Rheology of ice/rock systems and interfaces

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ABSTRACT: Behavior of both rock glaciers and conventional glaciers is highly affected by the character and temperature of their bottom deforming layer, usually composed of a mixture of ice and rock elements, the latter ranging from silty sand up to large boulders. Similarly as in the rock mass mechanics, large scale behavior of such mixtures is difficult to evaluate either by direct in-situ measurements or by taking samples. This paper uses some known soil and rock mechanics principles to simulate the large-scale behavior of such mixtures.

1 INTRODUCTION

Due to climate warming trends, there has been an increasing interest in recent years in the accelerating creep of rock glaciers and frozen slopes. The term "Rock Glaciers" has been used for various complex landforms of cold mountain areas, from debriscovered glaciers in permafrost-free areas, to steadily creeping perennially frozen and ice-rich debris on non-glaciated mountain slopes. In the field of glaciology, the creep of glaciers has been studied extensively, observed and analyzed for over 100 years. A brief overview of some principal findings in the last 50 years is given in the following section.

Many valuable and detailed theoretical models have been proposed through the years for simulating the creep behavior of glaciers, and this paper has no intention to propose another one. Its purpose is only to supply to these models some additional geotechnical perspectives, borrowed from the connected fields of frozen ground mechanics, rock mechanics and mechanics of mixtures. It is considered that this "crossbreeding" may be found useful for further development of glacier and rock glacier mechanics. In particular, this paper attempts to extend some known models of mechanical behavior of unfrozen soil and rock masses to those containing ice, including large-scale creep of ice/rock mixtures and the creep of ice/rock interfaces.

A brief description of basic constituents and the behavior of rock glaciers will be given in the following sections.

2 ROCK GLACIERS

According to Brown (1973), rock glaciers form under the influence of a periglacial climate in an area lacking the net accumulation of snow required for the formation of a conventional glacier. A permafrost environment must be present (mean annual air temperature lower than 0° C) to enable the snow and water, which trickles down into the interstices between the rock debris to freeze and to remain as ice. Barsch (1996) defines the term "rockglacier" as bodies of perennially frozen "normally consolidated" (in a geotechnical sense) material, supersaturated with interstitial ice and ice lenses, that move down-slope or down-valley by creep. This is thought to be a consequence of the deformation of ice contained in the rock glacier. Because of its relatively warm (close to the freezing point) temperature, Alpine permafrost is very sensitive to climate changes (Cheng & Dramis 1992, Haeberli 1993, Arenson & Springman 2000). Instabilities of Alpine permafrost are considered to be an issue of national importance in Switzerland (Haeberli et al. 1998). The loss of strength due to melting of the ice matrix might combine with an increasing supply of water, leading to stability problems within the rock glacier, which may also become a trigger for debris flow. In addition to Switzerland, rock glaciers are more widely distributed and most intensely studied in Alaska and in the Rocky Mountains bordering United States, Alaska and Canada.

In the case of rock glaciers, observations made in Switzerland by Hoelzle et al. (1998) in a borehole, DH2, drilled nearly to the depth of 40 m, show that the surface movement was mainly due to the creep deformation within the middle third of the rock glacier.

Two of probably the most intensively investigated rock glaciers, Muragl and Murtèl, are situated in the Upper Engadin, Switzerland. Recent reports on the drilling results and movement observations have been published by Haeberli et al. (1998), Arenson & Springman (2000) and by Arenson et al. (2002).

The Murtèl rock glacier contains extensive regions of nearly pure ice in the upper 10 to 25 m. The volumetric ice content in this zone is 80–90%, so that there is virtually no contact between solid particles. No such massive ice zone was found in the Muragl rock glacier. A layer of ice mixed with fine soil particles was found in the middle (29–31 m) of the Murtèl rock glacier, but predominantly larger particles were common in the Muragl rock glacier, ranging from silty sand up to large boulders of about 2 m diameter. The frozen soil was originally thought to extend to bedrock at about 50 m in Murtèl, but subsequent investigations (Arenson 2002) throw some doubt on this earlier assumption.

Borehole deformations measured with in-slope inclinometers during several years (Wagner 1992, Haeberli et al. 1998, Arenson et al. 2002) show similar movement profiles for both rock glaciers: 80-90% of the deformation happens in a bottom shear horizon, with a depth of about two to three meters (Fig. 1). In both cases borehole drilling shows that this deforming layer consists of frozen deposits, mainly of sand (Murtèl) or gravel (Muragl). As for temperatures, at the depth of shear zone, the temperature in Murtèl rock glacier is about -1.6°C (30 m depth), and only about -0.1°C in the Muragl rock glacier (17 m depth) (Arenson & Springman 2000).

As for the basal sliding of glaciers on a relatively clean but rough or undulating rock surface, it is considered to be due to the melting and regelation of ice when moving over rock asperities (Weertman 1957, Kamb & LaChapelle 1964, Kamb 1970, Nye 1970, Hallet 1981). However, due to very large normal pressures under thick glaciers, this regelation mechanism can also be active even at temperatures down to -2 to -5° C (Eichelmeyer & Zhongxiang 1987).



Figure 1. Deformation profile of Muragl rock glacier, borehole BH4 (from Arenson & Springman 2000).

In general, the total movement observed at the surface of glaciers is considered to consist of two components – internal deformation or "creep", and basal sliding, the latter being most often preponderant.

3 RHEOLOGY OF MIXTURES

In connection with slope creep and failure, including unfrozen saturated clay slopes, thawing frozen slopes, and already thawed mass sliding on frozen soil or rock beds, a basic ingredient in the analysis of the stability is the rheological behavior of mixtures rock/clay, rock/ice or rock/water. In the past, pioneering work in the area of rheology of mixtures was done in various fields, including general rheology, soil mechanics, rock-fill mechanics, and frozen soil mechanics. A general conclusion in all these studies was clearly that the behavior of a mixture depends on the character of its solid and liquid constituents and on their relative proportion in the mixture.

In the area of frozen soil mechanics, pioneering work on sand/ice mixtures was carried out by Goughnour & Andersland (1968), Kaplar (1971), and Baker (1979) (Fig. 2), and the findings were summarized by Ting et al. (1983) (Fig. 3). A conclusion from all these studies is that, up to a grain volume concentration of about C = 0.4, the pore ice governs the behavior, while for C > 0.4, frictional resistance mobilized between particles intervenes, and, finally, for C > 0.6, the dilatancy caused by interlocking of densely packed particles also adds to shear strength of the mixture.

It will be seen in Fig. 3 that the strength of icesaturated sand/ice mixtures, for 0 < C < 0.65, can be approximated by the formula (Ladanyi 2002):

$$q_{u,mix} = q_{u,ice}(1 + 14.1 C^4) \tag{1}$$



Figure 2. Effect of total moisture content on the unconfined compressive strength of a frozen fine sand at -12° C, and at a strain rate of $2.2 \times 10^{-6} \text{ s}^{-1}$ (after Baker 1979).

This formula implies that rate and temperature effects on the strength of a mixture are the same as those for pure ice.

On the other hand, as shown in Figure 3, when a dense sand/ice mixture becomes increasingly ice-poor,



Figure 3. Variation of unconfined compressive strength of frozen Ottawa sand at -7° C, and at a strain rate of 4.4×10^{-4} s⁻¹ (modified from Ting et al. 1983).



Figure 4. Relationship between uniaxial compressive strength and temperature for a frozen sand at different total water contents (after Kuribayashi et al. 1985).

the ice cohesion decreases, and eventually the strength of the mixture reverses to that of dry sand.

This is also clearly seen in the results of direct shear tests with sand/ice mixtures, published by Nickling & Bennett (1984), and in Figure 4 by Kuribayashi et al. (1985). Note that the experimental results shown in Figure 4 can be approximated by the empirical relationship (Andersland & Ladanyi 1994):

$$q_u(\text{kPa}) = 15.5 \ w(\%) \ (\theta + 15^{\circ}\text{C}) - 1373$$
 (2)

where q_u denotes the uniaxial compression strength, w is the total water or ice content, and $\theta = -T$ °C is the freezing temperature of the mass. Equation (2) is valid for 6 < w < 30%, and $10 < \theta < 30$ °C. These relationships may help in estimating the effect of temperature change on the behavior of both ice-saturated and ice-unsaturated frozen slopes and frozen bottom debris of glaciers.

4 FROZEN SLOPE CREEP DUE TO INTERNAL DEFORMATION

4.1 *Ice-saturated mass with low solids content* (C < 0.4)

Similarly to Nye (1970) and McRoberts (1978), Andersland & Ladanyi (1994, Chap. 8) have shown a solution of creep of an infinite frozen slope sliding down a plane, making an angle β to the horizontal. Assuming that the creeping layer is solidly attached to its bottom, and that creep of the mass is governed by the creep power law, valid for an ice-saturated mass (C < 0.4), with no internal friction:

$$\dot{\gamma} = \dot{\gamma}_c \left(\frac{\tau}{\tau_{c\theta}}\right)^n \tag{3}$$

where $\dot{\gamma}$ is the shear strain rate, τ is the shear stress, and $(\dot{\gamma}_c, \tau_{c\theta})$ are experimental creep parameters for shear creep. Note that, expressed in terms of uniaxial compression data, and assuming the validity of the von Mises flow rule, if $\tau_{c\theta} = \sigma_{c\theta}$, then (e.g. Andersland & Ladanyi 1994, p.134):

$$\dot{\gamma}_c = 3^{(n+1)/2} \dot{\varepsilon}_c \tag{4}$$

The effect of temperature on $\tau_{c\theta}$ can be expressed by Equation 5:

$$\tau_{c\theta} = \sigma_{c\theta} = \sigma_{co} \left(1 + \theta/\theta_o\right)^w \tag{5}$$

where $\sigma_{c\theta}$ is the reference stress at the reference strain rate, $\dot{\varepsilon}_c$ for temperature $\theta = -T^\circ C$, σ_{co} is the reference stress at strain rate $\dot{\varepsilon}_c$ when *T* tends to $0^\circ C$, $\theta_o = 1^\circ C$, and *n* and *w* are creep and temperature exponents, respectively. For example, according to Morgenstern et al. (1980) and Andersland & Ladanyi (1994), for polycrystalline ice, the above parameters would have the values: n = 3, w = 0.37, and $\sigma_{co} = 0.103$ MPa at strain rate $\dot{\varepsilon}_c = 10^{-5}$ h⁻¹.

The result, expressed as the creep velocity V_x at any depth 0 < z < D in the slope direction, can be written as (Andersland & Ladanyi 1994):

$$V_x(z) = \frac{\dot{\gamma}_c}{n+1} \left(\frac{g\rho_m \sin\beta}{\tau_{c\theta}} \right)^n \left(D^{n+1} - z^{n+1} \right)$$
(6)

where g is the acceleration of gravity and ρ_m is the density of the mass. This equation defines the creep velocity curve shown schematically in Figure 5 (Andersland & Ladanyi 1994). Differentiating Equation 6, the maximum shear strain rate at z = D is:

$$\dot{\gamma}_{max} = \dot{\gamma}_c \left(\frac{g\rho_m \sin\beta}{\tau_{c\theta}}\right)^n \tag{7}$$

It is assumed that slip will occur at the interface (z = D) when the maximum shear strain rate attains a critical value, $\dot{\gamma}_{max} = \dot{\gamma}_{crit}$.

4.2 Dense ice-saturated mass (C > 0.4)

The preceding theory is strictly valid only if the ice content of the mass is so high that the solid fragments do not interfere with the flow, i.e. C < 0.4. In the opposite case, when the solid fragments contribute to the shear strength of the mass, by giving it internal friction, the theory should be correspondingly modified.

As explained in Andersland & Ladanyi (1994), there are two simple methods for taking into account internal friction in a shear creep analysis. The first one, valid for a relatively warm frozen mass, containing a large percentage of unfrozen water, and which tends to consolidate under pressure, the friction angle ϕ may remain approximately constant, while only the



Figure 5. Notation for creep of an infinite slope on bedrock (after Andersland & Ladanyi 1994).

cohesion, c, will be affected by temperature and strain rate. In that case, a good approximation of the observed behavior, in terms of σ and τ , would be Coulomb's law, in terms of total stresses,

$$\tau = c(t, \theta) + \sigma \tan \phi \tag{8}$$

with:

$$c(t,\theta) = \frac{\sigma_{fu}(t,\theta)}{2N_{\phi}^{1/2}}$$
(9)

where:

$$N_{\phi} = \frac{1 + \sin\phi}{1 - \sin\phi} \tag{10}$$

and:

$$\sigma_{fu} \equiv q_u = \sigma_{c\theta} \left(\frac{\dot{\varepsilon}_1}{\dot{\varepsilon}_c} \right)^{1/n}$$
(11)

Translated into shear creep information, this allows Equation 3 to be replaced by:

$$\dot{\gamma} = \dot{\gamma}_c \left[(\tau - \sigma \tan \phi) 2N_{\phi}^{1/2} / \tau_{c\theta} \right]^n$$
(12)

Repeating the previous development for slope creep, the following equation is found, instead of Equation 5:

$$V_{x}(z) = \frac{\dot{\gamma}_{c} \left[2N_{\phi}^{1/2} / \tau_{c\theta} \right]^{n}}{g\rho_{m} \sin\beta(n+1)} \left[(Dg\rho_{m} \sin\beta - \sigma \tan)^{n+1} - (zg\rho_{m} \sin\beta - \sigma \tan\phi)^{n+1} \right]$$
(13)

This solution is valid only for $\phi > 0^\circ$.

Conversely, for an ice-rich, cold frozen soil, where both cohesion and friction are affected by temperature and strain rate, the shear strength τ can be expressed by (Andersland & Ladanyi 1994):

$$\tau = (\dot{\gamma}/\dot{\gamma}_c)^{1/n} (\tau_{c\theta} + \sigma \tan \phi_c)$$
(14)

where ϕ_c is the slope of the Coulomb line corresponding to the same strain rate as the one which determines $\tau_{c\theta}$, i.e. $\dot{\gamma} = \dot{\gamma}_c$. In that case, Equation 3 becomes:

$$\dot{\gamma} = \dot{\gamma}_c \left(\frac{\tau}{\tau_{c\theta} + \sigma \tan \phi_c}\right)^n \tag{15}$$

The slope creep solution, replacing Equation 6 is then:

$$V_x(z) = \frac{\dot{\gamma}_c}{n+1} \left[\frac{g\rho_s \sin\beta}{\tau_{c\theta} + \sigma \tan\phi_c} \right]^n \left(D^{n+1} - z^{n+1} \right)$$
(16)

and is valid for $\phi_c \ge 0$.

Following Vallejo (1981, 1989), for ice/rock mixtures, ρ_m in these equations should be replaced by ρ_{mix} , given by:

$$\rho_{mix} = \rho_i + C(\rho_s - \rho_i) \tag{17}$$

where ρ_i is the density of pore ice and ρ_s that of the rock fragments.

5 SHEAR STRENGTH OF ICE-FILLED INDENTED ROCK JOINTS

For basal sliding of glaciers, the studies made by Ladanyi & Archambault (1977, 1980) on filled, regularly and irregularly indented joints may be of some interest. In two recent papers, (Ladanyi 2002, Ladanyi & Archambault 2003), their theory was applied to ice filled rock joints to calculate the relationship among the shape of irregularities, the thickness of ice filling, the temperature and the shear strain rate. The solution makes it possible to predict the effect of temperature change on the rate of sliding along a rock joint or an irregular rock surface.

6 CONCLUSIONS

The purpose of this paper was to extend to frozen slopes and the frozen debris at the bottom of glaciers and rock glaciers, certain findings and methods borrowed from the fields of rock mechanics and the behavior of mixtures. The analysis shows how the ice component may be included in these methods, leading to the solutions potentially useful for analyzing the changes in the rate of movement of frozen slopes under temperature change.

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