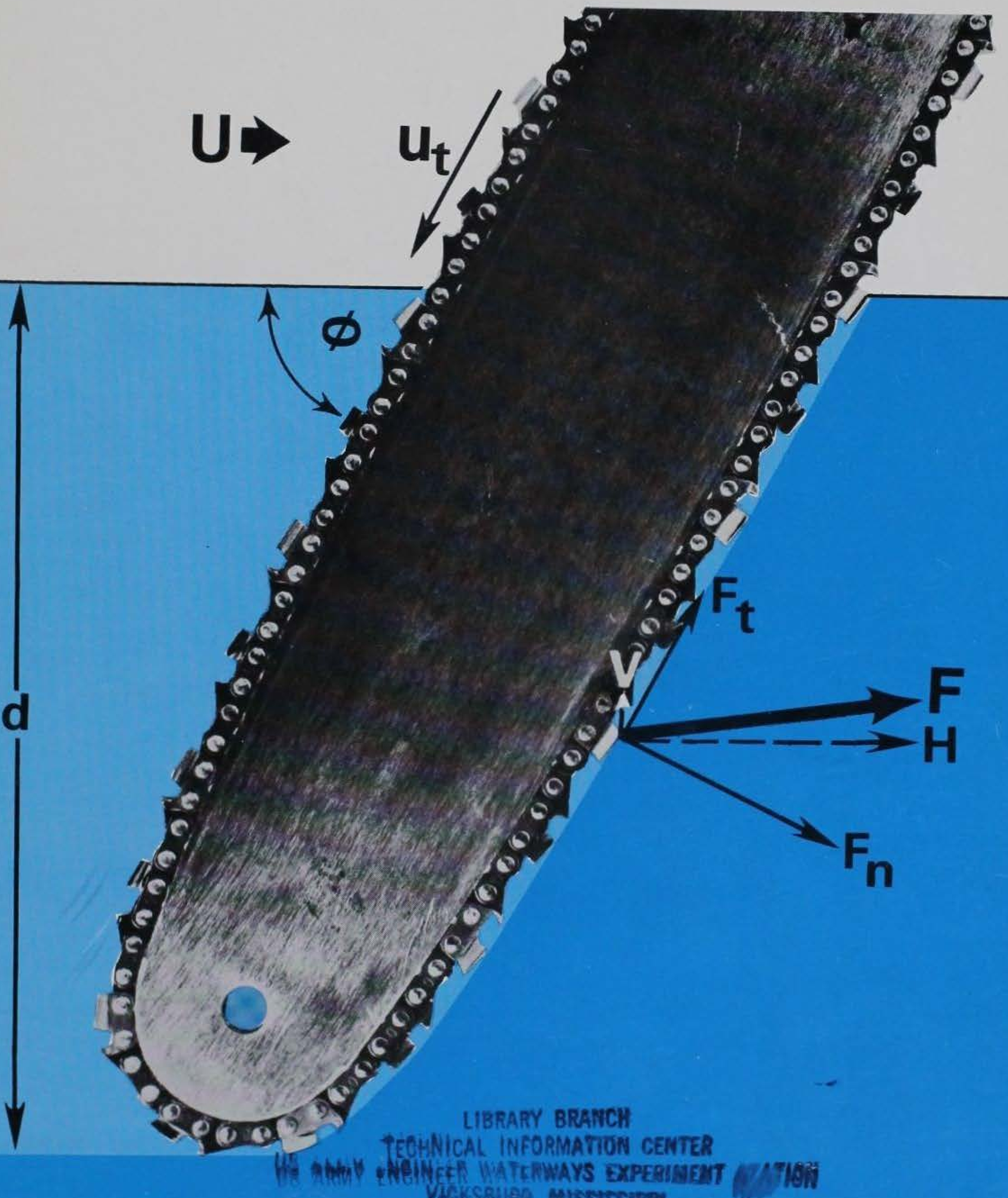
CRREL

REPORT 78-11

Mechanics of cutting and boring

Part VIII: Dynamics and energetics of continuous belt machines



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Mechanics of cutting and boring

Part VIII: Dynamics and energetics of continuous belt machines

Malcolm Mellor

April 1978

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The report deals with forces and power requirements for cutting machines of the belt type, as exemplified by large chain saws and ladder trenchers. The forces of single cutting tools are considered, and related to the overall forces on a cutter bar. Forces are related to power, and sources of loss are identified. Tractive thrust and normal reaction are analyzed and used to assess the traction, weight and balance factors for self-propelled machines. Specific energy consumption and performance index are treated, and concepts of power density and apparent belt pressure are introduced. Requirements for acceleration of cuttings are assessed, and the report concludes with a set of worked examples.

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PREFACE

This report was prepared by Dr. Malcolm Mellor, Physical Scientist, Experimental Engineering Division, U.S. Army Cold Regions Research and Engineering Laboratory. The work was done under DA Project 4A762719AT42, *Design, Construction and Operations Technology for Cold Regions,* Technical Area 02, *Soils and Foundations Technology,* Work Unit 004, *Excavation in Frozen Ground.*

The author is grateful to Donald Garfield and Paul Sellmann for their advice and insight on some of the technical matters discussed in the report. He has also benefited from numerous conversations with makers, dealers and operators of ditching and mining equipment. Technical review of the manuscript was provided by Paul Sellmann and Dr. Haldor Aamot.

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MECHANICS OF CUTTING AND BORING

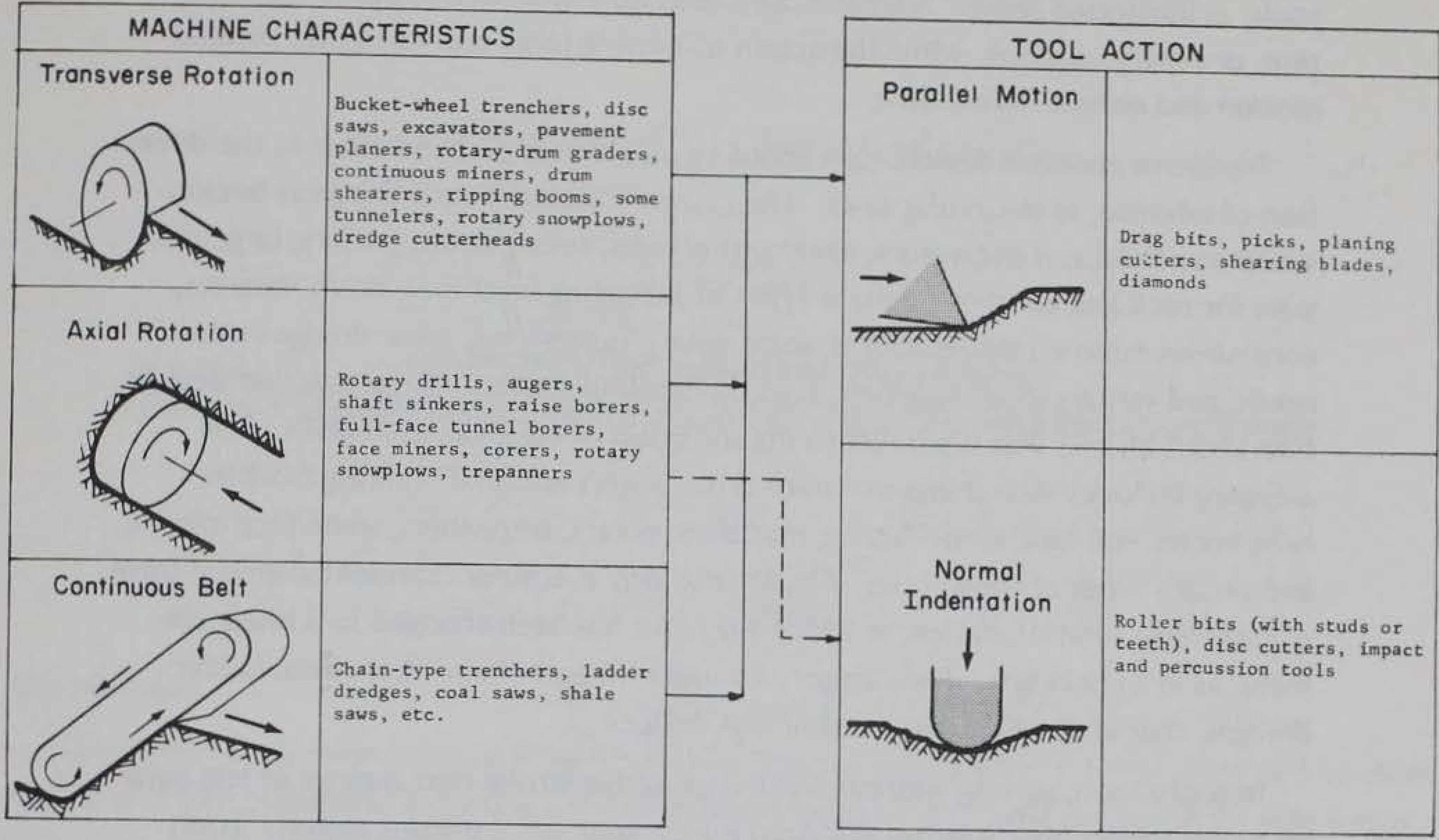
FOREWORD

There are a multitude of tasks that involve the cutting, drilling, or excavating of natural ground materials and massive structural materials. The required technology varies with the properties of the materials and with the scale of operations, but a broad distinction can be made on the basis of the strength, cohesion, and ductility of the material that is to be worked. In weak materials that have little cohesion (e.g. typical soils) the forces and energy levels required for separation and disaggregation are often small compared with the forces and energy levels required for acceleration and transport, and materials handling technology dominates the consideration. By contrast, in strong materials that exhibit brittle fracture characteristics (e.g. rock, concrete, ice, frozen ground) the forces and energy levels required for cutting and breaking are high compared with those required for handling the broken material, and the technical emphasis is on cutting and breaking processes.

CRREL has long been concerned with excavating and drilling in ice and frozen ground, and over the past decade systematic research has been directed to this technical area. The research has covered a wide range of established technologies and novel concepts but, for short term applications, interest has necessarily centered on special developments of proven concepts. In particular, there has been considerable concern with direct mechanical cutting applied to excavation, cutting, and drilling of frozen soils, glacier ice, floating ice, and dense snow. During the course of this work, numerous analyses and design exercises have been undertaken, and an attempt is now being made to develop a systematic analytical scheme that can be used to facilitate future work on the mechanics of cutting and boring machines.

In the industrial sector, rock-cutting machines are usually designed by applying standard engineering methods in conjunction with experience gained during evolution of successive generations of machines. This is a very sound approach for gradual progressive development, but it may not be appropriate when there are requirements for rapid development involving radical departures from established performance characteristics, or for operations in unusual and unfamiliar materials. A distinct alternative is to design more or less from first principles by means of theoretical or experimental methods, but this alternative may not be practically feasible in its more extreme form.

There are numerous difficulties in attempting a strict scientific approach to the design of rock-cutting machines. The relevant theoretical rock mechanics is likely to involve controversial fracture theories and failure criteria, and to call for detailed material properties that are not normally available to a machine designer. Direct experiments are costly and time-consuming, and experimental data culled from the literature may be unsuitable for extrapolation, especially when (as is sometimes the case) they are described by relationships that violate the basic physics of the problem. Comprehensive mechanical analyses for rock-cutting machines have not yet



Classification of machines and cutting tools for analytical purposes.

evolved, and while established design principles for metal-cutting machine tools may be helpful, they do not cover all pertinent aspects. For example, there are usually enormous differences in forces and power levels between machine tools and excavating machines, and force components that can be almost ignored in a relatively rigid machine tool may be crucial design factors for large mobile rock cutters that are highly compliant.

In dealing with cold regions problems where neither outright empiricism nor highly speculative theory seem appropriate, some compromise approaches have been adopted. While simple and practical, these methods have proved useful for analysis and design of cutting and boring machines working under a wide range of conditions in diverse materials, and it seems possible that they might form the basis for a general analytical scheme. The overall strategy is to examine the kinematics, dynamics and energetics for both the cutting tool and the complete machine according to a certain classification, adhering as far as possible to strict mechanical principles, but holding to a minimum the requirements for detailed information on the properties of the material to be cut.

Kinematics deals with the inherent relationships defined by the geometry and motion of the machine and its cutting tools, without much reference to the properties of the material being cut. *Dynamics* deals with forces acting on the machine and its cutting tools, taking into account machine characteristics, operating procedures, wear effects, and material properties. *Energetics* deals largely with specific energy relationships that are determined from power considerations involving forces and velocities in various parts of the system, taking into account properties of the materials that are being cut.

These mechanical principles are applied in accordance with a classification based on the characteristic motions of the major machine element and the actual cutting tools, as illustrated above. Machines are classified as *transverse rotation, axial rotation*, or *continuous belt,* while the action of cutting tools is divided into *parallel motion* and *normal indentation.*

Transverse rotation devices turn about an axis that is perpendicular to the direction of advance, as in circular saws. The category includes such things as bucket-wheel trenchers and excavators, pavement planers, rotary-drum graders, large disc saws for rock and concrete, certain types of tunneling machines, drum shearers, continuous miners, ripping booms, some rotary snowplows, some dredge cutter-heads, and various special-purpose saws, millers and routers. *Axial rotation* devices turn about an axis that is parallel to the direction of advance, as in drills. The category includes such things as rotary drills, augers and shaft-sinking machines, raise borers, full-face tunnel boring machines, corers, trepanners, some face miners, and certain types of snowplows. *Continuous belt* machines represent a special form of transverse rotation device, in which the rotor has been changed to a linear element, as in a chain saw. The category includes "digger chain" trenchers, ladder dredges, coal saws, shale saws, and similar devices.

In tool action, *parallel motion* denotes an active stroke that is more or less parallel to the surface that is being advanced by the tool, i.e. a planing action. Tools working this way include drag bits for rotary drills and rock-cutting machines; picks for mining and tunneling machines; teeth for ditching and dredging buckets; trencher blades; shearing blades for rotary drills, surface planers, snowplows, etc.; diamond edges for drills and wheels; and other "abrasive" cutters. *Normal indentation* denotes an active stroke that is more or less normal to the surface that is being advanced, i.e. one which gives a pitting or cratering effect such as might be produced by a stone chisel driven perpendicular to the surface. Tools working this way include roller rock bits for drills, tunneling machines, raise borers, reamers, etc.; disc cutters for tunneling machines; and percussive bits for drills and impact breakers.

A few machines and operations do not fit neatly into this classification. For example, certain roadheaders and ripping booms used in mining sump-in by axial rotation and produce largely by transverse rotation, and there may be some question about the classification of tunnel reamers and tapered rock bits. However, the classification is very satisfactory for general mechanical analysis.

Complete treatment of the mechanics of cutting and boring is a lengthy task, and in order to expedite publication a series of reports dealing with various aspects of the problem will be printed as they are completed. The main topics to be covered in this series are:

1. Kinematics of transverse rotation machines (Special Report 226, May 1975)
2. Kinematics of axial rotation machines (CRREL Report 76-16, June 1976)
3. Kinematics of continuous belt machines (CRREL Report 76-17, June 1976)
4. Dynamics and energetics of parallel-motion tools (CRREL Report 77-7, April 1977)
5. Dynamics and energetics of normal indentation tools
6. Dynamics and energetics of transverse rotation machines (CRREL Report 77-19, August 1977)
7. Dynamics and energetics of axial rotation machines
8. Dynamics and energetics of continuous belt machines.

MECHANICS OF CUTTING AND BORING
PART 8: DYNAMICS AND ENERGETICS OF CONTINUOUS BELT MACHINES

by

Malcolm Mellor

Introduction

This report deals with forces and power requirements in continuous belt machines such as chain saws and ladder trenchers. It parallels the content of Part 6 of this series, which deals with forces and power levels in transverse-rotation rotary cutting machines. In the following treatment it is assumed that the cutter bar has a simple geometry. Slack belts on widely separated suspension points are not treated explicitly, and effects of nose curvature are often neglected when the treatment is for a "long" cutter bar. If it should be necessary to analyze more complicated cutter bar geometry, the general principles outlined for the kinematics and the dynamics of transverse rotation machines and continuous belt machines can be combined to obtain solutions.

Kinematic factors pertaining to continuous belt machines are covered in Part 3 of the series. The dynamics and energetics of parallel motion tools, which are the kind invariably fitted to continuous belt machines, are dealt with in Part 4.

Terminology

Basic terminology of continuous belt machines is given in Part 3. A few additional terms are given below.

Tooth forces, tool forces or *cutter forces* are the forces developed by the individual cutting tools. The resultant force on a single tool fluctuates with time in response to formation of discrete chips in brittle material, but in this report the concern is with time-averaged cutting forces, without reference to pulsations. On the straight section of a cutter bar the tool force does not vary systematically with tool position in homogeneous material. The resultant force can be resolved into a tangential component f_t that is parallel to the face of the bar, and a normal component f_n that is perpendicular to the face of the bar (Fig. 1). These components are approximately equal to the components resolved parallel and perpendicular to the advancing work surface, and in most practical cases it is unnecessary to make a distinction. In some cases there may be a side force perpendicular to the plane of the overall cut.

Bar force is the resultant force F that acts on the bar as a consequence of the cutting action. Assuming that side forces balance out, the bar force F can be resolved into orthogonal components F_t and F_n that are respectively parallel and normal to the working face of the bar (Fig. 2). The bar force also can be resolved conveniently into components that are parallel and normal to the direction of travel; these components are designated F_h or H for the parallel direction, and F_v or V for the normal direction (Fig. 1). The resultant bar force can be assumed to act at the center of the working face.

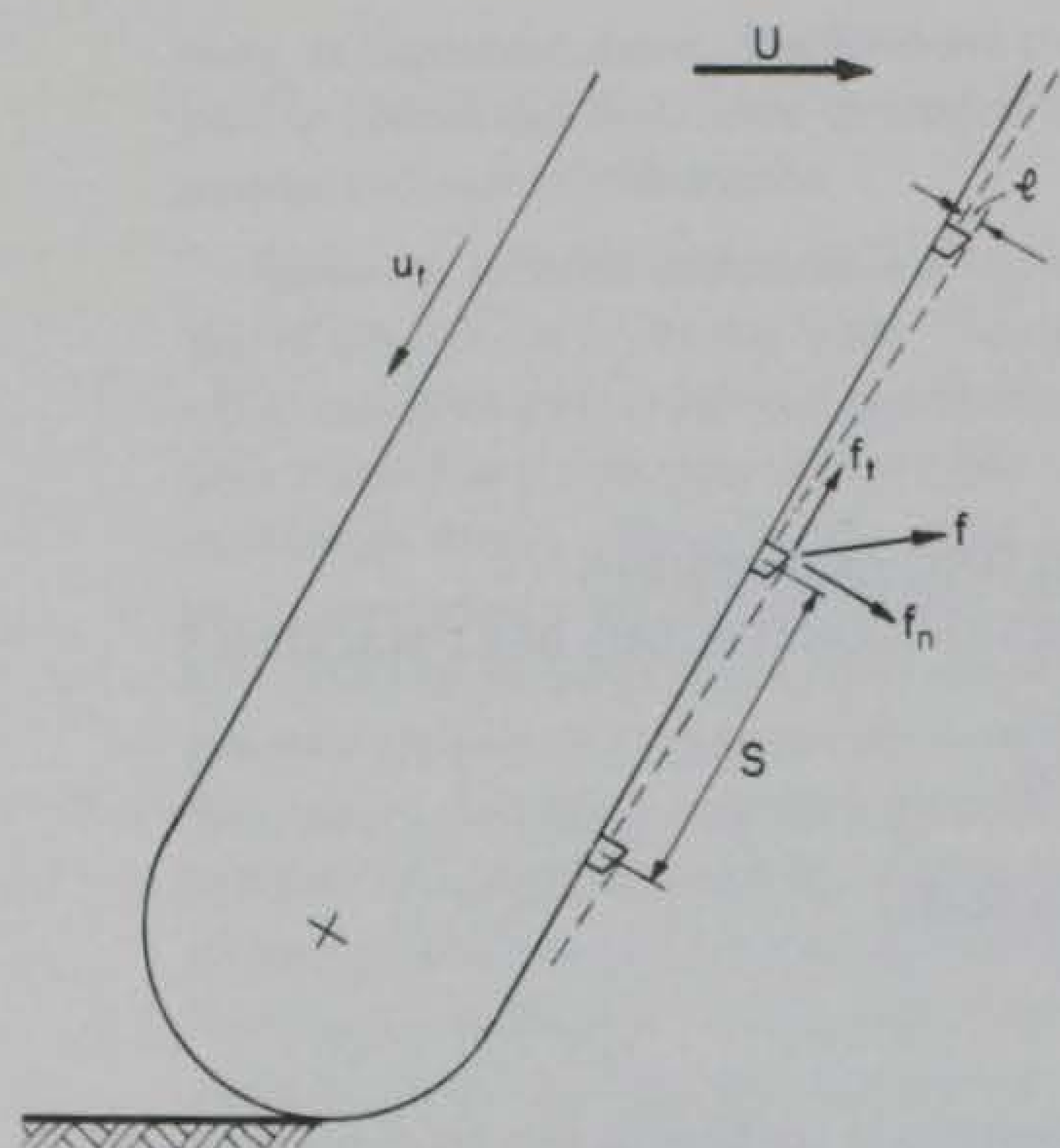


Figure 1. Resolution of tool forces.

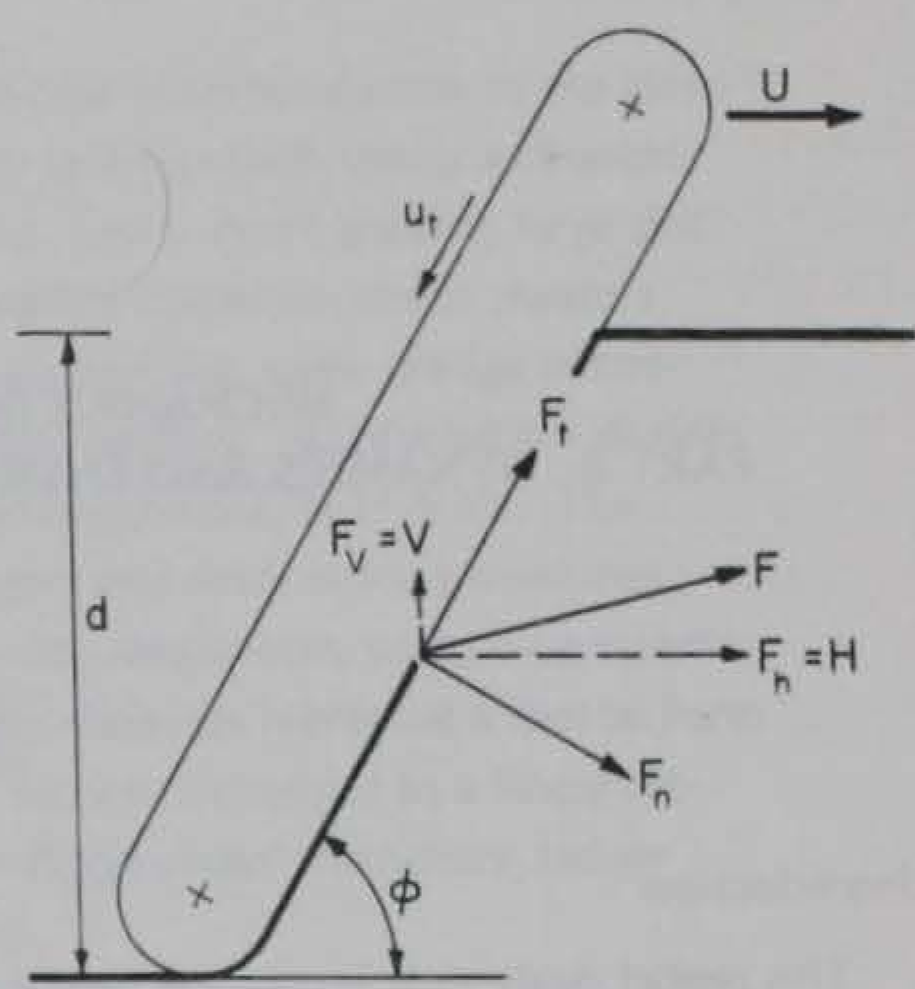


Figure 2. Illustration of forces on the cutter bar.

Cutter bar moments are the moments developed by cutting forces and by deadweight when the bar is mounted in the form of a cantilever. Moments about the pivot points of the bar actuating mechanism can be significant in design of that mechanism. Moments about various points on the carriage or carrying vehicle affect design of those elements.

Tractive thrust is the force required to propel the cutter bar in the traverse direction against cutting resistance. It is equal to the "horizontal" component of bar force H. When the cutter bar is mounted on a self-propelled vehicle, the tractive thrust is the net forward force developed by the wheels or crawler tracks, i.e. the "drawbar pull."

The *normal reaction* of the cutter bar is the force that acts normal to the traverse direction and the primary free surface in opposition to cutting forces. It is equal to the "vertical" component of cutting force V.

Apparent belt pressure for a long bar p_B is the normal component of bar force F_n divided by the approximate area of the working surface BL (or $Bd/\sin\phi$). It is proportional to the power density Q divided by the belt speed u_t.

Chain force F_c is the force developed in the chain or belt by cutting forces and internal friction. The maximum value of F_c, developed at the tension side of the drive sprocket, is made up of the tangential cutting force F_t plus the internal frictional drag produced by guides and rollers. The internal friction may be thought of as comprising a basic drag F_d plus a drag component that is proportional to the cutting forces, expressed here as μF_n.

Chain power P_{BT} is the total power supplied to the belt or chain at the drive sprocket. It is used partly to overcome internal frictional resistance within the belt system, and partly to cut and transport the work material against external resistance. Chain power is equal to chain force multiplied by chain speed, $P_{BT} = F_c u_t$.

Thrust power P_H is the power required to traverse the cutter bar against external cutting resistance. It is equal to the tractive thrust H multiplied by the traverse speed U.

Power loss in the present context is power that does not contribute directly to the cutting process. Power losses occur in the transmission systems of the bar and the carrier vehicle, and there are further losses in overcoming internal friction in the bar/belt system, and in vehicle tracks. Power used to accelerate cuttings, to transport cuttings against external resistance, or to run the belt against external fluid drag is usually considered as part of the cutting power. It is not always feasible to isolate all sources of power loss for analytical purposes.

Power density Q is defined here as power per unit area of the working surface. For a long bar, on which the effects of nose curvature can be ignored, the power density is approximately $Q = P \sin \phi / Bd$. For general comparison of different machines, it is convenient to define a nominal power density Q_N in terms of the gross belt power P_{BT} and the maximum rated area of the working surface $(BL)_{max}$, i.e. $Q_N = P_{BT}/(BL)_{max}$.

Specific energy for a cutter bar is the energy consumed per unit volume of material removed, or alternatively the power consumption divided by the volumetric excavation rate. An overall specific energy for a complete machine can be calculated from the total power divided by the volumetric excavation rate. However, it is more illuminating to estimate a process specific energy E_s for the cutting operation alone. The simplest procedure is to estimate E_s on the basis of total belt power P_{BT} such that $E_s = (P_{BT} + P_H)/\dot{v}$. Depending on analytical needs, process specific energy can also be based on net belt power P_B plus thrust power P_H, or on cutter power $F_t u_t$ plus thrust power P_H. Going further, power consumed in accelerating and transporting cuttings can be separated out, as can power consumed in overcoming any fluid drag that might exist.

Performance index is a dimensionless number obtained by dividing the specific energy E_s by the uniaxial compressive strength of the work material σ_c. The ratio E_s/σ_c is intended to characterize performance, such that the capabilities of different machines working in various materials can be compared.

Tool forces

A continuous belt machine invariably uses parallel motion tools, such as drag bits, shearing cutters, or abrasive grains. The dynamics and energetics of such tools are covered in detail in Part 4, and application to transverse rotation machines was discussed in Part 6.

Tool forces are usually resolved into orthogonal components that are normal and parallel to the direction of tool motion. On a continuous belt machine, this is approximately the same as resolution normal and parallel to the working surface of the belt if U/u_t is small. For present purposes this approximation will be accepted, but it should be kept in mind that conventional tool force resolution really ought to be with reference to the tool trajectory, as defined in Part 3. The distinction is important when U/u_t approaches unity; this condition arises in the analysis of slipping drive tracks on crawler vehicles, but it would rarely be applicable to cutting devices.

Following the lines of Part 6, the normal and tangential components of tool force, f_n and f_t respectively, can be related to chipping depth ℓ in a general way by

$$f_n = k_n(\ell/r)^a \qquad (1)$$

$$f_t = k_t(\ell/r)^b \qquad (2)$$

$$f_n/f_t = (k_n/k_t)(\ell/r)^{a-b} \tag{3}$$

where k_n and k_t are proportionality constants with dimensions of force (representing tool geometry and rock properties), r is the tip radius of the tool (used as a normalizing factor to make ℓ dimensionless), and a and b are dimensionless exponents (fractional).

Eq 1-3 are only approximate empirical relations, and in many cases the experimental data from cutting tests can be represented adequately by linear relations of the form

$$f_n = A_n + k_n(\ell/r) \tag{4}$$

$$f_t = A_t + k_t(\ell/r) \tag{5}$$

where A_n and A_t are constants with dimensions of force, representing tool force components as ℓ tends to zero.

For narrow tools, or tools that are cutting deeply in soft materials, it may be sufficient to assume simple proportionality between tool force and chipping depth:

$$f_n = k_n(\ell/r) \tag{6}$$

$$f_t = k_t(\ell/r). \tag{7}$$

The chipping depth ℓ varies with position as the tool sweeps around the curved nose of the cutter bar, reaching a maximum and constant value as the tool moves onto the straight working face of the bar. While the tool sweeps along the straight section of the bar or belt, the chipping depth is

$$\ell = (U/u_t)\, S \sin\phi \tag{8}$$

where U is traverse speed, u_t is belt speed, S is the spacing between tracking cutters, and ϕ is the bar angle (see Part 3 for more detail). Since ℓ does not vary along the straight section of belt, the mean tool force does not vary either (there are, of course, force fluctuations corresponding to formation and removal of discrete chips in brittle material).

Number of active cutting teeth

The average number of cutting teeth N_a that are actively engaged in the work is given by

$$N_a = \frac{mR}{S}\left[\frac{d}{R} - (1 - \cos\phi) + \phi\right] \tag{9}$$

where d is the depth of the cut, R is the radius of the curved nose, and m is the number of cutting tracks. When the nose radius R is small compared with S or d, the average number of teeth engaged in the work is given approximately by

$$N_a \approx \frac{md}{S\sin\phi}\ . \tag{10}$$

Tool force and chain force

The sum of the tangential components of tool forces, Σf_t, gives a tangential force F_t that has to be provided by chain tension. In the usual situation where curvature of the nose can be neglected:

$$F_t = \Sigma f_t = N_a f_t = \frac{md}{S \sin\phi} f_t = \frac{md}{S \sin\phi} f_1(\ell) \tag{11}$$

where $f_1(\ell)$ is one of the functions given by eq 2, 5 or 7. Accepting eq 7 for purposes of illustration:

$$F_t = \frac{md}{S \sin\phi} k_t \left(\frac{\ell}{r}\right) = \frac{md}{S \sin\phi} \frac{k_t}{r} \frac{U}{u_t} S \sin\phi = m\, k_t \frac{d}{r} \frac{U}{u_t}\, . \tag{12}$$

The sum of the normal components of tool forces, Σf_n, gives a normal force F_n that has to be resisted by the bar or belt. In the situation where the effects of nose curvature can be neglected:

$$F_n = \Sigma f_n = N_a f_n = \frac{md}{S \sin\phi} f_n = \frac{md}{S \sin\phi} f_2(\ell) \tag{13}$$

where $f_2(\ell)$ is one of the functions given by eq 1, 4 or 6.

The ratio of F_n to F_t is the same as the ratio of the force components for individual tools:

$$\frac{F_n}{F_t} = \frac{f_n}{f_t}\, . \tag{14}$$

The ratio f_n/f_t varies considerably with tool design and with the state of wear, but it is not very sensitive to variations of ℓ.

In many cases the normal force F_n creates a frictional resistance μF_n, where μ is an effective coefficient of friction for sliding or rolling of the belt against the supporting bar. This force adds to the chain tension, but it does not have an external reaction. The relative magnitude of μF_n varies with the ratio F_n/F_t, or f_n/f_t.

The total force in the chain or belt F_c is a maximum at the tension side of the drive sprocket. It is made up of 1) the basic frictional drag of guides and rollers F_d, 2) the additional frictional drag μF_n induced by the normal cutting force F_n, and 3) the direct tangential cutting force F_t:

$$F_c = F_d + \mu F_n + F_t. \tag{15}$$

Of these, only F_t has an external reaction, but the tensile strength of the chain or belt has to be at least equal to F_c. The drag of guides and rollers that are not *directly* influenced by F_n may well increase as F_t increases, but for present purposes this effect can be accommodated in the μF_n term, since F_t and F_n are more or less proportional.

Chain power

The total power supplied to the belt or chain by the drive sprocket is used partly to overcome frictional resistance within the bar/belt system, and partly to cut the work material and transport it against external resistance. If the total power at the sprocket is P_{BT}:

$$P_{BT} = F_c u_t = (F_d + \mu F_n + F_t)\, u_t. \qquad (16)$$

This immediately gives an estimate for the required strength of the chain F_c.

The power consumed in overcoming basic drag, $F_d u_t$, can usually be measured or estimated by running the machine without a cutting load. This then gives the net belt power P_B, which is

$$P_B = (\mu F_n + F_t) u_t. \qquad (17)$$

For some practical purposes, F_n/F_t can be taken as a constant for a given type of tool in a given condition:

$$\frac{F_n}{F_t} = \frac{f_n}{f_t} = K. \qquad (18)$$

Thus,

$$P_B = (\mu K + 1)\, F_t u_t. \qquad (19)$$

In this expression, μ might be of the order of 0.1, with K about 1.0 for good fresh tools or about 2 for partly worn tools.

Tool force and belt power

From eq 11 and 19, the time-averaged tool force component f_t can be expressed in terms of net belt power P_B. Using the approximate relation for the number of active teeth N_a and assuming F_n/F_t is constant:

$$f_t = \frac{F_t}{N_a} = \frac{P_B S \sin\phi}{(\mu K + 1) u_t m d} \qquad (20)$$

and the normal tool force component is

$$f_n = \frac{F_n}{N_a} = \frac{K F_t}{N_a} = \frac{P_B S \sin\phi}{(\mu + 1/K) u_t m d}. \qquad (21)$$

These relations are convenient for making a quick assessment of tool forces. They illustrate that tool forces are directly proportional to belt speed, and inversely proportional to the number of teeth engaged in the work.

Tractive thrust and normal reaction

In order to traverse the machine through the work it is necessary to exert a force H parallel to the traverse direction. It is also necessary to provide a normal reaction V in a direction perpendicular to the traverse direction.

The tractive thrust H is

$$H = F_n \sin\phi + F_t \cos\phi \qquad (22)$$

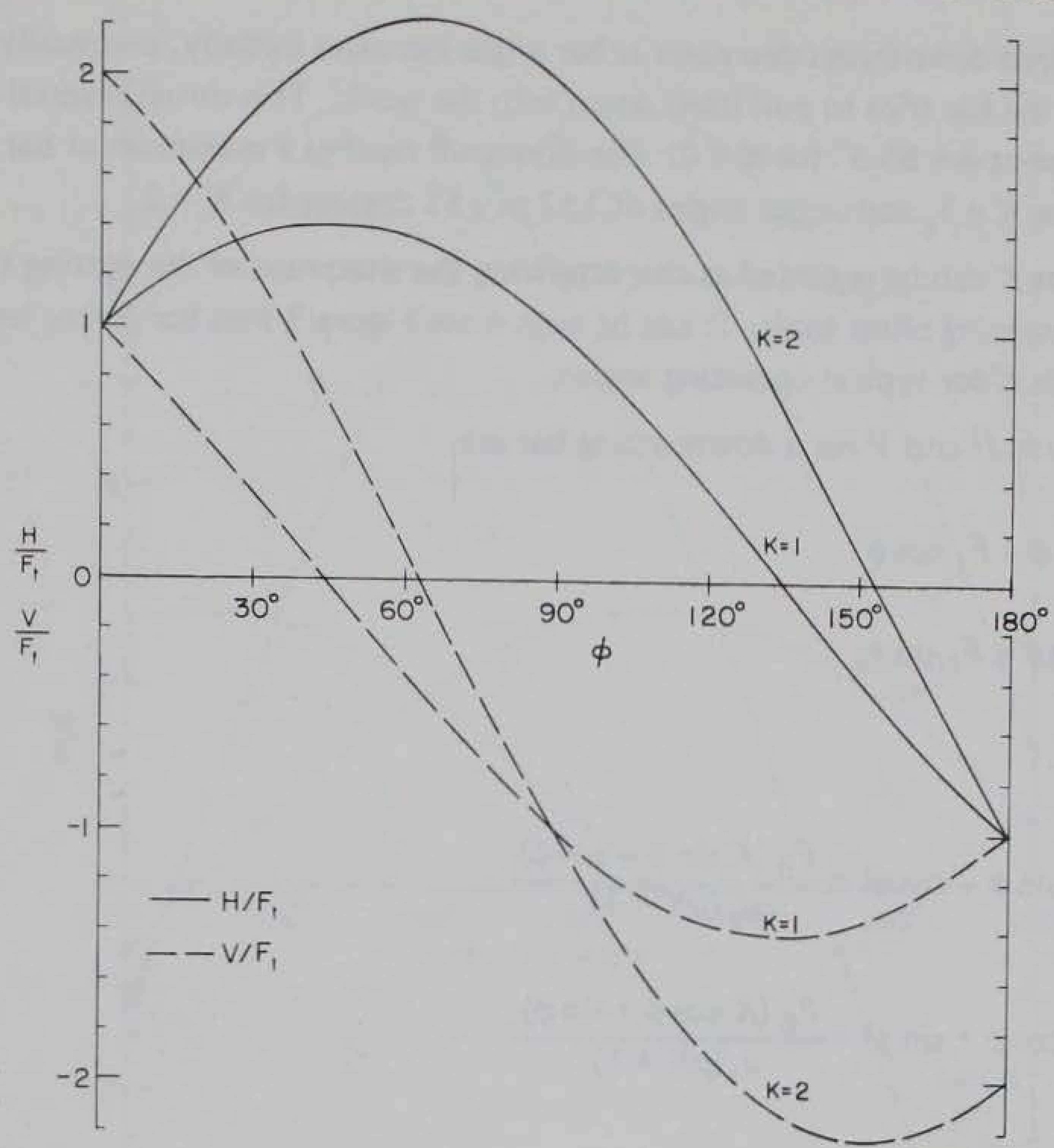


Figure 3. Bar force components H and V as functions of bar angle φ, for upcutting at two values of K.

where the forces are positive in the directions shown in Figure 1. If F_n/F_t is taken as constant,

$$H = F_t(K \sin\phi + \cos\phi) = \frac{P_B(K \sin\phi + \cos\phi)}{(\mu K + 1)u_t} \; . \tag{23}$$

The normal reaction V is

$$V = F_n \cos\phi - F_t \sin\phi \tag{24}$$

where the forces are positive in the directions shown in Figure 1. Taking $F_n/F_t = K$,

$$V = F_t(K \cos\phi - \sin\phi) = \frac{P_B(K \cos\phi - \sin\phi)}{(\mu K + 1)u_t} \; . \tag{25}$$

Figure 3 shows the variation of H and V with ϕ for two values of K when the bar is upcutting. With the bar trailing and cutting upward, in the manner usually adopted for ditching machines, the required propulsive thrust increases as the bar angle increases initially, but it eventually reaches a maximum. With $K = 1$ the maximum occurs between 40 and 50 degrees bar angle, and with $K = 2$ the maximum occurs at 62 to 65 degrees. As the bar angle gets steeper the required propulsive thrust decreases, and it keeps decreasing as the bar swings forward through the vertical position. The bar becomes self-propelling at an angle of $\phi = 135^\circ$ with $K = 1$, and at an angle of $\phi = 153.5^\circ$ with

$K = 2$. The required downthrust decreases as bar angle increases initially, eventually becoming negative, so that the bar tries to pull itself down into the work. This thrust reversal occurs at $\phi = 45^\circ$ for $K = 1$, and at $\phi = 63.5^\circ$ for $K = 2$. The downpull reaches a maximum at bar angles of 129 to 141 degrees for $K = 1$, and at bar angles of 152 to 155 degrees for $K = 2$.

The coefficient K can be regarded as characterizing the sharpness of the cutting tools, with high values of K representing blunt tools. It can be seen from Figure 3 that bar forces tend to increase in magnitude with K for typical operating angles.

The components H and V for a downcutting bar are

$$H = F_n \sin\phi - F_t \cos\phi \tag{26}$$

$$V = F_n \cos\phi + F_t \sin\phi. \tag{27}$$

When $F_n/F_t = K$,

$$H = F_t (K \sin\phi - \cos\phi) = \frac{P_B (K \sin\phi - \cos\phi)}{u_t (\mu K + 1)} \tag{28}$$

$$V = F_t (K \cos\phi + \sin\phi) = \frac{P_B (K \cos\phi + \sin\phi)}{u_t (\mu K + 1)}. \tag{29}$$

Traction of carrier vehicles

As explained in Part 6, the net tractive thrust (drawbar pull) of a self-propelled carrier vehicle has to be at least equal to the horizontal component of bar force H. For a given type of vehicle running on a given type of ground, the net traction or drawbar pull D_p is often taken as being proportional to the vehicle weight W:

$$D_p = C_d W \tag{30}$$

where C_d is the "drawbar coefficient." In order to traverse a cutter bar, D_p must be equal to, or greater than, H. However, the effective weight of the vehicle, or the normal force acting across the track/soil interface, is influenced by the vertical component of bar force V. In the simple case where the line of action of V is close to the vehicle's center of gravity or center of pressure:

$$H \leqslant C_d (W - V) \tag{31}$$

noting that V takes negative values when the cutter is pulling down against the vehicle. It is assumed here that the vehicle is "stiff," i.e. there is no bouncing on its springs during cutting. For an up-cutting bar, this leads to the condition

$$\frac{W}{F_t} \geqslant \left(\frac{K}{C_d} - 1\right) \sin\phi + \left(\frac{1}{C_d} + K\right) \cos\phi. \tag{32}$$

In Figure 4 this condition is displayed for a favorable set of circumstances when $C_d = 0.6$ and $K = 1$, and also for a less favorable set of circumstances when $C_d = 0.3$ and $K = 2$ (see Part 6 for notes on C_d).

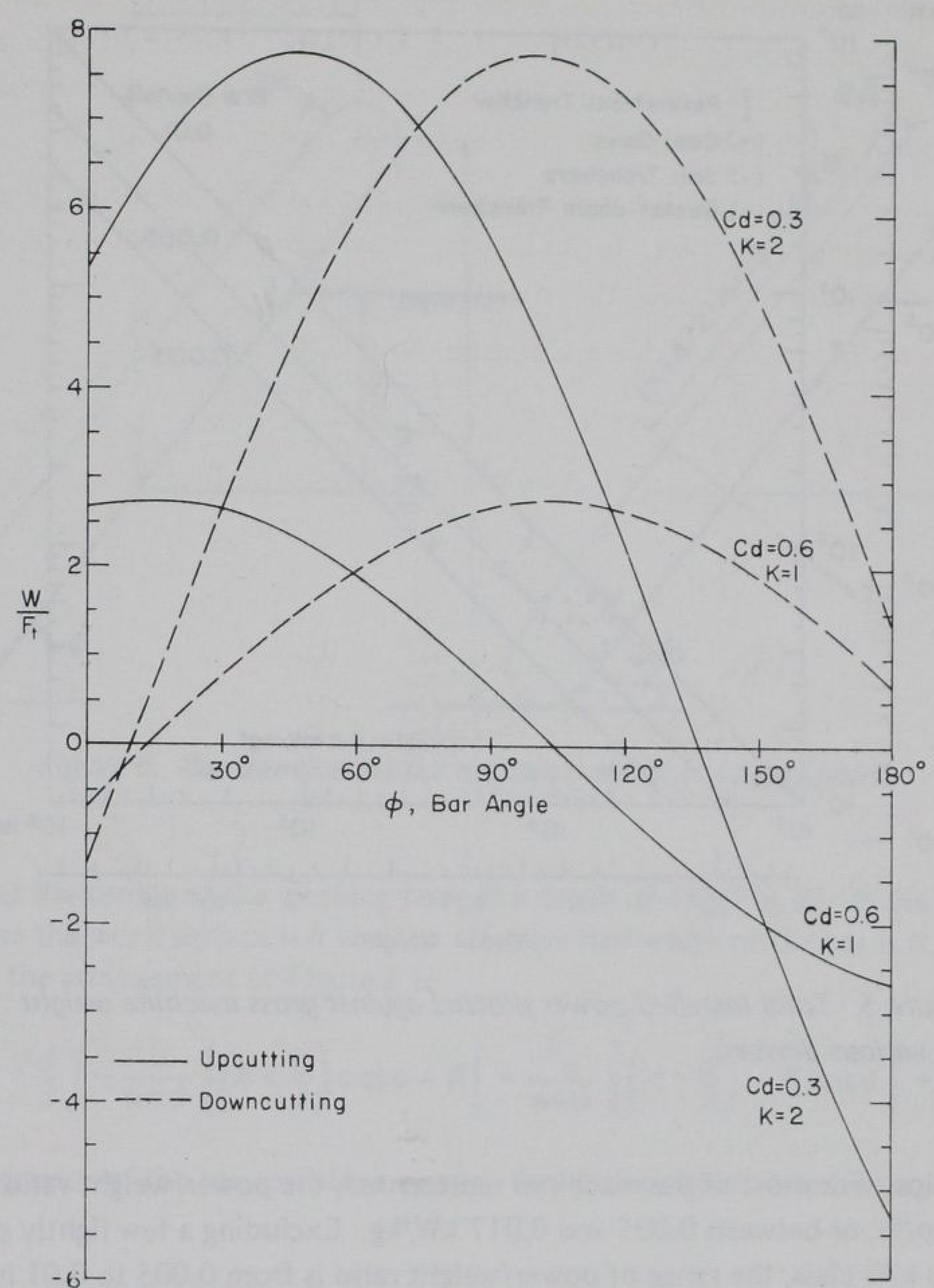


Figure 4. Examples of minimum weight requirements for self-propelled vehicles.

The condition for a downcutting bar is

$$\frac{W}{F_t} \geqslant \left(\frac{K}{C_d} + 1\right) \sin\phi + \left(K - \frac{1}{C_d}\right) \cos\phi. \tag{33}$$

Examples corresponding to those for the upcutting case are also shown in Figure 4.

Power/weight ratio

From the foregoing it is clear that a self-propelled machine has to have compatibility between the vehicle weight and the cutting forces, and this implies compatibility between vehicle weight and power. However, quite apart from such theoretical considerations, there is the practical fact that the weight of a given type of power plant and the weight of the structures needed to utilize the power increase as the power level increases.

In Figure 5 the total installed power has been plotted against the vehicle gross weight for a range of self-propelled continuous belt machines, and proportionality lines have been drawn to indicate

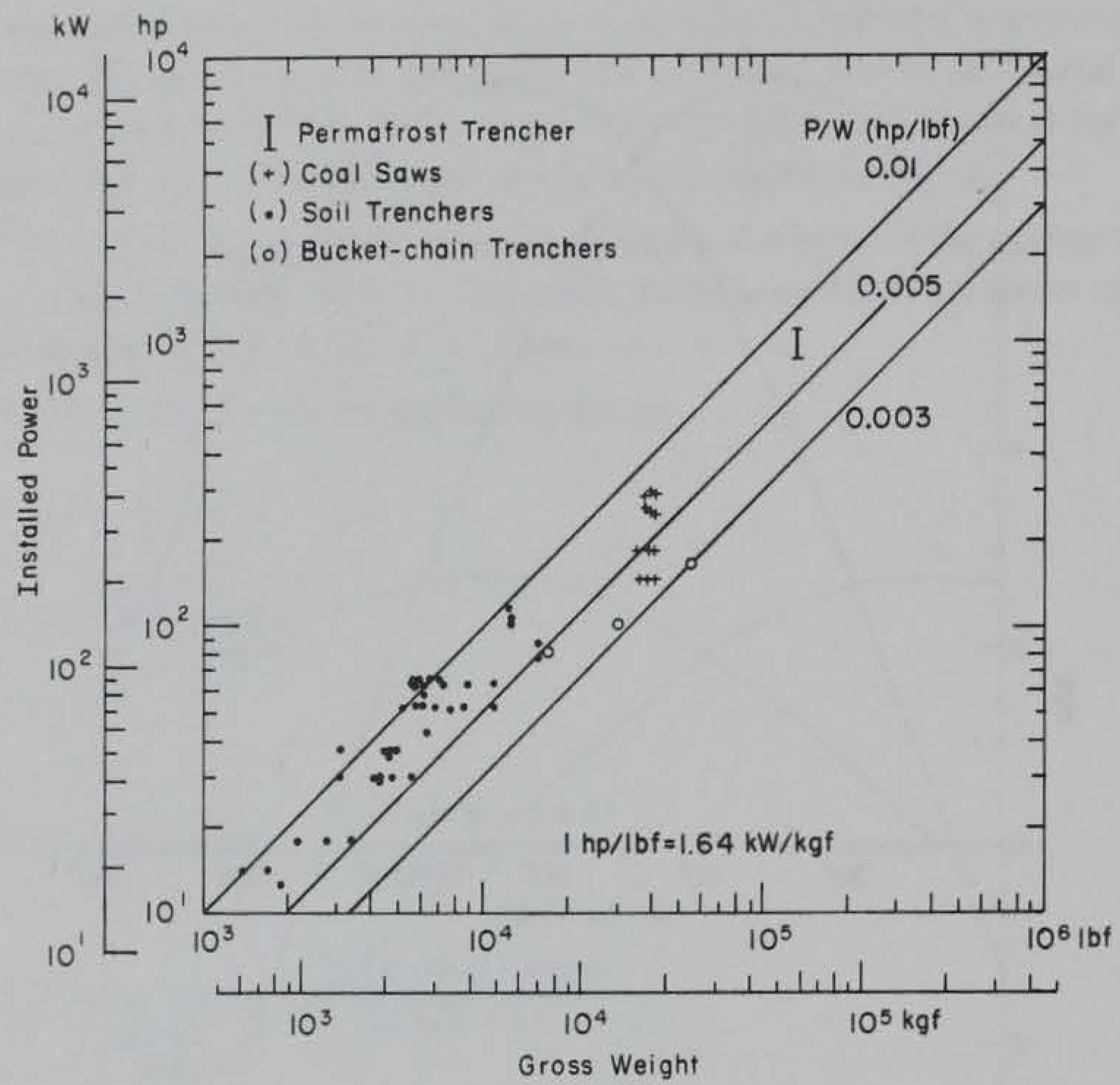


Figure 5. Total installed power plotted against gross machine weight for various devices.

power/weight ratios. For most of the machines represented, the power/weight ratio is between 0.003 and 0.01 hp/lb, or between 0.005 and 0.017 kW/kg. Excluding a few lightly powered machines in the 100 hp (75 kW) class, the range of power/weight ratio is from 0.005 to 0.01 hp/lb, or between 0.008 and 0.017 kW/kg. Power/weight ratios for continuous belt machines are very similar to those for transverse rotation machines, as can be seen by comparing Figure 5 of this report with Figure 8 of Part 6.

As a matter of interest, the power/weight ratios of tracked vehicles generally are not much different from the power/weight ratios of cutting and excavating machines. Values for modern high speed military vehicles such as tanks and armored personnel carriers are in the range 0.006 to 0.009 hp/lb (0.01 to 0.015 kW/kg).

Cutter bar moments

The cutter bar of a continuous belt machine is often mounted in such a way that forces on the bar have appreciable moments about points on the supporting system. Both the deadweight of the bar and the cutting force F have moments that need to be accounted for in the design of the carriage system and the manipulating mechanism. This can be important for a self-propelled machine that might have to operate on soft surfaces or with poor traction, since it affects the balance and pressure distribution of the carrier vehicle.

For illustration purposes, moments will be taken about the center of the drive sprocket at the supported end of the bar, assuming that this is the pivot axis for changes of bar angle. On a "long" bar, where the effects of curvature at the nose can be neglected, the resultant cutting force F can be

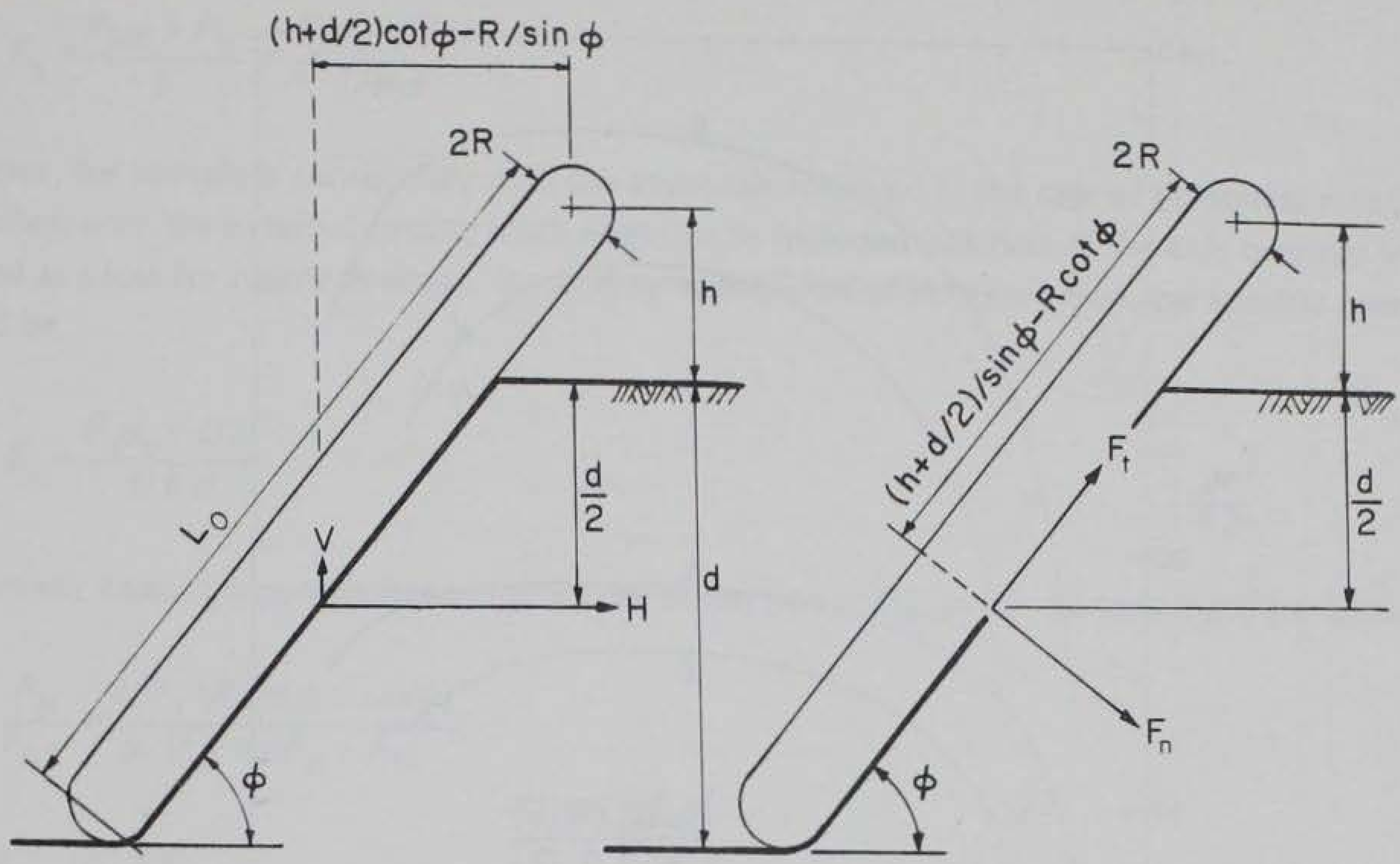


Figure 6. Bar dimensions for moments about the pivot point.

assumed to act at the center of the working face, at a depth of $d/2$ (Fig. 6). If the height of the pivot point above the work surface is h and the effective half-width of the bar is R, the cutting moment M_c for the arrangement of Figure 6 is

$$M_c = H\left(h + \frac{d}{2}\right) - \frac{V}{\sin\phi}\left[\left(h + \frac{d}{2}\right)\cos\phi - R\right] = \frac{F_n}{\sin\phi}\left[\left(h + \frac{d}{2}\right) - R\cos\phi\right] + F_t R. \tag{34}$$

If the center of gravity of the bar coincides more or less with its geometric center, the deadweight moment M_w is

$$M_w = W\frac{L_0}{2}\cos\phi \tag{35}$$

where $L_0/2$ is the distance from the pivot point to the center of gravity of the bar. In the case of a long bar where $L_0 \approx (h + d)/\sin\phi$

$$M_w \approx \frac{(h + d)W}{2\tan\phi}\ . \tag{36}$$

From the foregoing equations it is not immediately obvious that the cutting moment increases as the cutting depth d decreases when bar length L_0 is fixed and the machine is operated so as to draw full power. The effect is easier to see by using the long bar approximation, with $L_0 \approx (h + d)/\sin\phi$:

$$M_c \approx H\left(L_0\sin\phi - \frac{d}{2}\right) - \frac{V}{\sin\phi}\left[\left(L_0\sin\phi - \frac{d}{2}\right)\cos\phi - R\right]$$

$$\approx \frac{F_n}{\sin\phi}\left[\left(L_0\sin\phi - \frac{d}{2}\right) - R\cos\phi\right] + F_t R. \tag{37}$$

Assuming that $F_n/F_t = K$, this gives

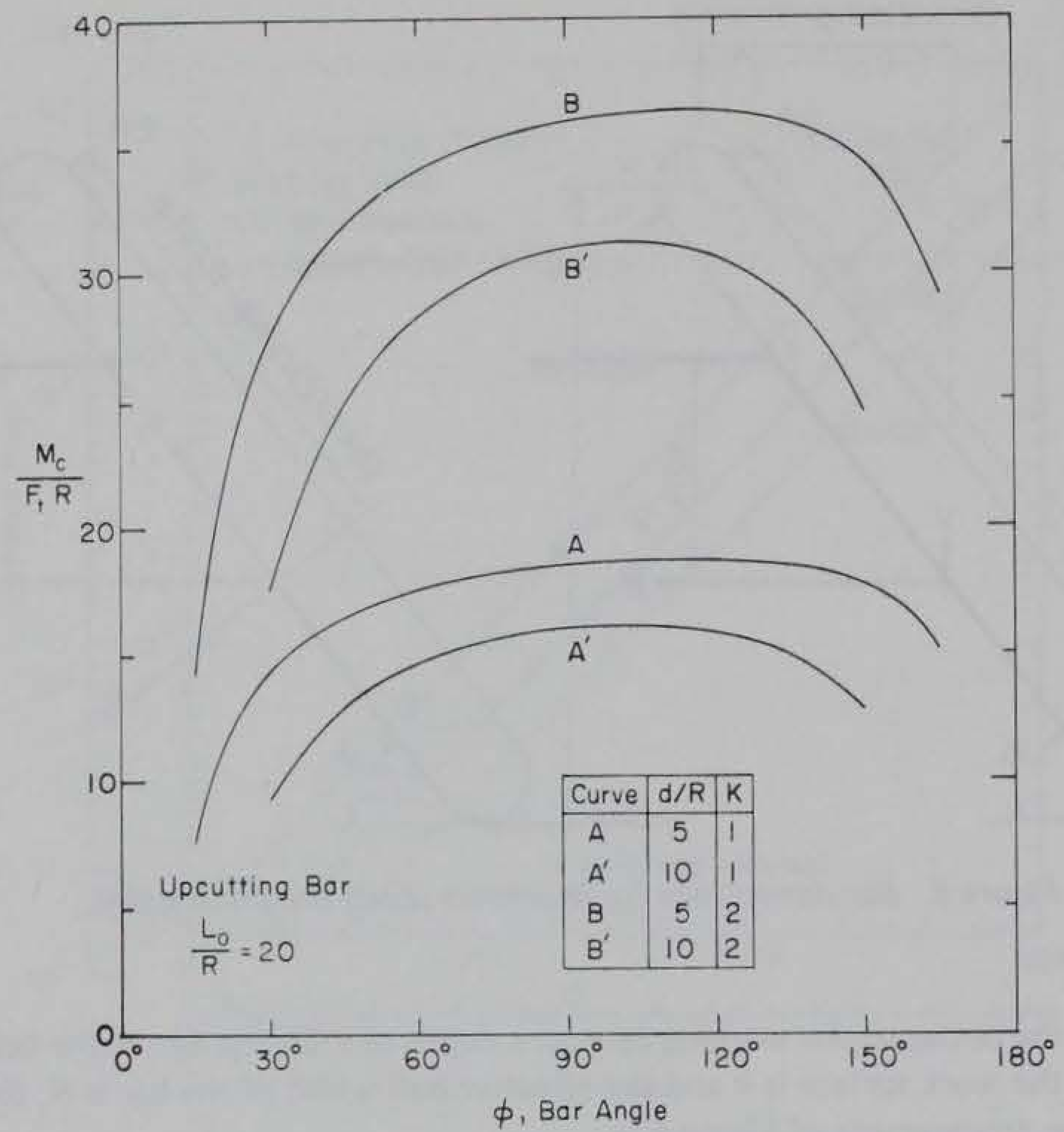


Figure 7. Moments about the pivot point as a function of ϕ for an upcutting bar of fixed length. Curves show cutting moments for two values of d/R and two values of K, assuming $L_0/R = 20$.

$$M_c \approx F_t\left(KL_0 - \frac{Kd}{2\sin\phi} - KR\cot\phi + R\right) \tag{38}$$

or

$$\frac{M_c}{F_t R} \approx \frac{KL_0}{R} - \frac{Kd}{2R\sin\phi} - K\cot\phi + 1 . \tag{39}$$

From this it can be seen that the full-power cutting moment increases linearly as d decreases with a given bar angle and constant L_0. The effect of varying the bar angle at constant depth is illustrated in Figure 7, taking assumed values of K, L_0/R and d/R. At steep bar settings, M_c is not very sensitive to ϕ in this example, but K has a very strong influence.

Specific energy

Specific energy ideas for a complete machine have already been outlined in connection with transverse rotation machines in Part 6. Here the discussion will be confined to process specific energy for the cutting operation alone, leaving aside the overall specific energy for a complete system. Even with this restriction, there is still considerable latitude in deciding which power losses should be included and which excluded. Probably the simplest and most useful procedure is to define the process specific energy E_s in terms of the total belt power P_{BT} plus the thrust power P_H, since these are readily measurable:

$$E_s = \frac{P_{BT} + P_H}{\dot{v}} = \frac{F_c u_t + UH}{UBd} \quad . \tag{40}$$

However, for complete consistency with the approach followed in the case of transverse rotation machines, only the external cutting work ought to be included (friction of the axle bearings was treated as a loss for rotary devices). According to this kind of interpretation, the specific energy would be

$$E_s = \frac{F_t u_t + UH}{UBd} \quad . \tag{41}$$

In many cases it is permissible to ignore the thrust power P_H, as can be seen from the following:

$$\frac{P_H}{P_{BT}} = \frac{U F_t \,(K \sin\phi + \cos\phi)}{u_t \,(F_d + \mu F_n + F_t)} \quad . \tag{42}$$

Taking $F_n/F_t = K$,

$$\frac{P_H}{P_{BT}} = \frac{U}{u_t} \; \frac{(K \sin\phi + \cos\phi)}{(F_d/F_t + \mu K + 1)} \quad . \tag{43}$$

The factor $(K \sin\phi + \cos\phi)$ in the numerator is likely to be between -1 and about $+2$ with reasonably good cutting teeth (see Fig. 3). In the denominator, F_d/F_t is always less than unity, while μK is quite likely to be also less than unity. Thus the complete factor applied against U/u_t is expected to be of order unity, and the ratio P_H/P_{BT} is approximately equal to U/u_t. The ratio U/u_t could be as low as 10^{-3}, and it is not likely to exceed 10^{-1} for typical cutting machines.

Neglecting P_H, the approximate expression for E_s is

$$E_s \approx \frac{P_{BT}}{UBd} \approx \frac{u_t}{U} \; \frac{F_c}{Bd} \, . \tag{44}$$

It might be noted that P_{BT} includes the power that is dissipated in transporting and ejecting cuttings, but in another section it is shown that this is usually a small proportion of the total power.

In Part 6 the inverse proportionality between unit production rate $(\dot{v}/P_R)$ and specific energy E_s was illustrated graphically. The same plot (Fig. 10 of Part 6) applies to continuous belt machines when P_{BT} is substituted for rotor power P_R.

Performance index

In Parts 4 and 6 the derivation of a dimensionless performance index for drag bit cutting was described. Because there is an overall linear correlation between the specific energy E_s and the uniaxial compressive strength of the work material σ_c, the ratio E_s/σ_c gives a measure of the efficiency of the cutting process.

It was explained in Part 6 how the performance index can be used in design or in performance analysis, and experimental data relating E_s and σ_c were plotted so as to define attainable levels of E_s/σ_c. The writer has not got a comparable body of data for continuous belt machines, but the few relevant results that are available are shown in Figure 8. Contributions of additional data would be welcomed.

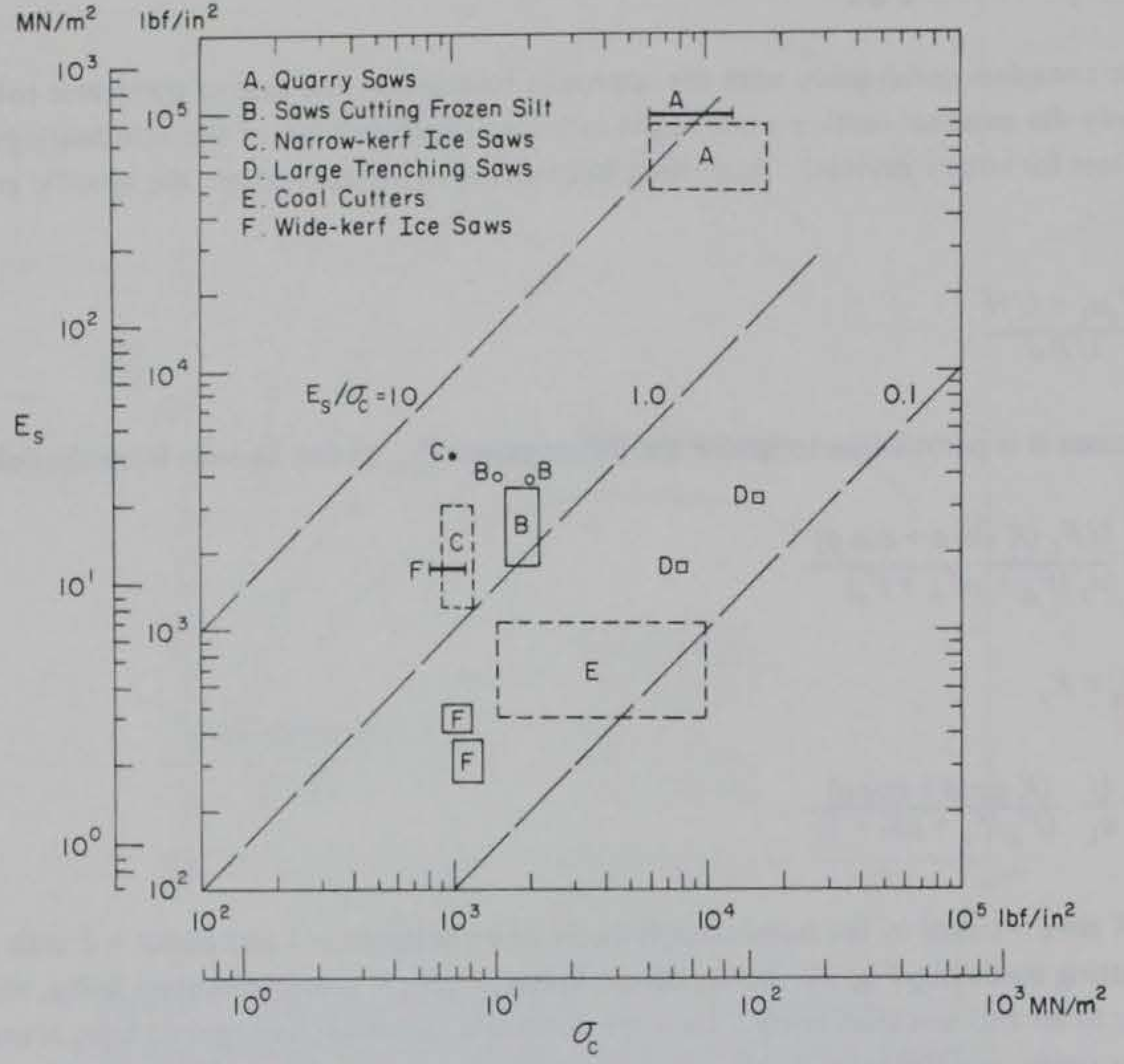


Figure 8. Specific energy E_s plotted against uniaxial compressive strength σ_c for various types of machines. Proportionality lines give different levels of the performance index E_s/σ_c.

From the limited data it appears that chain saw equipment has the potential to be almost as efficient in energetic terms as other types of drag bit machines, even though friction losses may be relatively high. However, the performances of some machines have been very poor, probably because of fundamental design faults or haphazard adaptation to nonstandard applications.

Power density

Power density is defined here as the power per unit area of the working surface of the belt. If the belt width is B, the cutting depth is d, the bar angle is ϕ, and the belt power is P, then the power density Q for a long bar is approximately

$$Q = \frac{P \sin \phi}{Bd}. \qquad (45)$$

For some practical purposes it is either convenient or necessary to take the belt power P as the gross power supplied by the drive sprocket P_{BT}, ignoring internal losses. In order to compare machines in general terms, it is convenient to define the working surface as the maximum working surface, i.e. $(Bd/\sin \phi)_{max}$, or $(BL)_{max}$, where $(BL)_{max}$ is the maximum usable area of the bar. This gives a nominal power density Q_N as

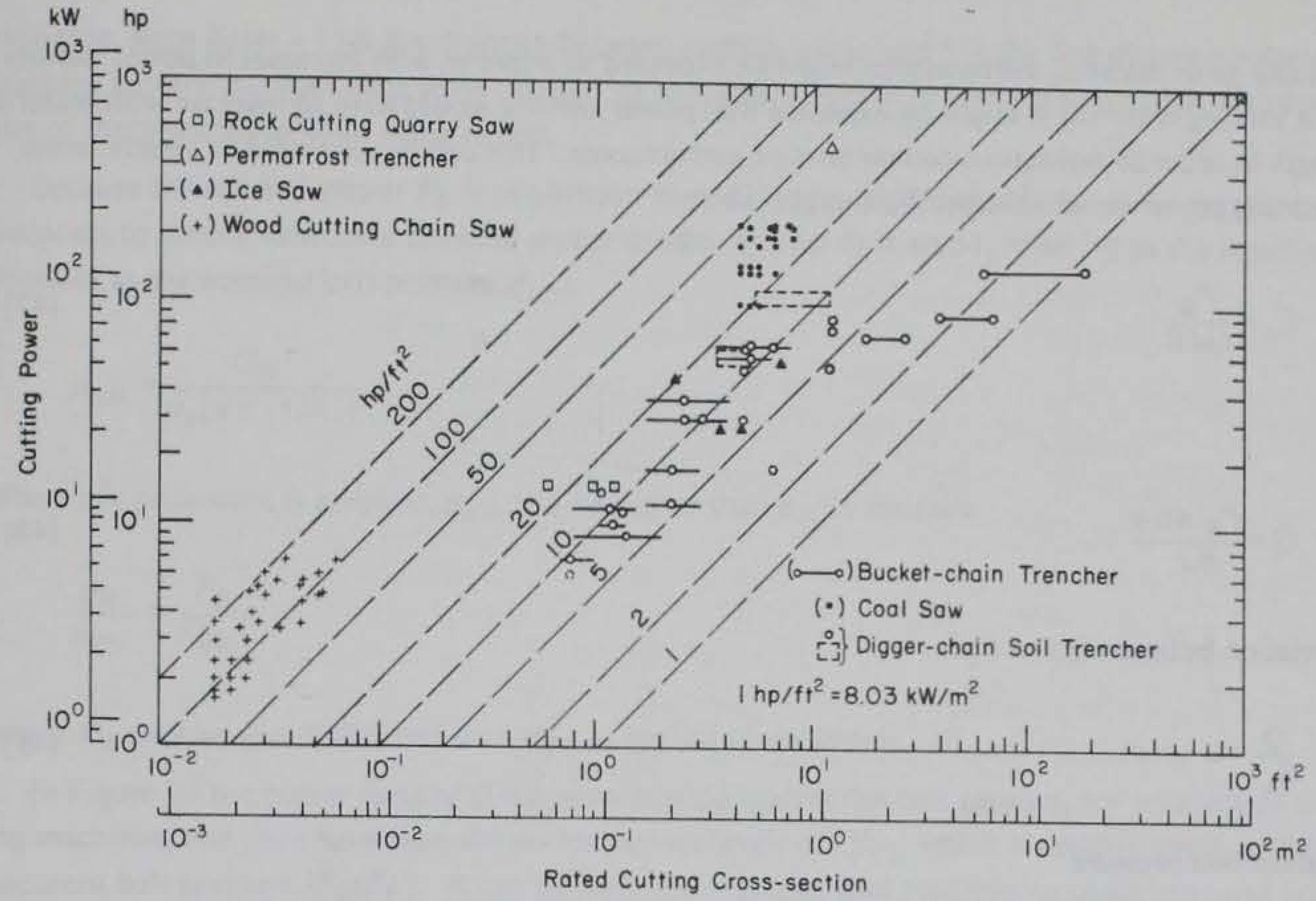


Figure 9. Cutting power plotted against the maximum rated value for the working area of the cutter bar. Data are for a wide range of existing machines, from hand chain saws to large bucket-chain soil trenchers. Proportionality lines give different levels of power density.

$$Q_N = \frac{P_{BT}}{(BL)_{max}} = \frac{P_{BT}}{(Bd/\sin\phi)_{max}} . \quad (46)$$

On some types of machines, most notably ladder trenchers, L does not vary much, since d is controlled by varying ϕ.

In Figure 9 the cutting power P_{BT} has been plotted against the rated cutting cross section $(BL)_{max}$ for a variety of belt machines, and lines representing different levels of the nominal power density Q_N have been drawn. Of the machines that are represented in the plot, hand-operated chain saws for cutting lumber are the most powerful group, with power densities in the range 100 to 200 hp/ft^2 (0.8 to 1.6 MW/m^2). Among machines designed to cut or excavate in earth materials, coal mining chain saws are the most powerful group, with power densities of 20 to 50 hp/ft^2 (0.16 to 0.40 MW/m^2). Digger-chain soil trenchers designed for cutting trenches of moderate width (up to 2 ft or 0.6 m) and moderate depth (up to about 7 ft or 2 m) commonly have power densities in the range 5 to 15 hp/ft^2 (40 to 120 kW/m^2). Very large ladder trenchers of the bucket chain type, which dig up to 6 ft (1.8 m) wide and up to 25 ft (7.6 m) deep, have very low power densities, somewhere between 1 and 5 hp/ft^2 (about 10 to 40 kW/m^2).

Power density is largely a matter of practicality in machine design. It is not normally feasible to furnish very high power density on large mobile machines that carry their own power source. For example, on a bucket chain trencher cutting 6-ft trench to a depth of 25 ft with a 60° ladder angle, it would take a power source of over 7000 hp to provide 40 hp/ft^2 at the ladder.

In any given material, performance might be expected to improve with increases in power density, and in varying materials it might be expected that power density would have to increase with material strength in order to maintain a certain level of performance. This can be expressed in simple terms by relating power density to specific energy. Since

$$E_s = \frac{P_B}{UdB} \tag{47}$$

and

$$Q = \frac{P_B \sin\phi}{Bd} \tag{48}$$

the relation between Q and E_s is

$$\frac{Q}{E_s} = U \sin\phi . \tag{49}$$

Apparent belt pressure

One way of looking at the thrust forces on a long cutter bar is to think in terms of an apparent belt pressure p_B, defined as the normal force F_n divided by the approximate area of the working surface BL (or $Bd/\sin\phi$). In a balanced design, the thrust force has to be commensurate with the belt power, or the apparent belt pressure has to be commensurate with the power density. If it is assumed that $F_n/F_t = K$,

$$p_B = \frac{F_n}{BL} = \frac{KF_t}{BL} = \frac{KP_B \sin\phi}{Bdu_t(\mu K + 1)} \tag{50}$$

and since $Q = (P_B \sin\phi)/Bd$

$$p_B = \frac{F_n}{BL} = \frac{Q}{u_t[\mu + (1/K)]} . \tag{51}$$

In eq 51 the factor $[\mu + (1/K)]$ is of order unity, so that the apparent belt pressure has to be more or less the power density divided by the belt speed. This is a useful rule-of-thumb for checking that thrust and power provisions are mutually compatible. The same information is conveyed implicitly by some of the earlier equations, but it is more convenient to keep in mind *specific* quantities, such as power density and thrust pressure, rather than absolute values of power and force for particular conditions. It should be kept in mind that the apparent belt pressure is not the same thing as the stress imposed by the cutting tools. Stated in terms of belt pressure, the tool forces are:

$$f_t = \frac{BS}{Km} p_B \tag{52}$$

$$f_n = \frac{BS}{m} p_B . \tag{53}$$

However, since $B/(m-1)$ is the distance between cutting tracks and S is the line distance between tracking cutters, p_B does have some relation to tool forces, which in turn have to match the properties of the work material to some extent.

Because the net belt power P_B is not always known, it may be acceptable for some practical purposes to simply substitute nominal power density Q_N for Q in eq 51, referring to the resulting pressure as the nominal belt pressure p_{BN}:

$$p_{BN} = \frac{Q_N}{u_t[\mu + (1/K)]} \cdot \tag{54}$$

When this procedure is adopted, p_{BN} will be higher than p_B in the ratio

$$\frac{p_B}{p_{BN}} = \frac{P_B}{P_{BT}} \tag{55}$$

where P_B may be about 20% less than P_{BT} at typical chain speeds.

In Figure 10 the power density Q has been plotted against the belt speed u_t for a variety of existing machines, and lines have been drawn for various levels of Q/u_t, which is roughly equal to the apparent belt pressure (F_n/BL). It can be seen, for example, that coal mining chain saws and lumber

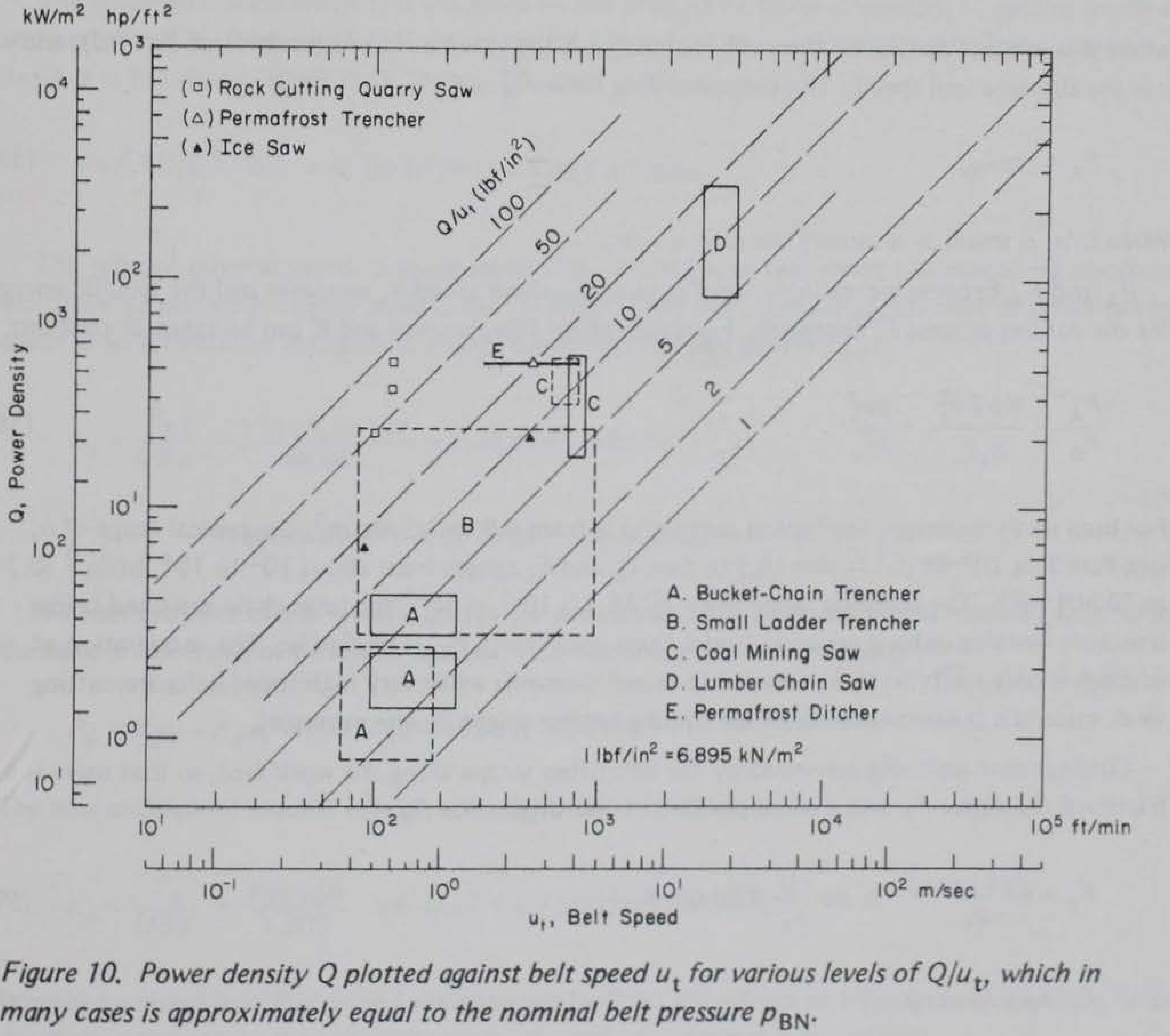


Figure 10. Power density Q plotted against belt speed u_t for various levels of Q/u_t, which in many cases is approximately equal to the nominal belt pressure p_{BN}.

cutting chain saws require nominal belt pressures around 10 lbf/in.^{2} (70 kN/m^2) in order to utilize their full power. The data for small ladder trenchers cover wide ranges because of variations in chain width, digging depth, and chain speed, even when some allowance is made for the fact that most cannot attain maximum rated width and maximum rated depth at the same time. The biggest ladder trenchers, those of the bucket-chain type which work mainly in weak soils, have very low values of Q/u_t, down to less than 2 lbf/in.^{2} (14 kN/m^2). At the other extreme are quarry stone saws, which have values of Q/u_t from 50 to 80 lbf/in.^{2}

As a matter of interest, the nominal belt pressure for lumber chain saws, coal mining chain saws, ice saws, and some ladder trenchers is comparable to the nominal ground pressure of military tracked vehicles. Other small ladder trenchers (with low power densities and/or high chain speeds) have nominal belt pressures comparable to the ground pressures of tracked construction machines. The big bucket-chain trenchers have nominal belt pressures comparable to the bearing pressures of very low ground pressure tracked vehicles.

Acceleration and transport of cuttings

The tangential tool force f_t almost always includes the force required to accelerate cuttings to the tool speed u_t, and the external chain force F_t also includes this inertial component. The minimum power needed for acceleration of cuttings P_A is given by the rate of supply of kinetic energy:

$$P_A = \tfrac{1}{2}\rho \dot{v} u^2 \tag{56}$$

where ρ is in-place density of the work material, $\dot{v}$ is the volumetric production rate (= UBd), and u is the absolute tool speed. The corresponding force F_A is

$$F_A = \tfrac{1}{2}\rho \dot{v} u. \tag{57}$$

When U/u_t is small, as is usually the case, $u \approx u_t$.

P_A and F_A become increasingly significant as the chain speed u_t increases and the specific energy for the cutting process E_s decreases. For cases where U/u_t is small and K can be taken as constant,

$$\frac{P_A}{P_B} = \frac{\tfrac{1}{2}\rho \dot{v} u_t^2}{E_s \dot{v}} = \frac{\rho u_t^2}{2E_s}\,. \tag{58}$$

For hard earth materials, the typical range of ρ is from 0.9 to 2.5 Mg/m^3, the typical range of u_t (see Part 3) is 10^2 to 10^3 ft/min (0.5 to 5 m/s), and E_s ranges from about 10^2 to 10^4 lbf/in.^{2} (0.7 to 70 MN/m^2). The common range of $(\rho u_t^2/2E_s)$ is 10^{-5} to 10^{-2} for large chain saws and ladder trenchers working in hard ground. Under these conditions, P_A is negligible. The acceleration of cuttings is only likely to make noticeable power demands when very high speed belts are cutting weak materials (a sawmill bandsaw for ripping lumber might be one example).

Cuttings that are being conveyed by the belt often scrape along the work face, so that there is a frictional resistance F_F and a corresponding power dissipation P_F:

$$F_F = \frac{\mu \rho \dot{v} g \cos\phi}{u_t} = \mu\rho \frac{U}{u_t} dBg \cos\phi \tag{59}$$

$$P_F = \mu \rho \dot{v} g \cos\phi \tag{60}$$

where μ is a friction coefficient for cuttings rubbing on the rough work face, ρ is in-place density of the work material, $\dot{v}$ is the volumetric production rate, u_t is the belt speed, ϕ is the bar angle, d is the cutting depth, and B is the cutting width (kerf width) of the bar. The relative significance of this frictional effect is given by:

$$\frac{P_F}{P_B} = \frac{\mu\rho\dot{v}g\cos\phi}{E_s\dot{v}} = \frac{\mu\rho g\cos\phi}{E_s}. \qquad (61)$$

If adequate provision has been made for storage and conveyance of cuttings on the belt (see Part 3), P_F is not likely to exceed 5×10^{-3} for hard earth materials, which is completely negligible.

Examples

Example 1. A coal cutter (chain saw) is undercutting a seam of hard coal with an 11-ft (3.35-m) length of the bar engaged in the work. The bar is angled forward 15°, such that the nose of the bar is leading. The kerf cut by the bar is 6.5 in. (165 mm) wide. The electric motor driving the cutter chain draws 130 kW (174 hp) when the saw is progressing at a traverse speed of 8 ft/min (2.44 m/min), and it draws 22 kW (29.5 hp) at zero traverse speed when the saw is no longer cutting. The chain speed is 700 ft/min (3.56 m/s). Calculate the specific energy consumption of the saw, and obtain a value for the performance index if the uniaxial compressive strength of the coal is 4000 lbf/in.^{2} (27.6 MN/m^2).

The volumetric production rate $\dot{v}$ is given by the product of traverse velocity U, cutting depth d, and kerf width B. With an 11-ft (3.35-m) length of bar into the work and $\phi = 105°$, the cutting depth d is $11\times\sin\phi = 10.63$ ft (3.24 m). Thus $\dot{v}$ is

$$\dot{v} = \frac{8\times10.63\times6.5}{12} = 46.06 \text{ ft}^3/\text{min} = 1.305 \text{ m}^3/\text{min}.$$

The ratio of traverse speed to chain speed U/u_t is 0.0114, so that errors introduced by neglecting the thrust power are probably around the 1% level. Thus the approximate formula of eq 44 can be used for an estimate of the specific energy based on total belt power P_{BT}:

$$E_s \approx \frac{P_{BT}}{UBd} = \frac{174\times3.3\times10^4}{46.06} = 1.25\times10^5 \text{ ft-lbf/ft}^3$$

$$= 866 \text{ lbf/in.}^2 = 5.97 \text{ MN/m}^2 \text{ (or MJ/m}^3\text{)}.$$

A closer estimate of the specific energy for the actual cutting process can be made by subtracting the basic friction loss of the chain and bar, thus obtaining the net belt power P_B:

$$P_B = P_{BT} - F_d u_t = 130 - 22 = 108 \text{ kW} = 145 \text{ hp}.$$

The new estimate of specific energy, still neglecting thrust power, is

$$E_s \approx \frac{P_B}{UBd} = \frac{108\times60}{1.305} \text{ kW-s/m}^3 = 4.97 \text{ MJ/m}^3 \text{ (MN/m}^2\text{)}.$$

It might be noted that this second estimate still includes the effects of friction losses resulting from the chain pressing into the bar guides under the action of normal cutting force.

The performance index is the specific energy divided by the uniaxial compressive strength of the work material, i.e. E_s/σ_c. The two estimates of E_s/σ_c corresponding to the two values of E_s are:

(a) $E_s/\sigma_c = 0.22$

(b) $E_s/\sigma_c = 0.18$.

It can be seen that for practical purposes the difference between the two determinations is not very significant. Either estimate would indicate relatively efficient performance by the machine (see Fig. 8 of this report or Fig. 12 of Part 6).

Example 2. A machine is required for cutting pipeline ditch in permafrost that has a uniaxial compressive strength averaging about 14 MN/m^2. The ditch is to be 0.5 m wide by 1.5 m deep, and a single machine is expected to progress at short-term rates of 1.3 to 1.8 m/min (excluding "down time"). The machine offered by one manufacturer is a modified ladder trencher fitted with a "frost chain" of appropriate width. Chain speed can be varied between 1.5 and 3 m/s without overstressing or overheating the drive and chain The ladder, or bar, will operate at an angle of 60° to the horizontal, the chain will be supplied with a drive power of 190 kW, and another 40 kW will be available for driving the tracks and conveyor. The gross weight of the crawler-mounted machine is 26,000 kgf. Make a quick preliminary appraisal of this machine.

The power/weight ratio of the complete machine P/W is

$$P/W = 230/26{,}000 = 0.0088 \text{ kW/kgf} = 0.0054 \text{ hp/lbf.}$$

Checking this value against Figure 5, it appears that P/W is toward the low end of the range that might be expected for a machine of its type.

The power density of the bar Q is the belt power divided by the area of the working surface:

$$Q = 190/(0.5 \times 1.5 \times \operatorname{cosec} 60°) = 219 \text{ kW/m}^2$$

$$= 27.3 \text{ hp/ft}^2.$$

Checking this value against Figure 9, it seems that Q is comfortably above the values for ordinary soil trenchers, but it is only near the lower limit of power densities for coal saws.

The apparent belt pressure cannot be calculated exactly from the available information, but Q/u_t will range from 73 to 146 kN/m^2 (10.6 to 21.2 $lbf/in.^2$), depending on belt speed. This is quite comparable to values for coal saws.

The required production rate $\dot{v}$ is between 0.98 and 1.35 m^3/min, with a power input of 190 kW. This is equivalent to specific energy values E_s of

$$\text{from } E_s = \frac{P}{\dot{v}} = \frac{190}{0.98/60} \; \frac{\text{kW-s}}{\text{m}^3} = 11.63 \text{ MJ/m}^3 \text{ (MN/m}^2\text{)}$$

$$\text{to} \quad E_s = \frac{190}{1.35/60} = 8.44 \text{ MJ/m}^3 \text{ (MN/m}^2\text{)}.$$

The estimated compressive strength σ_c for the work material is 14 MN/m^2 so that the range of values for E_s/σ_c will be

$E_s/\sigma_c = 0.6$ to 0.83.

Checking these values against Figure 8, and recalling that low values of E_s/σ_c denote high energetic efficiency, it is seen that 0.6 to 0.83 are attainable values for existing machines, but a bit beyond what has yet been achieved in frozen soils.

Taking these things together, there is nothing that rules out the machine from further consideration. However, an "educated guess" might be that the machine is somewhat underpowered, since P/W and Q are on the low side for hard rock machines, and the required values of E_s/σ_c seem optimistic compared with the performance of existing machines in frozen soils. With higher power, chain speeds at the high end of the available range would probably be needed.

It should be remembered that these are only "ballpark" guidelines for the dynamic and energetic factors, based on accumulated experience. For the machine to work properly it would also require suitable cutting teeth (see Part 4) and sound kinematic design, as discussed in Part 3.

Example 3. A large chain saw is to be used for cutting away collars of ice that adhere to the vertical walls of a ship lock during wintertime operation. The upcutting saw is to be carried on a rubber-tired tractor, with an offset mounting that allows the saw to hang down the face of the wall while the tractor travels on the esplanade parallel to the edge of the wall. The problem is to calculate the forces and moments that would tend to tip over the tractor towards the lock wall if no dolly wheel were provided under the offset mount. The centerline of the kerf cut by the saw will be 0.6 m (23.6 in.) beyond the outside edge of the tractor tires. The bar will normally be operated in the upcutting mode while angled forward from the vertical by 20°, i.e. ϕ will be 110°. The chain is to be fitted with aggressive teeth which can be assumed to have $K = 1$, but a check should be made to find the effect of blunting when $K = 2$. The power available at the drive sprocket will be 35 kW (46.9 hp), but 12 kW (16.1 hp) will be used in overcoming chain resistances that have no net external reaction (mainly friction in the chain links, the chain guides, and the nose sprocket). The chain speeds in the three available operating gears will be 0.8, 2.1 and 3.2 m/s (157, 413 and 630 ft/min). The static weight of the saw is counterbalanced by lead ballast on the opposite side of the tractor. The gross weight of the tractor is 2500 kg, and the tractor width is 1.63 m to the outside edges of the tires.

From eq 24, the vertical bar force V is

$$V = F_n \cos\phi - F_t \sin\phi$$

and if it is assumed that $F_n/F_t = K$, then

$$V = F_t\,(K \cos\phi - \sin\phi).$$

If the power available for work against external resistance is known, then it can be equated to $F_t u_t$. In this case

$$F_t u_t = 23 \text{ kW} = 23 \text{ kN-m/s}.$$

For chain speeds of 0.8, 2.1 and 3.2 m/s, the respective values of F_t are

$$F_t = 28.75, 10.95 \text{ and } 7.188 \text{ kN}.$$

The corresponding values of V are:

with $K = 1$ $V = -36.8$, -14.0 and -9.21 kN

with $K = 2$ $V = -46.7$, -17.8 and -11.7 kN.

The negative signs indicate that V is pulling down against the tractor.

With an offset vertical force pulling downward, the tendency is for the tractor to tip by rotating about the outside edge of the tires nearest the saw. The moment of V about this axis of rotation is 0.6 V kN-m when V is in kN. The moments corresponding to the above values of V are:

Moments with $K = 1$ 22.1, 8.4 and 5.53 kN-m

Moments with $K = 2$ 28.0, 10.7 and 7.02 kN-m.

The restoring moment provided by the weight of the tractor is $2500 \times (1.63/2)$ kgf-m = $24.52 \times (1.63/2)$ kN-m = 20 kN-m. From these numbers it is clear that the tractor would try to tip over with the saw developing full power at the lowest chain speed. Even at higher chain speeds the tipping moments are high enough to "bounce" the suspension of the tractor. A solution would be to fit a dolly wheel beneath the offset mounting, thus shortening the cantilever that supports the saw.

Example 4. It is proposed that a machine should be built to cut small underwater trenches in coral. Trench width is to be 180 mm, and the required trench depth varies from 1 m to 1.8 m. An upcutting chain saw is to be mounted on tracks similar to those used for crawler rock drills, and because the complete machine will be small and relatively light in weight, traction is expected to be a problem. The saw is intended to have a 100-kW drive motor, and tests indicate that 20 kW will be lost in chain friction and fluid resistance at the intended chain speed of 3.3 m/s. The effective friction coefficient for the chain sliding against the work side of the bar is 0.2. It is estimated that the submerged weight of the unballasted underwater unit will be 2700 kg (26.48 kN). Two preliminary design concepts consider: a) a vertical cutter bar ($\phi = 90°$) that slides vertically to adjust cutting depth, and b) a pivoting cutter bar that has a maximum bar angle of 60° at maximum cutting depth, and a pivot height of 1.0 m above ground level. Estimate the horizontal tractive force needed to propel the machine at the limits of the cutting depth range in each of these configurations, and estimate the required values of drawbar coefficient if the vehicle is to operate without ballast. Calculate the nominal power density and apparent belt pressure for the two machine configurations. Comment on the proposed designs.

The required horizontal tractive force H is, from eq 23,

$$H = F_t \,(K \sin\phi + \cos\phi) = \frac{P_B \,(K \sin\phi + \cos\phi)}{(\mu K + 1)\, u_t} \; .$$

In this equation, $P_B = 100 - 20 = 80$ kW, and $u_t = 3.3$ m/s, so that the dimensional part P_B/u_t = 80/3.3 = 24.24 (kW-s)/m = 24.24 kJ/m = 24.24 kN. For configuration (a) the bar angle ϕ remains constant at 90°. For configuration (b), a cutting depth of 1.8 m requires $\phi = 60°$, which implies that the effective bar length L_0 is 2.8/sin 60° m, i.e. $L_0 = 3.23$ m. For configuration (b) and a cutting depth of 1.0 m, the bar angle is $\phi = \sin^{-1}(2.0/3.23) = 38.26°$.

The factor K varies with the design and the condition of the cutting tools, but for calculation purposes some representative values of $K = 1$ and $K = 2$ can be taken, representing respectively good fresh teeth and dull worn teeth. The friction coefficient μ gives the added chain drag when the chain is thrust against its guides by the normal force F_n, and the given value is 0.2. Taking all of these values, the following estimates of H are obtained for full-power operation:

Configuration (a)				Configuration (b)			
$d = 1.0$ m		$d = 1.8$ m		$d = 1.0$ m		$d = 1.8$ m	
$K = 1$	$K = 2$	$K = 1$	$K = 2$	$K = 1$	$K = 2$	$K = 1$	$K = 2$
20.2 kN	34.6 kN	20.2 kN	34.6 kN	28.4 kN	35.0 kN	27.6 kN	38.6 kN

The drawbar coefficient C_d is the net drawbar pull of the vehicle divided by the effective vehicle weight (weight supported by the tracks). In this case, where the cutter bar is mounted in the center of the track system, the effective vehicle weight is the actual submerged weight plus or minus the vertical reaction of the cutter bar V. The values of V corresponding to the values of H given above are, from eq 25:

Configuration (a)				Configuration (b)			
$d = 1.0$ m		$d = 1.8$ m		$d = 1.0$ m		$d = 1.8$ m	
$K = 1$	$K = 2$	$K = 1$	$K = 2$	$K = 1$	$K = 2$	$K = 1$	$K = 2$
−20.2 kN	−20.2 kN	−20.2 kN	−20.2 kN	+3.35 kN	+16.5 kN	−7.39 kN	+2.32 kN

Negative values of V indicate that the cutter bar is pulling down on the vehicle carriage, and therefore these values are added to the vehicle weight W. Positive values indicate that the saw has to be pushed down into the work, so that these values have to be subtracted to obtain the effective vehicle weight. In order to meet the traction requirements for full power operation, the crawler must have a drawbar coefficient of

$$C_d = \frac{H}{W - V}.$$

The values of C_d corresponding to the above pairs of values for H and V are thus:

Configuration (a)				Configuration (b)			
$d = 1.0$ m		$d = 1.8$ m		$d = 1.0$ m		$d = 1.8$ m	
$K = 1$	$K = 2$	$K = 1$	$K = 2$	$K = 1$	$K = 2$	$K = 1$	$K = 2$
0.43	0.74	0.43	0.74	1.23	3.51	0.81	1.60

For track-laying vehicles operating on land, C_d is typically in the range 0.3 to 0.8 depending on soil conditions and vehicle design, and only under very favorable circumstances can it exceed 1.0. Judging from the values of C_d that were calculated above, it seems very unlikely that the proposed machine would be capable of consistent full power operation in configuration (b) even with sharp teeth. In configuration (a) it seems probable that the machine would be capable of consistent full power operation only when there is firm bed material and when the cutting teeth are in good condition and penetrating properly (i.e. in accordance with sound kinematic design).

The nominal power density Q_N, based on gross belt power P_{BT} and maximum usable bar length, is 309 kW/m^2 (38.5 hp/ft^2) for configuration (a), and 267 kW/m^2 (33.3 hp/ft^2) for configuration (b). The apparent belt pressure for configuration (a) working at maximum depth is, from eq 50, 62.4 kN/m^2 (9.04 $lbf/in.^2$) when $K = 1$, and 107 kN/m^2 (15.5 $lbf/in.^2$) when $K = 2$. The apparent belt pressure for configuration (b) working at maximum depth (= maximum usable bar length) is 54.0 kN/m^2 (7.8 $lbf/in.^2$) when $K = 1$, and 92.5 kN/m^2 (13.4 $lbf/in.^2$) when $K = 2$. *Nominal* belt pressures would be 20% higher.

Comparison of these values with Figures 9 and 10 indicates that the proposed power level for the saw is not unreasonable.

Neither of the proposed designs is properly balanced, and it is unlikely that the full power of the saw could be utilized because of traction limitations. Of the two proposals, the one using a vertical cutter bar shows the most promise. In principle, the traction limitation could be removed by increasing the weight of the machine or by decreasing the power of the saw. Decreasing the power of the saw would probably be inadvisable, since it would drop the power density and the nominal belt pressure below the levels that have been found necessary for machines such as coal saws and hard ground ditchers. Increasing the weight of the machine by ballasting would not be a good solution if the crawler has to operate over soft bottom materials, since the track bearing pressure would increase and cause the vehicle to sink deeply. A feasible solution would be to accept configuration (a) and to mount the vertical cutter bar on a carrier vehicle that is both larger and heavier, holding track pressure to a suitably low value for soft ground travel.

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