $h_1 = h$ occurs at $L = 0$. We conclude from this that a single-layer ice accretion is formed above the jam nucleus and this has the smallest longitudinal compression $\sigma_m < \sigma_k$ and a constant minimum thickness of the ice accretion equal to the block thickness $h_1 = h$. This case corresponds to the formation of the undeveloped jam.

3. For $M < 0$, the derivative in (4.1a) is positive, and the value h, in agreement with Eq. (4.2) monotonically increases with an increase in L from h to infinity. At the same time, the value L_0 is negative, as seen from Eq. (4.3). For such a case, static equilibrium is impossible due to the continuous increase in the ice thickness at the jam head. However, this process is accompanied by a decrease in the slope at the core, which, under conditions of an unlimited height of the jam may be reduced to $i_3 < p_0/\gamma$ B. In this case, the value M becomes positive and the further formation of the jam will proceed as a function of the ratio between A and h, according to the first or second of the examined cases.

4.2. Conditions of Static Equilibrium of Jams

The relationships A > h, i.e., B > p_0 - p_3B) h/p, and A $\leq$ h in Eq. (4.2) together with the inequalities obtained from Eq. (2.19), correspond to the concepts of a wide and a narrow river, respectively, introduced by Canadian researchers /15/, but are more general, determining the possibility of an increase along the length of the current of both stress and ice thickness, and also the type of jam, if in parameter A the value i is determined according to Eq. (3.9).

The equilibrium conditions of non-filtering ice accretions of a constant thickness of the front edge and at a distance from it have already been subjected to analysis /5, 15, 49, 50/. The equilibrium condition of jams as uniform formations of specific, size, structure, and properties comprised of individual units - blocks - is examined below. In this connection, the equilibrium conditions of a free-flowing body (1.6) - (1.12) are supplemented by regularities and special features of the properties ot the investigated object, reflected in Eqs. (2.9) - (2.11), (2.14), (2.19), (4.2), etc. In this case, the conditions of static equilibrium of the jam will include the following requirements:

/73/ 1. In each cross section of the jam, in agreement with Eq. (2.4), the condition M > 0 is fulfilled, where the pressure on the shore is determined according to the dependences: (a) $\sigma_k = \mu_1 h_1$ for developed jams; (b) $\sigma_k' = \eta a \Delta\gamma l_1$ for undeveloped jams; so that for $i_y = 0$, we have the expressions:

$$Bi_s < \mu_0 h_a; \quad (4.6a)$$

$$Bi_s < a_0 l_1, \quad (4.6b)$$

where $\mu_0 = 2\varphi\beta\zeta\mu_1/\gamma;\ a_0 = a_1\ (\gamma/\gamma_1 - 1);\ a_1 = 2\varphi_1\beta\eta\zeta a;$

a and l_1 are determined according to Eq. (2.20).

The average slope of the water surface in the developed jam, obtained from the geometric structures, is equal to

$$i_s = \frac{1}{L_0}\left(\frac{q}{r u_a} - h_m\right) + i_a. \quad (4.7)$$

and for the undeveloped jam, it is determined according to Eq. (3.9) with the substitution there of the average depth value according to Eq. (3.10).

2. On the lower surface of the jam body coming into contact with the current below the ice, the current speed is equal to:

$$u \leq u_a = u_{s,v} \quad (4.8)$$

where $u_{s,v}$ is determined according to Eqs. (2.17), (2.18), for $\tan\ \theta = \phi$.

3. At the upper boundary of the jam, where a single-layer block accretion is formed, the stability conditions are described by the following equations:

(a) The static state:

$$A_1 \leq h; \quad (4.9a)$$

$$\sigma_n < q_k; \quad (4.9b)$$

(b) The dynamic state (in the collision of blocks floating up to the edge of the jam):

$$u \leq u_k = u_{ps}; \quad (4.10a)$$

$$v < v_2, \quad (4.10b)$$

where $$A_1 = Bk_\sigma u_k^2 [p_n - \gamma_1 B (t_1 - \varphi t_v)];$$

σ_k is determined according to Eq. (2.20) and u_s according to Eq. (2.6).

We note that requirement (4.10) is already provided by condition (4.9), since in the derivation of Eq. (4.12) on the upper boundary of the jam, the depth H_0 at which $u = u_k$ was fixed. Eqs. (4.9) and (4.10) are examined in developed form in Section 4.4.

4. The jam stability (as a single body) vis-a-vis shifting along the current bed is guaranteed:

(a) By condition (1.12) for jams at transverse obstacles;

(b) By the relationship $\phi_1 \geq \phi$, while for $\phi_1 < \phi$ by equation:

$$L_p \geq \frac{B\mu_1 h_k}{\varphi_1 (\varepsilon\gamma_1 + i\beta_\omega^2 \mu_0 h_p)}, \tag{4.11}$$

/74/ for developed and freely filtering jams, where $L_p = 0.5m_1 a_2 L_1 + m_2 L_2$; $m_1 = 0.4 - 0.7$; $m_2 = 0.7 - 0.9$ are coefficients which consider the action of the filtration current through the jam; $a_2 = 0.9 - 1.2$ is the coefficient of the form of the bottom slope of the jam. $L_2 = 0$ in Eq. (4.11) for undeveloped jams.

It may happen that the equilibrium condition (4.10a, b) is not observed if the pressure of the single-layer block accretion adjacent to the jam increases to $\sigma_m > \sigma_k$ (with a current velocity of up to $u > u_k$) due to the large slope in this section or for other reasons. This causes a thickening of the ice at the upper jam section, as well as an additional rise in the water level, determined from the equilibrium condition by the equation:

$$\Delta h = \frac{\sigma_m}{\mu_0} - h. \tag{4.12}$$

The process of ice thickness increase by plunging, piling up, and shifting of the ice is gradually spread downstream - to the jam head. A new equilibrium position is established for the altered jam parameters. The nature and size of these changes depends on the jam form and the value Δh. In this case, the undeveloped jam acquires a transitional character, while the latter becomes developed. The packing of ice in the head section of a developed jam first leads to upstream shifting of the section L_0 and a respective lengthening section of L_2 by the value:

$$\Delta L = \Delta h / i_0. \tag{4.12a}$$

For $\Delta h > i_0 L_0$, the thickness of the ice accretion increases at the jam head, causing an additional level rise there:

$$\Delta h_m = \sigma_m/\mu_0 - i_0 L_0 \tag{4.12b}$$

At the same time, the slope of the water surface decreases on section L_0 which is found in a new position - shifted upstream. The value of this slope is determined according to Eq. (4.7) for $h'_m = h_m + \Delta h_m$ and H'_0 for σ_k and u_k, calculated according to the equations examined in Chapter 2.

When the slope is reduced to a critical value:

$$i_k = 2\eta\beta\sigma_c/\gamma_1 B \tag{4.13}$$

the process of ice thickness increase in section L_2 terminates, and only an increase in the length of this section can take place after this as well as a corresponding removal upstream of the section L_0. The noted phenomena of conversion of one type of jam to another, the growth of section L_2, and of the ice thickness at the head of the jam were recorded in model investigations.

/75/ 4.3. Level Head and Other Characteristics of Jams Under Stationary Conditions

It is not hard to see from the examined regularities of jam formation that the maximum level head H_m occurs in the head section of jams, and over a period of time coincides with the termination of their formation, i.e., with the occurrence of the equilibrium state. Special features in the form, structure, filtration properties, and equilibrium conditions of various types of jams evidently predetermine the necessity of applying a nonuniform procedure for evaluating the level head. For this is taken the data Q, i_0, p, ϕ, ϕ_1, the dimensions and form of the channel, the dependence B = f (H), and also the calculated block characteristics: l, h, h_{min} and $\sigma_{i,s}$.

In undeveloped jams, where the L_0 and L_2 sections are absent, the equilibrium conditions relate directly to the formed nucleus and the maximum level head will be determined by the water depth at the core. This depth is expressed by Eq. (3.10) for $\Theta = 0^{\circ}$, and the level head:

$$H_m = H_n - H_b = \frac{q}{v H_n} + \nu h + h_{min} - H_b, \qquad (4.14)$$

where H_b is the actual depth up to the jam formation.

The slope of the water surface at the jam and the longitudinal dimensions of the core may be determined from the dependences given in 2.4 and the equilibrium conditions (4.11).

Freely filtering, developed jams under equilibrium conditions comply with Eqs. (4.6) - (4.10). The maximum value of the level head here will correspond to the maximum thickness h_m of the submerged section of the ice accretion in the jam, which is situated at the boundary of sections L_0 and L_2. For determining this value, we have the basic equation (4.2) in agreement with which:

$$H_m = h_m - H_b = \frac{Rp}{M} - H_b, \qquad (4.15)$$

and also Eq. (4.7), and with a breakdown in the equilibrium conditions (4.10) - the relationship (4.13). For purposes of obtaining a closed system of equations for given hydraulic and ice-movement characteristics, it is necessary to have an equation for determining the filtration pressure. However, due to the insufficient study of this phenomenon, its value can be established only empirically or semi-empirically. The results of our tests show that in a first approximation, the filtration pressure can be evaluated according to the equation $p_5 = k'_0 u_f^2$, where it is assumed that the resistance coefficient $k'_0 = k_0$ is determined according to Eq. (1.18), and the average filtration velocity $u_f \approx \varepsilon q/h_m$.

The dimensions of the jam sections L_0 and L_2 are determined according to Eqs. (4.3), (4.5), and (4.7), established for the case of prismatic channels with constant bottom slopes

/76/ and roughness. In natural channels, changes in the morphological and hydraulic characteristics along the length of the river caused corresponding deviations of the dependence $h_1 = f(L)$ from an exponential type curve described by Eq. (4.2), creating local increases and decreases of the accretion. These latter may be called secondary cores. Such deviations were clearly traced in the investigation of jam formation in channel models with a variable depth, slope, and roughness.

In jam sections L_1 and L_2, the hydraulic slope l increases, forming a depression curve. The value of this slope in freely filtering jams should assure surmounting hydraulic resistance to the passage of the filtration discharge $Q_f = Q$. With an increase in the length of the L_2 section, the filtration path increases and evidently there exists a limiting length of this section which corresponds to the given values h_m and L_1, at which a sufficient filtration through the jam is still assured. The precise determination of this value is difficult for a number of reasons, even for jams with constant hydraulic and filtration characteristics. The approximate limiting value of section L_2 can be shown by the equation:

$$L_{2\,\mathrm{lim}} = \frac{h_m}{I_0} - \frac{H_3}{r I_0} - L_1, \qquad (4.16)$$

where I_0 is the averaged hydraulic slope.

A further increase in the L_2 length can lead to a decrease in the porosity coefficient, and consequently the filtration as a consequence of the packing of the ice accretion which arises under the condition $T > F_2$.

The determination of the porosity coefficient and consequently k_f under field conditions is one of the basic difficulties that arise in the jam calculation, since this value depends on a number of ambiguously and nonuniformly acting factors. Thus, under water and ice pressure in a jam, as a consequence of ice settling and fluidity, the coefficient ε decreases, while under the thermal and mechanical action of the filtering water, it increases, but decreases in the freezing of the blocks under conditions of negative air temperature, regelation, etc. For the short-term equilibrium existence of the jam with no freezing of the blocks, we may take in a first approximation $\varepsilon = 0.20 - 0.25 = \text{const}$, as a consequence of some interbalancing of the given phenomena. However, this question requires further investigation. A decrease in the filtration coefficient may lead to a conversion of the freely filtering jam to a slightly filtering one with the formation of sub-ice flows under the ice accretion (or within it) (see Section 3.5).

With slightly filtering jams, i.e., under the conditions $Q > Q_f > 0$, the value of the level head should be greater than the thickness of the block accretion jam determined according to
/77/ Eq. (4.15). For determining the level head in this case, as an initial position, which has been confirmed by experiments, it is

assumed that in the head section of the jam, the blocks are distributed at a density that guarantees the filtration of the entire water discharge for a given accretion thickness h'_m and section length $L_1 + L_2$. Then the value of the level head is determined from the equation:

$$H_m = h_m - H_b \qquad (4.17)$$

The value h'_m is based on the following arguments. The head section of the jam $L_1 + L_2$ may be examined from the viewpoint of hydraulic engineering as a filtering dam with a vertical upper slope and an acute-angled bottom slope - with angle α of slope to the bed. Filtration through the jam head has a turbulent nature and Chezy's formula may be taken in the following generalized form for determining u_f with some degree of approximation, as is found in the calculation of filtration through a stone talus:

$$u_f = C_{ob} \varepsilon \sqrt{h'_m I}, \qquad (4.17a)$$

where C_{ob} is the generalized Chezy coefficient which is a function of the dimensions of the block, the form, structure and density of their packing; I is the hydraulic gradient.

By comparing Eq. (4.17a) with the linear filtration formula $u_f = k_f I$, by introducing the conditional concept of a "nonlinear filtration coefficient"

$$k_f = (C_{ob} \varepsilon) \sqrt{h'_m / I}, \qquad (4.17b)$$

it is possible to formally arrive at the Darcy law in the form $u_f = k'_f I$ for a quasi-constant value k_f. By substituting Eq. (4.17b) for k_f in the formula for determing q_f through an earth dam with a vertical front slope /79/, we obtain the dependence for estimating h'_m in the form:

$$(h'_m)^2 = \frac{2q\,(L_1 + L_2 - h_a \cot \alpha)}{C^2_{ob}} \Big] \sqrt{\frac{I}{h_m}} - h_a^2, \qquad (4.18)$$

where h_a is the depth in the tapering of the depression curve on the bottom slope, which is determined according to known hydraulic methods /79/.

In a first approximation it is possible to take $h_a \approx (1.2 - 1.5)\ H'_b$, where H'_b is the actual depth below the jam. The coefficient C_{ob} in the assumed structure k'_f and $h_m > 0.2$ m depends primarily on the block forms and their packing method /28, 79/.

We obtained a value of C_{ob} from 1.5 for free filtration to 0.4 - 0.5 for slight filtration with paraffin plates. Such values should be somewhat higher for the field tests.

In developed non-filtering jams, the sub-ice flows (or flow) are retained, as has been mentioned, sufficient for passage of the entire water discharge at the current velocity under the

/78/ ice accretion u_k, which assures the condition of its strength (4.8) u_k, which assures the condition of its strength (4.8). With a wide rectangular channel, the sub-ice flow in the jam section L_0 is characterized hydraulically as pressure head movement. The depth of the current downstream is reduced as a consequence of the increase in the ice accretion thickness in section L_0, while the velocity increases (see Fig. 6c). The possibility of an increase in the velocity $u_k = u_{s,v}$ while observing Eq. (4.8) is assured in agreement with Eqs. (2.17a) and (2.18a) by an increase in the slope angle θ of the blocks and the density of their packing.

To sum up, on the upper boundary of a non-filtering jam, there is established a maximum sub-ice average flow depth H_0, determined according to Eq. (2.10) for $\theta = 0^{\circ}$, as in the case of filtering jams. The minimum value of this depth H'_0 is established in the line with $h_1 = h_m$, i.e., on the lower boundary of section L_0. It is possible to distinguish two cases here:

(1) In transitional-type jams, formed without shifts and packing down, i.e., for $h_m < H_1$, this depth is determined according to Eq. (3.8) at a critical current velocity, calculated for the block slope angle equal to the angle of internal friction (rupture) of the ice accretion under water, i.e. for $\tan \theta = \phi$;

(2) In the formation of developed jams as a consequence of the shifting and packing of the ice mass, the blocks in the head section of the jam can assume almost a vertical position, densely pushing against each other. The strength of the accretion considerably increases and may exceed the internal resistance determined according to the law of free-flowing bodies (1.26), and the stability of the lower surface of the accretion increases and may oppose current velocities considerably larger than the critical one for $\tan \theta = \phi$. Correspondingly, the depth of the sub-ice flow decreases. According to theoretical investigations /43, 50, 73/, the maximum degree of filling with ice of the useful cross section is diverse; from

0.4 to 0.8. However, filtration properties of the jams were not considered or were evaluated only very approximately.

The establishment of a reliable method for determining the minimum H'_0 in the examined case requires a special experimental-theoretical study. The maximum critical velocity u_k and the minimum depth H'_0 at the jam head may be estimated approximately from the equality of the initial and final specific energies of section L_0 taking into consideration Eq. (1.16) according to the following equations:

$$u'_k = \sqrt{2gh_m + u_k}; \qquad (4.19a)$$

$$H'_0 = \frac{q}{\varepsilon u'_k}, \qquad (4.19b)$$

where u_k is the critical velocity at the upper jam boundary at the depth H_0.

/79/ It follows from Eq. (4.19) that in developed non-filtering jams, there is a connection between the values h_m, L_0, and $\Delta H = H_0 - H'_0$. Elementary geometric structures permit it to be expressed in the form:

$$L_0 = \frac{h_m - \Delta H}{i_0 - i_3} \qquad (4.20)$$

A combined solution of Eqs. (4.2), (4.7), and (4.20) permit obtaining the value h_m and the head level:

$$H_m = h_m + H'_0 - H_b \qquad (4.21)$$

It is possible to note that in the case $H'_0 < H_k$ (where H_k is the critical depth), the sub-ice flow is converted to a turbulent state. In this case, directly below the jam or in section L_1 there arises a hydraulic jump, which joins the sub-ice flow with the water level in the jam under water.

4.4. Jams with a Limited Formation and Jams under Nonstationary Conditions

The jam characteristics examined above refer to jams with a complete (unlimited) formation (see Section 3.2) under stationary conditions of the water/ice-movement regime. Under natural conditions, the jam formation may terminate earlier than would follow from the examined regularities, for a number of reasons; these include:

1. The termination of ice movement or in the general case the reduction of its intensity to $S_p \leq S_0$, as a consequence of which, the jam development may terminate at any intermediate stage. For the given function $S_p = f(t)$, the value h_m may be determined according to the formulas given above, from the ice volume going into the jam formation.

2. The insufficient length L'_1 of the jam section, when $L'_1 < L_1$, where L_1 corresponds to conditions of a jam unlimited in length. Such jams can arise in shallow sandbanks, in ice fields, etc. The value h_m for jams with a bottom slope in this case is determined by the equation:

$$h_m = a_2 L'_1 \tau + h, \qquad (4.22)$$

where a_2 is the same as in Eq. (4.11).

3. The insufficient length of the flow section for positioning the developed jam body. For example, this length may be found in the presence of large channel turns directly above the jam section with $S_p > S_0$. In such a case, the maximum possible jam body length $(L_0 + L_2)$ will be equal to the length of the rec-

tangular section of the river above the jam core.

/80/ 4. The insufficient height of the obstacle (banks) in the jam section when an outlet path for water and ice is formed at some head level H'_m. Such a case was recorded, for example, in a jam on the Nieman R. /53/.

5. The insufficient strength of the obstacle which crowds the channel when Eq. (1.12) breaks down at some head level $H''_m < h_m$ (jam in structures of reduced outlay, in a section with the ice cover, etc.). The maximum value of the head in this case will correspond to the moment of the appearance of the disruptive pressure on the obstacle σ'_p, caused by the weight of the ice jam or the water and ice pressure. For freely filtering jams unlimited in length, the maximum thickness h_m is approximately determined according to the value of the maximum active pressure:

$$h_m \sim \frac{\sigma_p}{\mu_1}. \qquad (4.23)$$

Special formulas that determine the value of ice pressure on a structure provide a more exact solution /6, 25, 36, 39, 51/. For jams which form on the ice cover, not resting against the bottom, the dimensions of section L_1 and L_2 (and also the value H_m) should correspond to the condition

$$0{,}5L_1 + L_2 = \frac{0{,}4\sigma_p h_0^2}{b h_m}, \qquad (4.24)$$

where h_0 is the thickness of the ice cover.

Eq. (4.24) is corrected after establishing the physical constants in the mk (force) s system. It is investigated from conditions of stability of a loaded plate and holds true for such a speed of load buildup on the ice cover D_h (see Eqs. 1.1, 1.5), when σ_u is not a function of this factor - according to Dzhellinsk's data /59/ - for $D_h > 0.5$ kg/(cm^2·s).

6. The breakdown of the condition of stability (4.11) of the jam head upon dislocation along the flow bed as a consequence of: (a) An increase in τ with an increase in h_m (jams in bends, in channel constrictions, on ice layers, etc.); (b) A decrease in $\phi\sigma$ as a consequence of the rise of the jam from low-lying dams and other jams, or due to a decrease in filtration through the jams because of freezing and deformation of the blocks. For a preliminary evaluation of h_m in a channel constriction of length l_s with a shore convergence angle α_s, the approximate equation, which we suggest, may be used in the mk (force) s system:

$$h_m \sim \frac{L_s(B-2L_s \operatorname{tg}\alpha_s)}{91\,[B-1,3L_s\,(1-\sin\alpha_s)]}, \qquad (4.25)$$

and for other types of jams in the case of their limited equilibrium - Eq. 4.11.

/81/ 7. A change in the throughput of the jam section as a consequence of: (a) A change in the water discharge of the jam with a rise and fall of the high water; (b) The emergence or a change in the force of wind pressure on the blocks (piling up and floating of ice, including wind surges of the jam with reverse ice transit, observed, for example, in the Ob R. lower section).

The force of wind pressure may be taken into consideration by including it in the composition of active forces P in the equilibrium equation (4.1). The formation of this jam with a variable ice discharge is related to the case of a non-stationary regime examined below. It should be noted that the qualitative aspect of the influence of most of the causes examined above was noted earlier by F. I. Bydin /12/.

The special features of jam formation under the conditions $dq/dt \neq 0$ are clarified by an analysis of Eq. (3.4), which will take the following form with constant values for S_p, n, u_v, k_f, and $H_{n?} = 0$:

$$dS_{0n} = \frac{\partial S_{0n}}{\partial H_p} dH_p + \frac{\partial S_{0n}}{\partial q} dq. \qquad (4.26)$$

As shown in Chapter 3, with an increase in H_p the quantity of ice entering into the jam S_{0n} decreases, reaching zero when its formation is terminated, due to a reduction in the energy of pressure T and an increase in the work of resistant forces $A_{2,3,4}$, i.e., in Eq. (4.26), the derivative $\partial S_{0n}/\partial H_p < 0$, while the derivative $\partial S_{0n}/\partial q > 0$, since with an increase in discharge, the energy of pressure T increases and work $A_{3,4}$ decreases. Summing up, it can be said that:

1. At $dq/dt > 0$, three cases can occur:

(1) $\partial S_{0n}/\partial H_p > S_{0n}/\partial q$ - the slow formation of the jam at higher levels H_p and a larger volume of ice in it, W, than in the case q = const.;

(2) $\partial S_{0n}/\partial H_p = \partial S_{0n}/\partial q$ - unlimited (continuous) development of the jam at constant intensity;

(3) $\partial S_{0n}/\partial H_p < \partial S_{0n}/\partial q$ - the breakdown of an earlier formed jam as a consequence of the increase in throughput of

the jam section according to the condition $S_P < S_0$.

2. For $dq/dt < 0$ an acceleration in the jam development will occur with smaller level rise and ice volume in the jam. The effect of the following conditions:

$$du_v/dt \neq 0; \quad dk_f/dt \neq 0; \quad dS_P/dt \neq 0.$$

on the jam formation can be examined in a similar way.

With a wind surge (or intensifying) downstream, the energy of block pressure T increases and the jam is formed for large values of level rise and ice volume, while with a reversal of its direction - slowly with smaller values of H_P and W, and with a large u_v value, the sign of T can change to the negative,
/82/ which causes the breakdown of the jam with the emergence of a reverse ice transit - upstream.

With a decrease in the filtration coefficient as a consequence of freezing and fluidity of the ice, the level rise increases at the jam, the work $A_{2,3,4}$ is reduced, and the throughput of the jam increases or its rise increases. For $S_0 > S_P$ or $T > A_{2,3,4}$, the jam breaks down. An increase in the filtration coefficient due to ice thawing and other phenomena causes a reduction in H_p, which increases $A_{2,3,4}$ and leads to an accelerated formation of the jam with small level rise and ice volume (see also section 4.3).

Chapter 5

Practical Assumptions for Calculation and Regulation of Jam Formation

5.1. Hydromechanical Principles of Counteracting Jams and Methods for Jam Control

The question to be examined is a part of the total problem of controlling the ice movement for water management purposes. The struggle against jams to prevent the harmful effects that accompany them is of basic importance; such phenomena are:

(1) The rise in the water level with the flooding of the territory around the bank;

(2) The static and dynamic ice pressure against the shore and hydraulic structures;

(3) The formation of a "tidal wave" saturated with the ice mass to the extent of a mud-laden torrent during a rapid breakdown of the jam;

(4) A reduction in the water discharge and level below the place of jam formation, which may cause interruptions in the work of water management equipment in certain cases.

At the same time, in specific circumstances it is expedient to form artificial jams in certain sections of the flow or to limit the jams by certain dimensions for the purpose of inundating flood plains, maintaining or reducing the intensity of ice movement, etc. Such measures belong to the field of jam regulation and encounter additional difficulties, since their theory and practice necessitates considerable development.

The measures used for controlling jam formation and for counteracting the harmful consequences of various types of jams under diverse conditions cannot be identical. The drawback to this position is that unsatisfactory and even contradictory results tend to appear, such as has been shown, for example, by F. I. Bydin /12/ applicable to the case of poor human intervention of the ice cover at bridges without taking into consideration the action on the dynamics of ice movement under the actual conditions of the given object.

/84/ A haphazard, not reasonably thought out action directly on the jam using explosives and bombing may not cause a breakdown, but rather a strengthening of the jam due to packing and subsequent ice filling, an increase in the total area of crushed ice coupling with the bed, and a decrease in filtration through the jam. This in turn may cause an additional level rise, which decreases work $A_{2,3,4}$ and facillitates the piling up of new blocks. To sum up, the strength and dimensions of the jam increase according to Eq. (1.9).

Correctly placed explosives, primarily at the jam head may be very effective under certain conditions /3, 12, 68/. For example, if there is a sharp decrease in filtration through the jam with the use of explosives, then it can break down as a consequence of the rise and loss of stability of the bottom slope for $\tan \alpha > \phi$ or of its entire head section in the breakdown of Eq. (4.11).

The general principle for combatting jams and controlling them, as can be concluded from the regularities examined above and the mechanism of jam formation, consists of a fully controlled change in the ratio between the factors (forces and parameters), which cause the emergence and development of jams on the one hand, and the jamless ice movement regime on the other. The question of which parameters, where and when, to what degree and what form of action to take, should be solved independently in each case. This principle of the struggle against jams in essence was expressed by Bydin in 1961 /12/.

For general purposes and for overall control, the measures for combatting jams may be classified into three groups:

(1) Preventive - banning jam formation;

(2) Liquidating measures - breakdown of formed (or forming) jams;

(3) Control measures - The limited development of a jam by allowing a certain level rise or ice pressure to exist, changing the rate of formation (or breakdown) and the place of jam formation, as well as an artificial jam formation at a specified place.

The determination of a more effective method of applying these measures should be based on an analysis of the causes and mechanism of formation of the examined jam, the flow hydraulics, the channel structure, the ice movement conditions, etc., and also the expected damage from the jam or its beneficial effect taking into consideration technical-economic possibilities.

There is now available a relatively large number of publications concerning questions with regard to testing, effectiveness, and principles of various engineering measures for counteracting jams.

The established quantitative regularities of jam formation allow some generalizations to be made and hydromechanical principles for the examined measures to be given as a part of

/85/ the theory for managing ice jams. The problem is solved by working out a system of hydromechanical criteria that determine the conditions of the emergence, development, and termination of jam formation properties. The fully controlled change in the ratio between values entering into the criteria may permit controlling the process in the desired direction.

Three groups of criteria have been developed, which determine the conditions of emergence (I), the degree of development (II) and the jam size (III), as well as the conditions of their limitation and breakdown.

I. The emergence (prevention) of jams in accord with the rheological equations (1.1) - (1.2) and equilibrium conditions (1.2) - (1.12), is determined by the ratios between the values entering into Eqs. (3.2a, b, c). They are expressed in developed form in the following three criteria subgroups.

(1) Criteria of flow section throughput with respect to Eqs. (1.12), (2.12), (3.2) and the requirements given in Chapters 2 and 3:

$$\frac{S_0}{S_p} = \frac{S_0}{(qBv)_p} \leq 1; \qquad (5.1)$$

$$\frac{h}{H_0} \leq 1; \qquad (5.1a)$$

$$\frac{\sigma_m}{\sigma_B} \leq 1; \qquad (5.1b)$$

$$\frac{\sigma_m}{\sigma'_p} \leq 1, \qquad (5.1c)$$

where σ_m is the maximum longitudinal stress (pressure) of the blocks; σ'_p is the strength limit of the jam obstacle.

The value S_0 can be determined by the theoretical formula (2.3); in a number of cases other methods are also suitable. Thus in the passage of ice through hydraulic equipment including breakdown of blocks, the throughput may be determined by the methods examined in special investigations /16, 35, 37, 47, 63/. For ice floating according to the first or second regime of ice movement (see Section 2.1), i.e. for $\sigma_m / \sigma_s < 1$, in an open channel, Eq. (2.2) may be taken with the determination of v according to Eq. (2.5). In a closed channel (movement of blocks under the ice cover, through ice bridges, the blocked up ice field, etc.), the value S_0 may be approximately determined according to Eq. (3.6).

The different combinations of values of the different criteria correspond in a given section to free (floating) ice, to partial and complete retardation (stopping) of the blocks.

(2) Criteria of emergence (prevention of formation) of a jam according to the static diagram (due to the pressure of block accretion):

/86/ (a) In the presence of a connection between the blocks and the shore according to Eqs. (2.6) and (2.19) or (4.9a)

$$\frac{2(p_0 \pm p_4)}{(k_2+F)\,C^2 H r i} \gtrless 1; \qquad (5.2a)$$

(b) In the absence of a connection with the shore, with the investigation of the equilibrium conditions for $p_0 = 0$ and $p_4 = 0$:

$$\frac{1}{[r^2 (k_1 h + k_2 L_{pr})\, C^2 H + 2\gamma_1 h L_{pr}]\, i} \gtrless 1, \qquad (5.2b)$$

or in a simpler approximate form, the calculation according to which for the Dniester R. gave a satisfactory correspondence with field data:

$$\frac{h + 0{,}02 l}{\left(\frac{1}{n} H^{2/3} + h L_{pr}\right) i_0} \gtrless 1, \qquad (5.2c)$$

where L_{pr} is the length of the straight section of the river above the place of jam formation.

(3) The criteria of emergence (prevention) of a jam under dynamic conditions (due to losses in block stability in collision) for the cases:

(a) Plunging and piling up of blocks - according to Eqs. (2.9) and (2.10):

$$\frac{a_3 \rho_2 h^2 + 0{,}6 \rho_1 h l^2}{0{,}5 r^2 \rho_1 C^2 H l i} \gtrless 1; \qquad (5.3a)$$

(b) The tightening of the blocks - according to Eq. (2.14):

$$\frac{2 n_p h l}{r^2 (A l_2 - k_2 h)\, C^2 H i} \lesseqgtr 1. \qquad (5.3b)$$

Therefore, for the emergence of a jam with the arresting of blocks that is expressed by certain ratios according to criterion (5.1), it is necessary that one of the criteria (5.2) or (5.3) be less than unity. With the corresponding values of some of these criteria, the jam is formed according to that diagram which corresponds to the least value for the criterion.

Instead of slope, the rate of flow or the discharge may be introduced into Eqs. (5.2a, b, c) and (5.3a, b), using the Chezy formula for conditions of a useful cross section. Obviously, these criteria may be expressed by the Froude number Fr, which in certain cases may prove expedient. Thus, criterion (5.2a) in such form will be:

$$\mathrm{Fr} \gtreqless \frac{p_0' \pm p_4'}{gH(k_3 + F)}.$$

II. The type of jam according to the extent of its development with respect to the rheological equation (1.3) and the equilibrium condition (1.8) is established by specific ratios between
/87/ A and h in Eq. (4.1a) or (4.9a) with a substitution there of the slope value i according to Eq. (3.8) and the depth - according to Eq. (3.10). The corresponding criterion, which may be obtained also directly from Eq. (2.6) takes the form:

$$\frac{2(p_0' \pm p_4')}{r^3(k_0 + F)C^2H_0 l_1} \gtreqless 1. \qquad (5.4)$$

When the criterion is less than unity, the undeveloped jam forms with the limiting value $\sigma_m < \sigma_k$.

III. The jam dimensions, the limit to its development according to the given value h_m and the condition of its breakdown, in agreement with the initial equations (1.3) - (1.12) are established according to Eqs. (2.5), (2.19), (4.2), and certain additional relationships examined in sections 4.1 - 4.4. They permit obtaining obtaining a system of the following total and partial criteria:

(1) The criterion of jam throughput with respect to Eqs. (5.1) and (2.5) with the substitution there of variables

corresponding to the jam regime under limiting or given (calculated) dimensions for it:

$$\frac{S_{0z}}{S_p} = \frac{\varphi_m B_z}{(\psi U v)_p}\left[u - \left(\frac{p' - p u^2}{k_z}\right)^{0.5}\right]_z \lesseqgtr 1, \qquad (5.5)$$

where S_{0z} is the jam throughput ($S_{0z} > 0$ for an incomplete retardation of the blocks in the jam, while the index "z" corresponds to the characteristics of the forming jam. For $S_{0z}/S_p > 1$, the jam dimensions decrease; $S_{0z}/S_p = 1$ corresponds to the equilibrium state of the jam, while for $S_{0z}/S_p < 1$, the jam may increase, grow upstream, or remain unchanged. For $S_p = 0$, the jam formation terminates.

(2) The criterion of hydromechanical stability (equilibrium) of the upper boundaries - "the tail section" - of the jam with respect to Eq. (4.9):

$$\frac{h(a_1 \Delta\gamma l_1 - \gamma_1 l_1 B)}{Up} \lesseqgtr 1, \qquad (5.6)$$

where $a_1 = 2\phi_1 \eta \beta \zeta a$; for a and l_1 - see Eq. (2.20).

(3) The criterion of jam body stability in each cross-section obtained from Eq. (4.6a):

$$\frac{p_{c} h_{z}}{t_{z} B} \lesseqgtr 1. \qquad (5.7)$$

For values of Eqs. (5.1a), (5.5), (5.7) of less than unity, the stability breaks down and the jam force increases. In this case, an undeveloped jam may acquire the character of a transitional jam, and a transitional jam, that of a developed one. If Eq. (5.5) is greater than unity, a packing of ice will occur (shift) with a decrease in the length of the jam body, the piling up of ice in the head section and the

/88/ possible breakdown of the jam for the corresponding values of criteria that determine the stability and strength of the jam head (see below).

Under the conditions when Eqs. (5.1a) and (5.7) are greater than unity, the jam formation discontinues or there is a decrease in the head due to ice settling. The case when Eq. (5.7) is equal to unity corresponds to the equilibrium condition of the jam body for any ice accretion thickness (breakdown in continuity Eq. (4.2)).

4. The criterion of hydromechanical stability of the jam head is established according to Eq. (4.2) in the form:

$$\frac{\mu_1 h_m^2 - t_s \gamma_1 B h_m}{B p} \lessgtr 1, \qquad (5.8)$$

where $t_s' = (t - \varphi_1 t_p)$.

When this criterion is greater than or equal to unity, the ice accretion in the head section of the jam is statically stable, while if it is less than 1, it is unstable. In this case, the jam power may increase or it breaks down, depending on the type of jam and jam obstacle, conditions of ice movement, etc.

(5) The criterion of jam head strength with respect to Eqs. (1.5), (1.12), and (1.25):

$$\frac{\sigma_3}{\mu_1 h_m} \gtrless 1, \qquad (5.9)$$

where σ_3 is the limit of block strength or jam obstacle strength. When this criterion is less than one, the jam breaks down.

(6) The partial criterion of jam stability with a rapid increase in water discharge in the form of a straight packing wave with respect to Eq. (4.6a)

$$\frac{H\,\Delta i}{\mu_0 h_m - H i_0} \lessgtr 1, \tag{5.10}$$

where Δi is the additional slope of the water surface created by the water discharge admission. If this criterion is less than unity, the equilibrium jam body is broken down, which may lead to an increase in the jam power or its breakdown, as is noted in the examination of criterion (5.8).

(7) The partial criterion of stability against head shifting of a freely filtering jam with a bottom slope, resting on the flow bed conforming to Eq. (4.11)

$$\frac{L_p(\varepsilon\gamma_1 + 2\beta\zeta\mu_2 h_m)}{\mu_1 h_m B} \lessgtr 1. \tag{5.11}$$

The stability of the jam against shifting is assured if this criterion is greater than or equal to unity, while if it is less than 1, the jam breaks down.

/89/ (8) The partial criterion of limiting the possible jam height with respect to Eqs. (4.5), (4.22) - (4.25) due to the causes:

(a) Insufficient height of the jam obstacle:

$$\frac{h_m}{H_m} \gtrless 1; \tag{5.12a}$$

(b) Its insufficient length

$$\frac{a_2 L_1 \varphi + h}{h_m} \lessgtr 1: \tag{5.12b}$$

(c) The insufficient strength of the base for jams at the edge of the ice cover

$$\frac{\varphi B\left[0.5L_0\left(1 - \frac{H_1}{h_m}\right) + \frac{h_m}{a_2\varphi}\right]}{0.4\sigma_i h_m^2} \lessgtr 1: \tag{5.12c}$$

(d) The insufficient strength of the base for jams in a constriction.

$$\frac{\beta t h_m [H - 1.3 L_3 (1 - \sin \alpha_3)]}{L (H - 2 L_3 \operatorname{tg} \alpha_3)} \gtrless 1, \tag{5.12d}$$

where H_m is determined as a function of the type of jam according to the equations given in section 4.3; H_1 - according to Eq. (3.8).

Criteria (5.12c, d) are given in the mk(force)s system. The height of the jam can increase only to the criteria values equal to unity, and the jam breaks down with larger values for criteria (5.12c), and (5.12d).

In addition to the partial criteria examined here, if necessary, additional criteria, which limit the dimensions, strength, and stability of jams for various other reasons may also be established by a similar method (see section 4.4).

5.5. Some Practical Considerations for the Calculation and Control of Jams

A quantitative analysis of jam formation in the general case may include the determination of the place, time, emergence, and dimensions of the jam. For given morphometric, hydraulic, and ice movement characteristics, such a calculation with the use of the quantitative dependences determined above will include the following sequence of operation:

(1) The determination of the location of the jam section and the moment of emergence of the jam regime for a given function $S_p = f(t)$ according to the relationships given in section 2.1, or according to values for criteria (5.1) - (5.1c), which correspond to the cases of holding back or stopping the ice blocks;

/90/ (2) The determination of the possibility of jam emergence in the jam section according to dynamic or static diagrams, according to the data in sections 4.3, 4.4, or according to the respective values of criteria (5.2a, b, c) and (5.3a, b);

(3) The determination of hydraulic characteristics in the jam section for the period of jam formation including $H = f(L, B)$;

(4) The establishment of a jam type according to the degree of its development on the basis of data given in section 4.1 or according to the value of criterion (5.4);

(5) The determination of the maximum block accretion thickness h_m and the height of the level head H_m under conditions of an unlimited jam development according to the equations given in section 4.3 or taking into consideration criterion (5.8) with its value equal to one;

(6) The establishment of the stability and strength of the jam according to the data of section 4.2 or criteria (5.6)-(5.9);

(7) The determination of the possibility of limiting the jam formation to causes examined in section 4.4 by means of an investigation of the values for criteria (5.10) - (5.12a, b, c, d);

(8) The establishment of limited jam dimensions corresponding to values equal to unity, those of criteria (5.6) - (5.12), according to which the instability of a jam is established or its development is limited;

(9) The determination of the calculated value of the head level and other characteristics (which are established in the presence of some limitations), as a minimum value of those obtained by calculation.

For a nonstationary regime, the establishment of the level head and other jam characteristics requires introducing and analyzing the respective functions $Q = f_1(t)$, $u_k = f_2(t)$ with the determination of dependence $H_m = f(Q, u_k)$, etc.

The regularities examined in Chapters 3 and 4 also permit giving a theoretical basis to the established field observations, cases, causes, and "factors" of large jam formation /2, 11, 29, 44, 68/. In the general case, jams are larger, the greater the energy and strength of the blocks in the approach to the section with an insufficient floating capacity (jam section) and the smaller this capacity, the longer the jam section, and the higher and steeper the bank.

The regularities of jam formation obtained in the present study, which characterizes them according to physical indicators and a developed system of hydromechanical criteria, as well as overall experienced gained in ice technology permit making some generalizations with regard to measures for counteracting jams.

The regularities examined in Chapter 2 and expressed in the form of the system of criteria (5.1) - (5.3b) can serve as a theoretical basis for preventive measures. It follows from the above that a jam does not arise in a given place with certain
/91/ combinations of values of the given criteria, which guarantee the

free passage of blocks or their holdup (stoppage) with the formation of a single-layer accretion.

A free jam-less ice passage, in agreement with criterion (5.1), requires the fulfillment of condition $S_0 \geq S_p$ which is supplemented for $b/B_0 > 1$ by the requirement of sufficient pressure energy at the blocks for the breakdown of the obstacle or shearing of the blocks in constricted areas. The ice passage in a closed channel (ice bridges, etc.) requires the fulfillment of the additional conditions: $u \geq u_{s,v}$ (see section 2.3) and $l \leq H$, which ... ents the wedging of tightened blocks (see section 3.3). The holding back (stopping) of blocks in a given section for $S_0 \leq S_p$ does not cause the formation of a jam for values of criteria (5.2) and (5.3) not greater than one, and also with the current velocity corresponding to Eqs. (2.10) and (2.14).

Therefore, the prevention of a jam (for $S_p > 0$) may be achieved with a decrease in S_p or an increase in the throughput of the section, $S_0 = f\ (u, H, B, R, u_v, h, n, \phi_1, \eta, \beta, \zeta)$ by means of influencing the values on which it depends, and also by changing, if necessary, the respective criteria. functions of the hydraulic characteristics of the current, dimensions, and strength of the blocks, the obstacle, etc. (see Eqs. (2.6), (2.9), (2.14), (2.20), (5.1), etc.).

Attention should be drawn to the fact that the same measures may under diverse conditions lead to opposite results. Thus, an increase in the current velocity may assure the passage of ice in constrictions due to an increase in the pressure energy, but with sufficient strength of the obstacle, it may lead to the formation of a jam due to a decrease in criteria (5.2a, b) or (5.3a) to values less than one. It follows from this that a careful analysis of the concrete conditions and the special features of the jam formation in the given section should precede the carrying out of preventive measures.

Some of the means for preventing jams, derived from the examined overall aspects are given below.

1. A decrease in the intensity of ice movement to $S_P < S_0$ may be fulfilled by holding back the ice in traps, semi-dams, etc., including artificial jams created upstream in low-risk places.

2. An increase in the throughput of the section to $S_0 \geq S_P$ by: 1) Increases in the ice thrust front B_0, the current speed, the hydraulic drop at hydraulic structures, by means of corrective works, admissions from above-situated reservoirs, etc.; 2) Decreases in the resistance of the blocks to movement by means of correcting the banks, and also decreases in the angle of their convergence and increases in the radius of channel turns in artificial currents, etc.; 3) Decreases in block size - by
/92/ their direct mechanical breakdown, injury, or indirectly - by holding back the growth of block thickness in the winter, reducing the strength of the ice cover by using radiational heat, applying chemicals, etc. /47/, by the breakdown or reduction to $\sigma_m/\sigma_p' \geq 1$ of the strength of the jam obstacle (for example, ice bridges, ice

layers, etc.).

3. A reduction in the block pressure energy $\ni$ and σ_m and an increase in the value of critical pressure σ_k for the formation of a single-layer block accretion on the jam section, including preventing the jam directly at the broken ice cover by means of (1) Reductions in the current speed and inclination at the approach to the jam section by means of correcting structures, reduction in the water discharge, formation of a level head from lower-lying head equipment, the creating of artificial obstacles, etc.; (2) Increases in the blocks' resistance to movement due to an increase in the degree of non-prismatic form and decrease in the radius of channel turn in artificial channels, etc.; (3) Increases in the longitudinal dimensions and strength of the blocks by means of a manmade premature breaking through of the ice cover, etc.

The principles for measures with regard to breakdown of the formed or still forming jams are established from the regularities examined in sections 4.1 - 5.1 and criteria (5.5) - (5.12d). It follows from an analysis of these regularities that jams will break down in the case when there is:

(1) An increase in the jam throughput in agreement with (5.5) to $S_{0z} > S_p$, including its being due to a thawing of the ice in the jam;

(2) A breakdown in the stability of the jam head according to Eq. (5.8) with the subsequent loss of ice strength in the jam or jam obstacle, determined by criteria (5.9), (5.12b),

and (5.12d), or losses in stability to shifting according to Eq. (5.11) - for jams with a bottom slope. The loss of stability of the jam head itself may be caused by a loss in stability of the jam body due to an increase in pressure forces, as well as an increase in water discharge according to Eq. (5.10);

(3) Losses in the strength of the jam head caused by other factors: manmade breakdown of the jam obstacle, collapse of ice in the bottom slope of the jam, etc.;

(4) A rise in the jam due to an additional influx of water, producing a rise in the water level under the jam.

Therefore the jam can be broken down by action on it as on a single solid body (stability against shifting, etc.) or on a block accretion (free-flowing body). In the latter case, the stability and strength of the jam, as derived from Eqs. (5.5) and (5.12) depends analogously on the characteristics presented in the examination of jam prevention.

A breakdown of undeveloped jams and of the core of forming developed jams requires a smaller energy consumption than the breakdown of fully developed jams. However, in the dynamic method of breakdown with the movement of ice along the jam,
/93/ for example, in the water passage, the developed jam breaks down, as has been shown by models, with a lower energy consumption. This is associated with the fact that the large ice mass in body of the developed jam, having obtained some acceleration and inertia, causes the jam breakdown due to an avalanche-like

movement of the ice in the direction of its head.

Of the developed jams, those possessing the greatest stability are those formed at $H < h$ and $dq/dt \leq 0$ as a consequence of the expanse of section L_2 (see Fig. 6) over the channel clogged with ice and the stability of the bottom inclination, which has a slope of $\tan \alpha < \phi$. On the other hand, jams that form at $dq/dt > 0$, $\ni$ and $T \geq A_{2,3,4}$ have less stability due to a shorter L_2 section and the limiting value $\tan \alpha = \phi$.

With a fall in the head level, caused by a decrease in the discharge under water or an increase in the filtration coefficient k_f, jams, by settling at the bottom, possess a high stability. Correspondingly, an increase in the water level above the jam, due to a change in the water discharge and filtration decreases its stability, which at $\partial S_{0n}/\partial H_p < \partial S_{0z}/\partial q$ leads to its breakdown. The same causes may provoke the breakdown of jam stability due to ice floating or an increase in the jam throughput.

The nonuniform distribution of the current speed and depth, as well as the special features of the bed relief and other causes create a nonuniform distribution of pressure and the degree of stability, and, in a number of cases the cross sectional jam strength. Therefore, in the head section of jams found completely under conditions of stable equilibrium, there are often local sections with a stressed state that is limiting or close to it where, in agreement with Eqs. (1.8) - (1.12), $\sigma = \sigma_k$ for $\tau_k = \phi\sigma$ or $\tau_{k?} = \phi_1\sigma$ and $\tau_k = k_0 u^2_k$ or $\sigma = \sigma_k$.

Here, the application of a relatively small force causes a local breakdown in equilibrium, which may be converted to a total breakdown for a jam in such a case when it would develop further due to a natural pressure increase through the flow of water and ice. Such key sections sometimes are called "jam locks". The searching out of such a section and destroying it correctly in order to evoke a "chain reaction" in the jam is the most effective and economical means of jam breakdown, confirmed by corresponding studies /48/.

Therefore, the jam may be broken down by destroying its stability and strength on the whole or on key sections by various methods derived from the analysis that has been made. We will now note some of these methods.

I. "Jam rinsing" by passage of water from an upstream reservoir as a function of the type and special features of the jam may bring about its destruction due to: (1) a breakdown in jam stability as a single body according to Eq. (5.10); (2) the rising of and floating of all the jam ice with the loss of sta-
/94/ bility against displacement according to Eq. (5.11); (3) a reduction in the internal resistance of the block accretion jam according to the condition $\tau_k > \phi\sigma$, due to block floating and a decrease in the joining of the blocks, which corresponds to the condition $dq/dt > 0$ for $\partial S_{0n}/\partial H_p < \partial S_{0n}/\partial q$.

The model investigations showed that this method is more effective for breaking down developed jams formed in sections with insufficient depth (shallow water, sandbanks). For jams at the edge of the ice cover "rinsing" is expedient only after a breakdown in the ice cover below the jam. There is established an inversely proportional and approximately linear relationship between the water discharge speed buildup rate and the maximum discharge value at which the jam breaks down.

II. A rise in the jam due to a rise in the water level underwater ΔH_b, which acts on jam stability in the same direction as the "rinsing", but creates a certain reduction in T. The necessary value for ΔH_b for jams with a bottom incline may be established from the conditions of breakdown of static jam stability. In a simpler approximate form, the respective equation is in the form:

$$\Delta H_b \geqslant H_m\left(\frac{1.1\varphi}{\tan\alpha} - 1\right). \qquad (5.13)$$

According to the data of experimental investigations, the examined method is more effective for jams of the type $H < h$ in sections with small bottom slopes or with a small L_2 section length.

III. The maintenance (regulation) of ice movement above the jam, which forms with a rise in high water for the purpose of raising the jam and breaking down its stability at $\partial S_{0n}/\partial H_p < \partial S_{0n}/\partial q$. The method may be effective for jams which form for $H < h$ and at the edge of the ice cover with a sufficiently intense increase in the water discharge.

IV. An increase in the hydraulic drop to guarantee the condition $T \geq A_p$ for jams of the type $b > B_0$ chiefly at artificial constrictions in the channel by the respective technological measures.

V. Breakdown of blocks in jams of the same type to dimensions of $b < B_0$ by mechanical, impact (explosives, bombing, etc.), acoustical, and hydraulic methods.

VI. Disruption of strength (breakdown) of the jam obstacle according to Eq. (5.9) by one of the methods named above for jams at the ice cover, in layers of ice, in channel constrictions, canals, etc.

VII. A reduction in filtration through the jam for an increase in H_p to fulfill the conditions examined in Paragraph I, in jams of the type $H < h$, in a closed channel, etc., that also may be brought about by the methods given in Par. V.

/95/ VIII. A breakdown in the stability of the bottom slope according to Eq. (5.11) by one of the methods examined in Par. V. This method will obviously be more effective, the closer $\tan \alpha$ is to ϕ and the shorter the L_2 jam section is.

IX. Breakdown in jam stability according to the same equation by action on key sections at the jam head or the formation of longitudinal channels in the jam, the direction of which correspond to the special features of the channel topography and coincide with the direction of vector $\vec{T}_m$. Such a condition may be fulfilled by the method named above. For example, a method of burning through channels by thermite loads was successfully applied by Kh. Barnes /3/ on the St. Lawrence R. The arrangement of the channels may be made appropriately under the conditions of separate stable jams of the type H h.

The following are particularly important in controlling the processes of jam formation: (1) Artificial formation of the jam in a specific flow section, and (2) A limiting of jam development to specified dimensions. There may also be an interest in controlling the time of the beginning of formation, the continuance of formation, and the existence of the jam, etc.

The theoretical basis for measures of artificial jam formation are dependences that determine the measures for predicting jams, but in an inverse numerical ratio of criteria (5.1) - (5.3). Correspondingly, the methods of action on the process of jam formation in the given case will also be directly the opposite when compared with conditions of jam prediction.

The basic measures for limiting the development of jams will be dependences examined on the basis of methods of jam breakdown; criteria (5.5) - (5.12), which determine possible methods of both breakdown and limiting of jam development, also apply here. The following can be attributed to these methods:

(1) A decrease in the intensity of ice movement to $S_P \leq S_0$ by holding the ice above the jam in semi-jams, artificial jams, etc. This method is applied in principle to jams of all types;

(2) A limitation of the length of the jam obstacle L_1 for jams with a bottom slope by corrective, explosive, and other measures to a magnitude determinable from the condition cor-

responding to the form of the bottom section of the jam:

$$L_1 = 0.9 H_m / a_3^2 \psi; \qquad (5.14)$$

(3) The arrangement of an outlet path for water and ice if the head reaches a value of H_m by clearing the banks and flood plains, the making of an outlet canal with $S_{can} \geq S_P - S_0$. With an appropriate flood plain structure, this method may be sufficiently effective and reliable. Thus, results of the investigation of jam formation at the
/96/ mouth region of the Nieman R., published by I. Ya. Nechai /53/ showed that rises in the water level caused by the formation of ice jams on the basic branch of the Niemen are insignificant due to the disperal of water from above the jam along the undammed up right-bank flood plain;

(4) A reduction in the water discharge by holding the discharge to a value(corresponding to Eq. (5.8) of less than unity, for which an increase in the level head terminates. However, in this case, the jam may continue to grow in length - upstream. This method is in principle applicable to dams of all types;

(5) The crushing of blocks that enter into the jam to a value of $b < B_0$ and $\vartheta < A_3$ with the simultaneous increase in q and a decrease in k_f or without this for the purpose of block passage along the surface of the ice accretion jam. Other methods examined above in the measures for jam breakdown are also fully possible.

It is evident that for purposes of controlling, predicting, and breaking down jams, a combination of the examined methods and measures may be applied.

5.3. Longitudinal Dimensions and Block Form

Calculating the dimensions and form of the blocks is necessary for solving a number of problems of jam formation. For example, the block dimensions enter into formulas of block and jam stability loss (2.9) - (2.18), (2.20), and (5.6), while the block form is implicitly considered by the probability coefficients which determine the block contact conditions (the λ coefficients, etc.).

At the same time there are as yet no analytical methods for calculating the dimension and form of the blocks; this results from the insufficient study of the phenomenon. The data from field tests, generalized in the form of probable or approximate analytical dependences should lie at the basis of methods for evaluating the examined values. The results of some treatments along these lines are given below.

In examining spring ice blocks as a product of the breakdown of the ice cover (or ice fields), the following should be noted:

1. In the general case, the ice cover on rivers is a complex, polycrystalline aggregate of stratified structure of nonuniform texture and strength. Four different types of layers are characteristic. The uppermost layer, the ice-snow layer, is characterized by the disorderly array of crystals and the low resistance to opening /9/. Therefore, it is of little importance

in the block mechanics, although its thickness may reach 30% or more of the total ice thickness.

The second layer, the "primary congealed ice" (according to P. A. Shumskii's terminology /85/), also of an unorderly structure, is the primary autumn formation (slush, caked ice,
/97/ etc.) of small thickness, comprising 10-15% of the total. Below this is the third layer of the secondary (orthotropic) ice formed by the crystallization of water under the primary ice in its transport through the river. The forced crystallization and geometric separating here lead to the formation of an ordered columnar structure. However, in the presence of slush under the primary ice layer, this orderliness breaks down. The thickness of the given layer can reach one-half or more of the total.

The fourth and lower layer, formed by direct crystallization of the river water, is a polycrystalline body of columnar structure. The third and fourth layers have the greatest strength that is almost the same in both cases /9, 19/. The ice cover in various rivers and river sections and in different winters may have fewer or more layers - alternate combinations of both examined above (for example, in sections of autumn ice jams), depending on hydrometeorological and other conditions. A high contamination of the liquid with various impurities causes many defects in river ice crystals, which facilitates the development of dislocation movements.

2. In the spring, before opening, the ice cover is subjected to an internal and external thawing and to an increased water pressure, which is reflected on its texture and strength. At this time, the ice prepares for its new form of existence as blocks. There takes place a partial breakdown of the intercrystalline layers in the lower ice layer due to processes of dissolution and absorption by solar radiation, which leads to a decrease in the specific surface energy. Here, heat energy alone suffices for the breakdown of a solid body along a defect network. Therefore, the lower surface of the ice cover and the blocks as well as the lateral surface is covered with developing cavities.

3. The thickness of the ice cover in rivers regularly decreases from the banks to the dynamic current axis and from the reaches to sandbanks. This is associated primarily with the nonuniform amount of heat energy, which evolves in the process of hydraulic-resistance energy dissipation. In other words, there is an inverse relationship between the thickness of the ice and the current speed, as a measure of the energy of hydraulic resistances. The ratio of the ice cover thickness on sections of the stream with different flow velocities may be expressed according to the following equation /41/:

$$\frac{h_1}{h_2} = \frac{\vartheta_1 + \frac{1}{427C_1^2}u_1^3}{\vartheta_2 + \frac{1}{427C_2^2}u_2^3}, \qquad (5.15)$$

where ϑ_1 and ϑ_2 is the heat transfer of the bottom; C_1 and C_2 are the Chezy coefficients. Eq. (5.15) may be used for evaluating possible fluctations in h.

/98/ 4. The appearance of compressive, tensile, and bending stresses in the ice cover is associated with the action of forces that are variable with respect to time and place. These include:

(1) The attractive force (1.18), which creates a tensile stress on the upper boundary of a certain ice cover section (striving to tear the cover from the above-lying section), while it creates a compressive stress at the lower boundary;

(2) The static ice pressure, which arises due to thermal expansion - contractions with daily fluctuations in temperature and those which create a uniform compression - elongation in the longitudinal and transverse directions, and the formation of thermal cracks in the limiting case;

(3) The vertical water pressure force (from bottom to top) on the ice cover, which arises with a raising up of the ice (Archimedes' force) and with an increase in the hydrostatic head, due to an increase in the water influx into the river and the formation of a temporary head. The latter leads to the appearance of bending moments with their maximum value at the shore in places where the ice cover is fastened to it;

(4) The wind pressure force, which creates tensile and compressive stresses in the longitudinal and transverse directions;

(5) The force of dynamic and static ice (block) pressure on the upper edge of the ice ice cover in the investigated

section.

Under the action of the given forces, the ice cover may be broken up due to: (a) its rupture or compression in the longitudinal and crosswise direction; (b) bending in the crosswise direction, including under the action of the gravity in the "sag" of the ice cover; (c) losses of longitudinal stability. The separate sections which form in the breakdown of the ice cover (ice fields, primary blocks) acquire a known mobility, which leads to the appearance of two new forces: (1) Inertia, and (2) Collision forces (dynamic pressure) of blocks during their collision. The further formation and reformation of dimensions and forms of the blocks, which continues up to the end of ice movement, occur under the action of these forces.

The dimensions of the individual blocks at this time are determined by the joint action of a large number of variable factors and may be examined as random. The average dimensions of the blocks under given hydrodynamic conditions are a more definite value, yielding to a known degree of quantitative analysis. For sufficiently long periods of time under conditions of the developed dynamics of the process, i.e., when the given forces play a leading role, it is not difficult to establish the existence of a close, almost functional, connection of the type $l = f(\vec{v}, h, \sigma_{i,s})$.

The presence of such a relationship, confirmed by field observations, permits expressing the horizontal dimensions of blocks as a function of thickness and strength, which are assumed

/99/ to be known. The principies of the theory of materials' resistance as well as special investigations, on the basis of which is determined the length of the blocks, preceding from the condition of their not breaking down upon collision with each other and with the banks, are used for this purpose. It is assumed that part of the impact force F_1 is consumed in the partial breakdown of the block edges according to the diagram examined in section 2.2. As a result of this, conditions of the creeping (plunging) of one block on another are facilitated. The blocks experience both bending and compressive stress in which there is consumed a second basic portion of impact strength $F = \sqrt{F_2^2 + F_3^2}$, where F_2 is the bending force, and F_3 is the simple compressive force. Other forces will be excluded from the examination, due to their known insignificance /61/, and then we can write:

$$\frac{mv^2}{2} = F_1 + F. \tag{5.16}$$

where m is the block mass.

Using a method similar to that for deriving Eq. (2.9), it is established that for average forces, with a reliability of 50%, the impact force $F_1 = h^2 \sigma_s$. The value F_2 is found from the relation known from materials' resistance:

$$F_2 = \sigma_i bh^2/6l.$$

According to investigations of block impact against an inclined surface, conducted by B. V. Zylev /27/, the results of which are applied to the examined case, the maximum compressive and bending forces may be associated with the relationship

$F_{3m} = 1.1F_{2m} \tan \beta$. Substituting in Eq. (5.16) the values of mass and forces F_1, F_2, F_3 with $\beta = 45^{\circ}$, $l = b$, and solving the equation with respect to block length, we obtain:

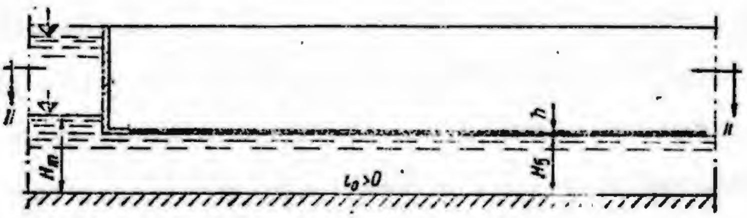
$$l = \frac{0.145}{\lambda_1 v} [h(\sigma_s + 0.24\sigma_i)]^{0.5} = \frac{0.151}{\lambda_1 v} (h\sigma_i)^{0.5}, \qquad (5.17)$$

where λ_1 - see Eq. (2.9).

A check of Eq. (5.17) showed that it gives the most satisfactory results for current speeds of 0.6 - 0.8 to 1.3 - 1.5 m/s and for ice movement of average intensity. For $\lambda_1 = 1$ and average values of h and σ, Eq. (5.17) gives average block dimensions. As field experimental investigations have shown (Fig. 12), the quantity of such blocks within deviation limits of $\pm$ 15-20% may comprise 30-60% of the total quantity of blocks in a given ice movement. Substituting in Eq. (5.17) the extreme values for λ_1, h, and σ, the limiting dimensions of blocks may be obtained. However, one must bear in mind that the physically limiting block dimensions will obviously be: the maximum $l_{max} = B_0$ and the minimum $l_{min} = h$. In addition, as shown by a statistical treatment of photographs of ice movement,
/100/ the distribution of block sizes is close to asymmetric curves for distribution of random values (with left asymmetry), while the form of the curves, like the block dimensions, changes with the passage of time.

The form of the individual blocks and their size may be examined as random values. In averaging these values, a known regularity, which has a theoretical basis, may also be examined. As has already been mentioned, the thickness of the ice cover in rivers usually decreases from the banks to the middle of the stream and from the reaches to the sandbanks. By applying in this case the tenets of the theory of plate deformation, it can be shown that with pressure on the ice cover, choked ice fields and, in general, formations of low mobility on the lower surface and upstream of the edge have the most probable breakdown: (a) parallel to the banks and (b) at an acute angle to it, along the diagonal. It follows from this that the most probable form of the blocks in a plane is close to an irregular trajectory with more acute angles for large gradients of ice thickness variation along the width of the stream. The nonuniform ice thickness along the length of rivers and various local abnormalities finally complicate the picture, but the general regular nature is evidently maintained.

Along I-I



H_b

Along II-II

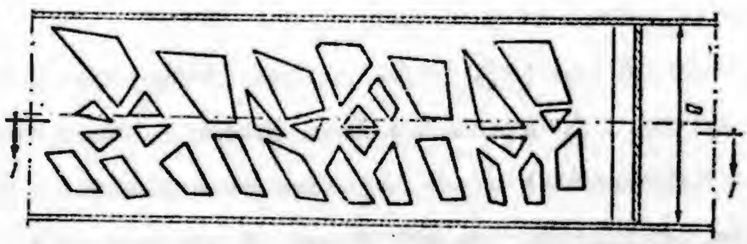


Fig. 12. Stylized diagram of the formation of blocks with a rapid (dynamic) breakdown of the ice cover due to longitudinal compression and crosswise bending.

Thus, the results of our investigations of more than 1500 average-size and large blocks (according to photographs) at the beginning of ice movement on rivers with flow velocities of 0.7 - 0.3 m/s showed the following:

/101/ (1) 50-75% of the blocks have a form in the plain that is close to an irregular trajectory, 15-25% are triangular, and 10-20% are rectangular, polyhedral, etc.;

(2) The acute angles of the blocks in 75-80% of the cases lie within limits of 40-60^{o}; 10-15% have angles of 20-40^{o}; and 5-10%, greater than 60^{o}. For obtuse angles, 75% of the blocks have 90-120^{o}, 20% have angles of 120-140^{o}, and 10% - greater than 140^{o}.

The results of field investigations (observations) agree with the data of certain experimental investigations on the breakdown of the ice cover of natural ice in a long hydraulic trough. The noted features are examined in Fig. 12, where a stylized diagram of the distribution of form and relative block dimensions is shown; these are forms in the trough with a sufficiently rapid dynamic breakdown of the ice cover with the simultaneous action of compressive and bending forces.

The given data may be utilized for probability evaluation of block forms. It should be borne in mind that these relate to "young" blocks (at the beginning of ice movement). As time goes by, the dimensions and form of the blocks change. As a rule, blocks acquire a more curvilinear profile, gradually approximating circular and ellipse forms with curvilinear angles of 90° and more. We can conclude that it is in principle possible to construct in future the calculation dependences for determining the dimensions and form of blocks for specific mechanical properties of the ice and conditions of block formation. Such a structure may be conducted along lines of elastic and plastic theory, introducing probability characteristics. Such constructions require a rather large number of special field-experimental and theoretical investigations.

/102/ CONCLUSION

Investigations of ice jams, based on principles of general physical laws of mechanics and rheology of both continuous and discrete media, indicates on the one hand the fruitfulness and the many possibilities of this method, which allows the solution of a number of important problems examined here, while on the other hand, indicates the urgent need of studying many new complex problems derived from actually combatting jams and controlling them.

While the regularities established here permit conducting a more or less reliable state of analysis of jam formation under stationary water conditions and known characteristics of ice movement, the question regarding calculations of jams under conditions of sharply expressed unstable conditions remains to a great extent unsolved. The same may be said of determining quantitative characteristics of ice movements, the conditions of its development, and changes over time, etc.

In order to solve such problems, it is necessary to consolidate the methods and laws of mechanics of continuous and discrete media with the methods and procedures of hydrology, meteorology, and also special divisions of the sciences, such as physical chemistry, ice technology, etc. This complex approach to methods of investigating ice jams should also apply to solving problems of controlling them.

It is obvious that future studies should be directed to improving and detailing mathematical models of ice jams, field and experimental methods of investigation, the treatment of methods for solving various applied problems, as well as to checking the accuracy of certain hypotheses and calculation models.

In conclusion, we think it necessary to again state that the present study is not exhaustive. The authors viewed their task only with respect to the possibility of characterizing the most important questions of the examined problem from the point of view of general physical laws, mechanics, and rheology of discrete media, comprised of individual particles - ice blocks. We note also that in following through with this concept, the particular biases and opinions of the authors have appeared to a certain extent.

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