by Robert F. Carlson

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ICE FORMATION ON RIVERS AND LAKES

INTRODUCTION

Northern hydrology, although sharing many common features with the hydrology of southern latitudes, is dominated by ice-covered surfaces for a large portion of the year. Many engineering projects such as dams, bridges, water supply intakes, pipelines and reservoirs must account for ice-covered waters on both rivers and streams; the ice cover may present an environmental hazard to the project, or the solid surface may offer an opportunity for winter transportation.

This paper discusses the hydraulic and thermal aspects of ice cover formation. The topics summarize the entire life of a seasonal ice cover and lend a perspective to understanding a wide variety of northern engineering problems in the fall, winter and spring of the northern year. This paper makes extensive use of Michel as a reference.[1] Other excellent general reviews include Ashton, Gerard and Starosolzky.[2,3,4]

THERMAL ENERGY TRANSFER AT THE WATER'S SURFACE

Thermal energy transfer at the water's surface plays an important role in determining the thermal energy balance of the entire water body. The four main energy transfer components for most circumstances include solar or short wave radiation, long wave radiation, convection and evaporation-condensation.

Solar Energy

Energy from the sun enters the earth's atmosphere with a flux of 442 BTU/ft^2/hr on a normal plane. In passing through the atmosphere it is influenced by a number of factors, and therefore its value must be modified at the earth's surface. Some of the factors which must be addressed include:

1. Entering at an angle other than normal.
2. Integration with direction and time throughout the day.
3. Turbidity, scattering, and path length.
4. Cloudiness of the sky.
5. Reflectivity or albedo of the surface.
6. Transmission into the surface.

A complete calculation of all these factors is long and tedious, and is best left for another paper. Michel[1] gives two expressions for solar radiation entering the water's surface.

For direct and diffuse radiation at a given time:

$$Y_{rs} = (B_1 + 0.83f)\ S\ C \sin (0.25 + 0.75\ M/N) \quad (1)$$

(expressed as BTU/ft^2/hr)

Where

- B_1 = Absorptivity of water.
- f = Ratio of indirect to direct radiation.
- S = The solar constant (442 BTU/ft^2/hr).
- C = A reduction parameter taking into account the factors listed above; the range is from 0.159 to 2.753 for 60°N latitude. An abbreviated list (abstracted from Michel) is given in Table 1.
- M/N = The fraction of clear skies during daylight.

When the above equation is integrated for the entire day we have:

$$Y_{rs} = 640\ S\ C(0.25 + 0.75\ M/N) \quad (\text{BTU/ft}^2/\text{day}) \quad (2)$$

TABLE 1

Values of C, a Parameter in Equations 1 and 2, for 60°N Latitude

First Day of	C
Sept.	1.97
Oct.	1.23
Nov.	0.54
Dec.	0.20
Jan.	0.16
Feb.	0.39
Mar.	0.99
Apr.	1.88
May	2.52

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Long Wave Energy Transfer

All physical bodies emit radiation energy with a wavelength proportional to the temperature of the body. For practically all natural earth bodies the temperature is much lower than that of the sun and therefore the wavelength is much longer. Because the longer wavelength is more readily absorbed by water vapor, a detailed thermal balance in the vicinity of any earth body is very difficult to calculate. For our purposes, the rate of long wave energy emission from a body is given by

$$Y_{re} = E_1 \sigma T_s^4 \qquad (\text{BTU/ft}^2\text{/hr}) \qquad (3)$$

Where

E_1 = The emissivity factor for the surface water, 0.97; most other natural surfaces, 1.0.

σ = The Stefan-Boltzmann constant, 0.173×10^{-8} BTU/$\text{ft}^2/\text{hr}/^o\text{R}^4$.

T_s = Surface temperature, ^{o}R (0^oF = 492^oRankine).

The long wave energy emitted from the atmosphere to a body is a complicated function of humidity, temperature and cloudiness. Michel gives an equation which sums up all these effects as

$$Y'_{re} = E_2 \sigma B_2 T_a^4 \qquad (\text{BTU/ft}^2\text{/day}) \qquad (4)$$

Where

E_2 = The emissivity factor for the atmosphere = 0.96, cloudy sky; $E_2 = 0.55 + 0.33\sqrt{P_a}$ for a clear sky, where P_a is the vapor pressure measured in terms of inches of mercury (in Hg).

B_2 = An absorptivity factor, given by Michel as 0.83.

T_a = The air temperature near the water surface, ^{o}R.

The total radiation gain at the water's surface can then be calculated from a combination of equations 3 and 4.

Convective Energy Transfer

The convective energy transfer mechanism operates when air molecules near a water surface are heated or cooled and then transported away from the surface by the turbulent motion of the air. The actual mechanism is quite complicated and relies on a sound knowledge of turbulent boundary air theory. In its simplest form, conductive energy transfer is given by

$$Y_c = H_c(T_s - T_a) \qquad (\text{BTU/ft}^2\text{/day}) \qquad (5)$$

where H_c is called the convective transfer coefficient. In a natural situtation it is a complex function of the wind speed, surface roughness and air properties. Michel gives a greatly simplified version as

$$H_c = 4.4\, V_{50} \qquad (\text{BTU/}^o\text{F/ft}^2\text{/day}) \qquad (6)$$

Where

V_{50} = The wind velocity at 50 feet above the surface, ft/sec.

Evaporation Energy Transfer

The fourth prime mechanism of energy transfer at a water surface occurs when water molecules pass to or from the surface, causing condensation or evaporation. Evaporation energy transfer occurs in a manner closely analogous to conductive transfer. The water molecules leave the surface because of a vapor pressure difference between the air at the water surface and the overlying air. As the mass transfer takes place, a corresponding heat transfer must also occur because of the change in state from liquid to gas. The heat transfer rate that accompanies this change of state is given by

$$Y_e = H_e(P_a - P_s) \qquad (\text{BTU/ft}^2\text{/day}) \qquad (7)$$

where H_e is the evaporation heat transfer coefficient and:

P_a = The vapor pressure of the air, in Hg.

P_s = The saturated vapor pressure of the air near the water surface taken at the water surface temperature, in Hg.

Again a simple equation for H_e is given as

$$H_e = 400\, V_{50} \qquad (\text{BTU/}^o\text{F/ft}^2\text{/day}) \qquad (8)$$

An equation replacing P_a and P_s by more useful temperature terms gives

$$\begin{aligned} Y_e &= H_e(16 - 0.5\, e\, T_a) && \text{for } T_a > 0 \\ &= H_e(16) && T_a \leq 0 \end{aligned} \qquad (9)$$

Where

e = Relative humidity.

Change in Water Body Temperature

In order to predict the date of freeze-up, the rate of change of the water temperature, and the rate of ice formation, the total energy balance of a water body must be understood. The important components of the complete thermal energy balance are illustrated in Figure 1.

The terms are defined as:

Q_{ai} = Heat advected in
Q_{ao} = Heat advected out
Q_{sur} = Heat out at the surface ($\Sigma Y = Y_{rs} + Y_{rl} - Y'_{rl} - Y_c - Y_e$)
Q_{as} = Heat advected in by precipitation
Q_{st} = Heat subtracted by change of thermal storage
Q_{gw} = Heat added by groundwater flow
Q_g = Heat added by ground conduction
Q_f = Heat added by flow friction
Q_i = Heat released to the water body by ice growth

For a given water body, then, the complete thermal balance can be calculated according to the following equation:

$$Q_{stor} = (Q_{ai} + Q_g + Q_{gw} + Q_f + Q_{as} + Q_i) - (Q_{sur} + Q_{ao}) \qquad (10)$$

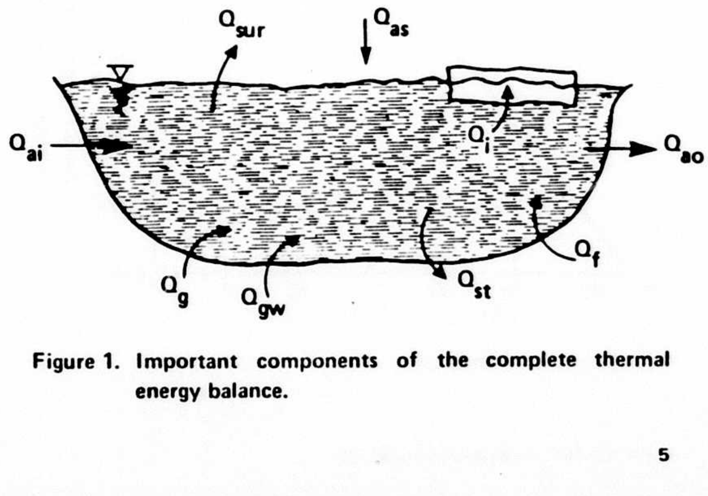

Figure 1. Important components of the complete thermal energy balance.

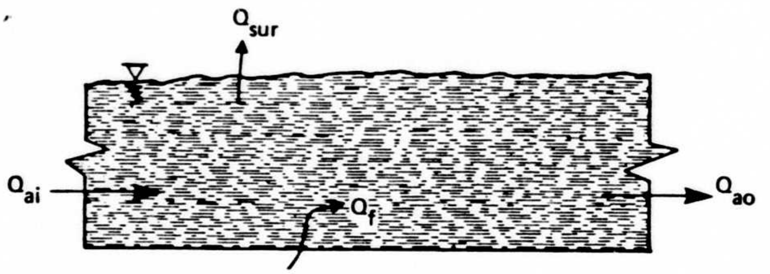

Figure 2. An illustration of the terms needed for analysis of stream temperature.

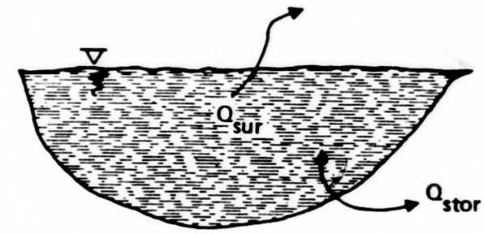

Figure 3. An illustration of the terms needed for temperature analysis of a lake or large pond.

The complete equation as given above would not be used in every case; the portion would depend on the problem of interest. For example, calculation of the thermal balance of a stream needs to include only heat additions of advection in and friction, heat extractions of loss to the surface and advection out (Fig. 2). When these terms are included in equation 10, one can determine the time to freeze-up of that section of the stream, the distance to an ice cover formation following a thermal discharge from a power plant, or the temperature distribution along a reach of the stream.

Michel shows an approximate solution for the change in water temperature, ΔT, in the longitudinal direction of a stream between points l_o and l:

$$\Delta T = \frac{1}{\rho_w C_p} \int_{l_o}^{l} \frac{\Sigma Y \, dl'}{q} \tag{11}$$

Where

ΔT = Temperature difference between two points in the stream, l_o and l, °F and ft.

ΣY = The sum of the surface heat transfer components, BTU/ft^2/day.

q = River discharge per unit width, ft^3/sec/ft.

C_p = Specific heat, BTU/lb.

ρ_w = Unit weight, lb/ft^3

In addition to the calculation of the separate surface energy transfer components, two empirical formulas for ΣY from Michel are:

$$\Sigma Y = 85\, T_a - 3000 \qquad T_a > 0 \tag{12}$$
$$\Sigma Y = 64\, T_a - 3000 \qquad T_a \leq 0$$

Another example is given by the temperature decrease of a lake or large water body. Because of the large surface area, most of the heat transfer is through the surface (Fig. 3). However, in this case, one also needs to recognize the complex temperature-density relationship which may be present because of the fact that water has its maximum density at 4°C. (A typical temperature density curve for water is shown in Figure 4.) As a result of this anomaly, warm surface water in the summer will float on top of the colder water, to an extent depending on depth and other factors. According to Michel, three kinds of temperature profiles can result for a very deep lake, a deep lake, and a shallow lake or water body (Fig. 5). In the fall, as air temperature begins to fall below that of the water surface, the top water particles cool and sink until they reach a layer of like temperature. This is illustrated in Figure 6, where isotherms are shown for consecutive days in a water body cooling very rapidly. First the top layer (the epilimnion) mixes down to the thermocline where there is a sharp break in temperature. Then the water cools more slowly until mixing extends clear down to the bottom of the lake. When the entire water body is cooled to 4°C, ice formation may occur very rapidly depending on the amount of wind mixing near the surface.

The prime factors which determine water temperature at a given date are the depth of the water body, wind conditions and, of course, a prediction of the future air temperatures. In a river, if the total depth is known, a prediction can be made of the likely time of freeze-up, since the entire water column reaches 0°C at the same time because of the mixing which ordinarily takes place. In a lake, the depth significant for calculating the date of freeze-up would be determined by the mixing down to the 4°C isotherm. This value is very difficult to calculate and probably can be known only from experience in a given lake or other large water body.

ICE FORMATION

Up to this point, the processes of heat exchange at the surface and redistribution of temperature within a lake or stream have served to reduce the surface water to near 0°C, at which time the ice formation process can begin. The processes for lakes differ somewhat from those for rivers but the basic features remain the same. At the initiation of ice formation two processes can take place — frazil ice growth in the center of a stream

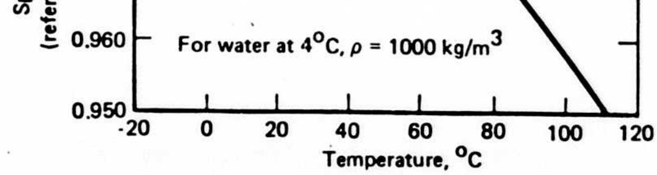

Figure 4. Specific gravity vs. temperature relationship near 4°C for water.

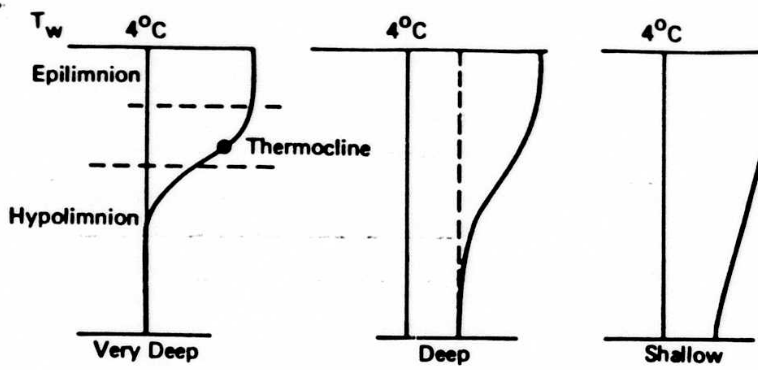

Figure 5. Three examples of lake stratification prior to freeze-up.

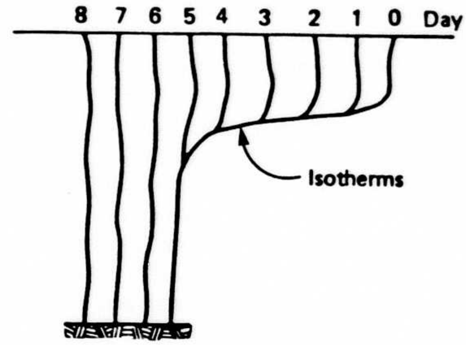

Figure 6. Equal temperature line on consecutive fall days as a stratified lake cools rapidly.

or lake, and shore ice growth along the borders of a stream or lake.

Frazil ice growth takes place with the formation of ice crystals near the water surface. Just prior to that time, the surface water temperature usually decreases to a bit below 0°C so the water is somewhat supercooled. In a small stream the entire water column will become slightly supercoooled; in a large lake, only the top surface will cool below 0°C. As the supercooling continues, ice crystals begin to form around nuclei at the surface of the water. The exact process of nucleation is not entirely clear, but the result has been widely studied and reported.[5] The ice crystal grows very quickly in a disc-shaped form; if the water is even slightly turbulent as a result of stream flow or wind waves, the disc is swept down into the water column. In the past this led to the mistaken notion that the frazil crystals formed spontaneously throughout the water column and perhaps even on the bottom of the stream. Once ice formation begins, supercooling ends because of the heat released by the freezing process, and the stream or lake surface temperature very quickly rises to 0°C.

Once the frazil formation is actively underway, two important things happen to the stream from a hydraulic point of view. One is the possible appearance of so-called anchor ice on rocks, steel, and other objects of high thermal conductivity during the early portion of frazil formation. Because these objects were supercooled during the brief period before frazil formation, they now become a medium to which the frazil particles can readily attach themselves. There also can be some direct ice formation in supercooled water impinging directly on the solid surface. Second, of more importance, is the contribution of the frazil ice to the formation of a solid ice cover at the surface. The frazil particles have a great affinity for each other and very quickly form first small flocs, then larger flocs, until finally they become so large that their buoyant lift overcomes the downward transport of the turbulent motion of the water. The flocs then rise to the surface and tend to continue congealing together until floating ice pans form on the surface. Although Figure 7 shows a time sequence of frazil to pan formation, the actual event is probably not quite as clear. In fact all three of the formation processes, frazil, floc, and ice pans, may be occurring at the same time.

The final sequence of the transformation of the frazil particles to solid ice cover occurs when the pans push together to form a solid cover (Fig. 8). If the flow is slow enough, new pans floating into position from upstream will push against the already-formed solid ice cover and continue to extend it in the upstream direction. If the flow is too fast, either the pans are swept under the leading edge of the ice cover and stick there, producing a hanging dam and thickening the ice cover at that point, or the pans continue to sweep on downstream. Therefore, frazil particles can form in steep areas but no ice cover results, while a solid ice cover forms in slower areas.

In smaller streams the formation of ice at the shore and its extension and growth outward can be the major process in forming an ice cover. On medium to large streams both shore and frazil ice formation probably occur nearly simultaneously so both are equally dominant processes. After the solid ice cover has formed, a top view of the stream would look like Figure 9, with the shore ice reaching out and forming a very smooth ice surface, while the frazil pans would be pushed and somewhat jumbled together. In winter, this feature is clearly evident on Alaskan streams where the boundary between frazil and shore ice often forms a series of ridges as the freezing season progresses and recedes for several sequences before the final ice cover forms. Also, one can clearly see the predominance of frazil in the fastest part of the river, in the outside of bends, while shore ice predominates on the quieter parts on the inside

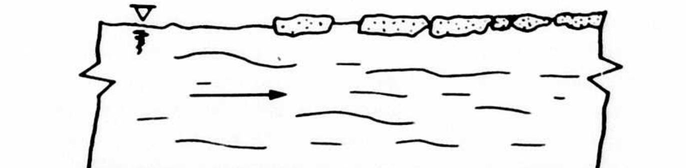

Figure 7. Sequence of frazil formation in a swift stream or wind-swept lake.

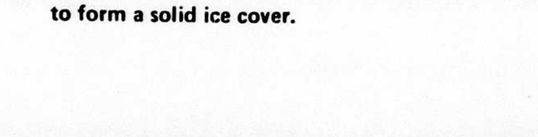

Figure 8. The frazil pans finally fill the stream and push together to form a solid ice cover.

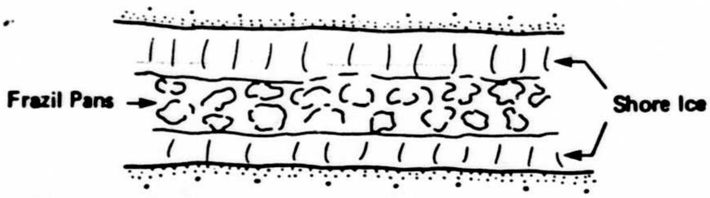

Figure 9. The plan view of an iced stream with shore ice on the sides and the frazil pans in the middle.

of bends. The ice cover provides an interesting snapshot of the river morphology for nearly the rest of the winter.

One interesting engineering design problem should be pointed out. During the period of active frazil formation, when the particles are still small enough to be swept down into the entire stream, frazil particles are capable of clogging water supply intakes. It is not at all uncommon for entire power plants or water supply and treatment systems to be rendered entirely inoperable at this time. If the frazil formation season lasts for many days, some remedial measure may be needed, such as building a barrier and causing the water to form a quiescent pool so the frazil flocs can float to the surface and water can be withdrawn from the ice-free levels underneath. If the frazil formation lasts for only a few hours, it may be more expeditious simply to shut off the water supply until a solid ice cover forms. The water can then be withdrawn from under the ice cover for the remainder of winter.

The determination of whether the reach will stay open at the leading edge or freeze progressively upstream with the arrival of new ice pans depends on the hydraulic condition at the leading edge of the ice cover. The important factors are velocity, depth of flow, and ice thickness which combine to produce a downward force from an airfoil effect exerted by the water flowing underneath. This downward force is met by an upward buoyant force; if equilibrium is reached, the leading edge is right at the water's surface. (A slightly different criterion exists for the whole leading edge but the same principle applies.) Michel summarizes this phenomenon and gives as a governing equation:

$$V/gY = N_F = 0.08 \qquad (13)$$

Where

- N_F = A Froude number.
- V = The upstream velocity, ft/sec.
- g = The gravity parameter, ft/sec^2.
- Y = The upstream depth, ft.

If N_F, the upstream Froude number, exceeds the critical value of 0.08, the floe or leading edge will be turned under and the ice cover will not proceed upstream.

If the velocity is too high to permit an ice cover to form on a steep section, that area of the stream may stay open for the entire winter and continually produce frazil ice. This, in turn, can be swept along and accumulate in large deposits downstream. Michel has devoted an entire chapter to frazil problems and a discussion of their engineering features. Another excellent review has been given by Osterkamp.[5]

ICE GROWTH

Once a solid ice cover is formed by either shore ice growth or frazil accumulation, the ice cover may continue to thicken throughout the winter. The most dominant driving force, of course, is the cool overlying air that removes heat from the ice-water interface, causing additional ice to freeze. Ice formed in this manner is termed "black ice" because of its relatively clear appearance. Its rate of accumulation can be calculated by the well-known laws of conduction of thermal energy through solids. The complicating factors are the amount of ice present as an initial condition, the amount of snow cover as it occurs, accumulates and changes depth, and the rate of heat transfer at the surface. Unfortunately, although the mechanism of heat transfer in solids is well understood, its application to actual field conditions is very difficult because of the limits on the possible amount of instrumentation and understanding of the heat transfer climatology at a particular site. As a result, a simplified equation is often used to calculate the depth of a stream's ice cover that has occurred since the onset of the freezing season:

$$d_i = a\sqrt{S} \qquad (14)$$

Where

- d_i = Ice depth, inches.
- a = A coefficient; its value depends on climatological conditions, ranging from 0.8 for lakes with no snow to 0.2 for a small river with rapid flow.
- S = The freezing degree-days since fall.

A balance must exist between the downward weight of the ice and snow and the upward pressure of the water on the bottom of the ice (Fig. 10). The diagram shows the upward force for a unit area column, as in equation 15.

$$F_w = W_i + W_s \qquad (15)$$

or

$$\rho_w g h_w A = \rho_i g d_i A + \rho_s g d_s A \qquad (16)$$

and solving for h_w

$$h_w = \rho_i/\rho_w\, d_i + \rho_s/\rho_w\, d_s \qquad (17)$$

Where

- F_w, W_i, W_s = Force of water, ice, and snow, lb.
- ρ_w, ρ_i, ρ_s = Mass density of water, ice, and snow, slugs/ft^3.
- d_i, d_s = Depth of ice and snow, ft.
- A = Area of force balance.

At a point of just flooding where the height of the water column, as in a crack, is equal to the depth of the ice, and, if we assume the density of the snow to be 0.2, we can derive an expression for the depth of snow which will cause the ice surface to flood:

$$d_s = \tfrac{1}{2} d_i \qquad (18)$$

If new snow exceeds this amount, water is forced through cracks, floods the ice surface and presents an entirely new thermal problem. Now, rather than ice being added through heat removal at the original ice-water interface underneath the solid ice cover, the new water freezes down from the top of its surface to the top of the old ice surface. Then the thermal problem again moves back to additional ice at the bottom of the ice cover. If the surrounding climatology of a stream allows for repeated additions of snow, especially in the early part of the winter when the ice cover is still thin, the formation of ice on top

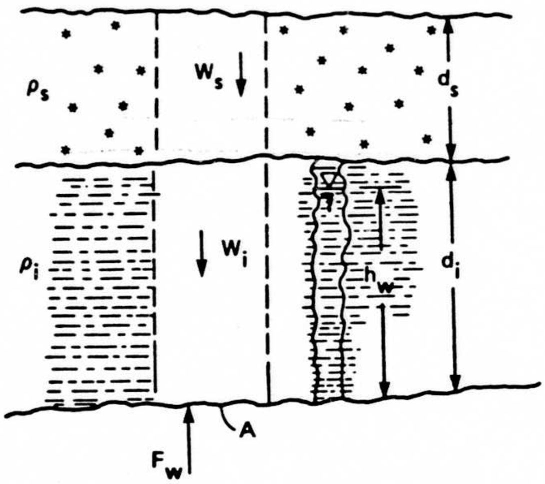

Figure 10. Diagram of the balance between the downward force of snow and ice, and the upward pressure of water on the bottom of the ice.

of the original ice cover can be the predominant process. Ice formed in this way is called snow ice and has a much milkier or more opaque appearance than the underlying clear or black ice.

An interesting problem results if a sufficient amount of snow has accumulated to cause flooding of the original ice cover. Because two ice-water surfaces are now present, the region between the two becomes isothermal, with no heat transfer. Since there is no longer an appreciable rate of heat removal from the bottom face, only a slight amount of thermal energy in the water (such as additions by power plants, groundwater, or flow friction) can cause very rapid ablation of the ice. This effect can be rigorously calculated but a field application is very difficult. Quite often in the spring, because of new snow or the sudden warming of the air, the bottom heat transfer can become important; ablation rates of one to two inches per day are not unusual. During this time unsuspecting travelers crossing a stream in the same location as they have all winter may suddenly exceed the strength of the ice and break through.

One can easily see that the formation of ice can be quite complicated and is not easily predicted, even with good information.

ICE BREAKUP

In the spring, increasing air temperature and solar radiation end the process of ice growth and begin to cause ice ablation. This may occur either from the top, first through snow melt and then by direct ablation of the ice surface, or it may occur directly from the bottom, because the water temperature is slightly above 0°C and heat is no longer being removed from the ice-air interface. Often the ice becomes very porous at this point and forms crevasses along the original growth planes, producing candle ice.

The final process of breakup or movement of the ice occurs with increasing flow and the disintegration of the ice cover and is very difficult to predict. If the river flow begins to rise while the ice cover is still very solid, huge pans of ice may break loose from their shore mooring and begin to float downstream. An ice jam may result if a constriction is present which does not allow a sufficient amount of ice transport. A notable example of this is the nearly annual occurrence of ice jams on the Yukon River in Alaska. However, if the local ice becomes considerably disintegrated by the time fresh flow arrives, the ice floes may become too weak to form any effective jam or be transported any distance downstream. Probably the only reliable prediction of the breakup is the observation of the process in a reach for a number of years. Predicting the ice breakup period is important for transportation planning, because very little use can be made of either lake or stream surfaces during this time.

SUMMARY

The hydrologic features of northern regions are dominated by ice cover for much of the year. The annually-renewed solid surface offers special opportunities for transportation, greatly alters the natural environment, and presents serious design problems for northern engineering structures. Before northern engineers or designers can properly account for ice covers, they must have an adequate understanding of the processes of heat transfer, water body temperature change, ice cover formation and growth, and ice breakup. The important components of surface heat transfer can be estimated with several simple concepts and equations, at least to the accuracy necessary for most field calculations. When the surface heat transfer regime is understood, thermal energy balance concepts can be applied to common engineering situations to calculate the temperature change rate in ponds, lakes and rivers. Beginning with a known summer or fall initial condition, the time of the ice formation process or winter open water equilibrium condition can be calculated. Once the water temperature is lowered to 4°C in a large water body or 0°C in a river, the ice formation process can begin. The ice formation process occurs as frazil ice growth and shore ice growth, with a solid ice cover being formed in a river if the water velocity is slow enough. The rate of ice thickness growth can be calculated but heavy snow deposits can often greatly complicate the estimation process. The breakup season in the spring completes the ice cover season. Nearly impossible to forecast from basic principles, the breakup season is best understood by annual observation on a particular river.

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