AIR-WATER FLOW IN HYDRAULIC STRUCTURES

UNITED STATES DEPARTMENT OF THE INTERIOR
WATER AND POWER RESOURCES SERVICE
The purpose of this report is to summarize the work that has been completed on air-entrainment and air-demand in both open- and closed-conduit flows. The intent was to produce a concise reference source from which design manuals, monographs, and charts for specific applications could be prepared. Areas that need additional research have been identified. The report was prepared from available reference material. In several areas, data from several references have been combined to produce generalized curves. Includes 64 figs., 74 ref., 3 app., and 155 pp.
AIR-WATER FLOW IN HYDRAULIC STRUCTURES

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United States Department of the Interior
Water and Power Resources Service
FRONTISPICE.—High velocity jet from a slide gate. P801-D-79273
As the Nation's principal conservation agency, the Department of the Interior has the responsibility for most of our nationally owned public lands and natural resources, protecting our fish and wildlife, preserving the environmental and cultural values of our national parks and historical places, and providing for the enjoyment of life through outdoor recreation. The Department assesses our energy and mineral interests of all our people. The Department also has a major responsibility for American Indian reservation communities and for people who live in Island Territories under U.S. administration.

ENGINEERING MONOGRAPHS are published in limited editions for the technical staff of the Water and Power Resources Service and interested technical circles in Government and private agencies. Their purpose is to record developments, innovations, and progress in the engineering and scientific techniques and practices which are used in the planning, design, construction, and operation of water and power structures and equipment.
Preface

The material assembled in this report is the result of studies extending over many years by a large number of engineers. Ellis Picket at the U.S. Army Engineer Waterways Experiment Station in Vicksburg, Mississippi, supplied a reference list dealing with air-water problems. Personnel of the Water and Power Resources Service E&R Center, Water Conveyance Branch made their files and drawing on air design criteria in pipelines available for publication in this report. Prior to publication, the report was reviewed by Ellis Pickett and Ted Albrecht with the U.S. Army Engineers; and by engineers in the Dams, Mechanical, and Water Conveyance Branches, E&R Center, Water and Power Resources Service. The many constructive comments by these individuals and the assistance of Richard Walters who provided continuity and technical editing is greatly appreciated.
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<td>Total depth of underlying and air free zones</td>
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<td>$E$</td>
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<td>Eötvös number $E = \frac{\gamma D^2}{\sigma}$</td>
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<td>Euler number $E_t = \frac{\Delta p}{\rho V^2}$</td>
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<td>Distance Reynolds number $R_e = \frac{V_x}{\nu}$</td>
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<td>Ratio of the circumference of any circle to its radius, 3.14159...</td>
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$\infty$ Infinity
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Introduction

In many engineering projects a strong interaction develops between the water flowing through a structure and the air which is adjacent to the moving water. Sometimes the interaction produces beneficial effects. However, more often than not, the effects are not beneficial and the remedial action required to reduce the effects can be costly.

Cases in which air-water interaction develop include:

- Open channels with fast flowing water that require depths adequate to contain the air which is entrained within the water
- Morning-glory spillways that must have a capacity to convey the design flood and its entrained air
- Vertical shafts that entrain large quantities of air at small water discharges
- Measuring weirs that need adequate ventilation to prevent false readings and to eliminate surging
- Outlet gates that require adequate aeration to prevent the development of low pressures—which can lead to cavitation damage
- Emergency gates at penstock entrances that require ventilation to prevent excessive negative internal pressures during draining or emergency gate closures
- Sag pipes (inverted siphons) that can be damaged due to blowback of entrained air
- Long pipelines that require air release and vacuum relief valves

From these cases it is noted that air-water flows can be generalized into three basic flow types:

1. Air-water flows in open channels,
2. Air-water flows in closed conduits, and
3. Free-fall water flows.

The first type usually is called air-entraining flow because air is entrained into the water mass. The second basic flow type generally is referred to as air-demand. The term air-demand is both misleading and technically incorrect, since an air vent does not demand air any more than an open valve demands water. However, since the term has been in common use for over 20 years, efforts to improve the nomenclature seem rather futile. The third type is referred also to as air-entraining flow.

"'siphon, inverted—A pipe line crossing over a depression or under a highway, railroad, canal, etc. The term is common but inappropriate, as no siphonic action is involved. The suggested term, sag pipe, is very expressive and appropriate." Nomenclature for Hydraulics. Comm. on Hyd. Str., Hyd. Div., ASCE, 1962.
Purpose and Application

The purpose of this report is to summarize the work that has been done on *air-entrainment* and *air-demand* regarding the most recent theories and to suggest ways in which the results can be applied to design. The intent was to produce a concise reference of material from which design manuals, nomographs, and charts for specific applications could be prepared.

Although many generalizations of the data can be made, some types of flow conditions that are encountered in practice can be treated only by individual studies with physical models. These cases are identified when they occur.

Additional studies are needed in many areas. Some of the most critical areas requiring further research include the following:

- Effects of turbulence and air concentration on bubble dynamics
- Fluid dynamics in the developing aeration regime of free-surface flow
- Effects of hydraulic and conduit properties on probabilistic description of water surface in free-surface, high-velocity flow
- Effect of pressure gradients on air flow in partially-filled, closed conduits
- Bubble motion in closed-conduit flows for conduit slopes exceeding 45-degrees
- Effects of ambient pressure levels on cavitation characteristics of gates and valves discharging into a closed conduit
- Interaction between the air and a free jet
Summary and Conclusions

Methods have been developed to predict the mean air concentration and the concentration distribution with open channel flow. A new description of the free water surface in high velocity flow is proposed which more accurately represents actual conditions in high velocity flow. The effect of air entrainment on the performance of a stilling basin can be estimated using a bulked flow concept. A computer program (app. II) is presented with which the mean air concentration in steep chutes and spillways can be estimated.

With exception of a falling-water surface and decreasing flow in pipelines, closed conduit flows require model studies. When properly conducted and analyzed, model studies will yield accurate data for estimating air-flow rates. Experimental methods are discussed. A computer program (app. III) is presented which can be used to predict the airflow rate with a falling-water surface. Design charts are presented for sizing air relief valves and vacuum valves on pipelines.

The airflow rate in vertical shafts was found to be extremely dependent upon the flow conditions at the shaft inlet. Equations are included for estimating the airflow rate having various inlet conditions.

Factors influencing the airflow rate around free falling jets are discussed. This area is identified as one needing additional research. Equations are presented from which the air entraining characteristics of a jet entering a pool can be estimated.
INTRODUCTION

In observing flow in a chute or on an overflow spillway, one normally observes a region of clear water where the water enters the chute or spillway. Then—at some distance downstream—the water suddenly takes on a milky appearance. Lane [46] suggested that the "white water" begins when the turbulent boundary layer from the floor intersects the water surface. The validity of this assumption has been verified by many researchers. The cases in which the boundary layer creates the air entrainment are referred normally to as self-aerated flows. However, this is not the only way in which air entrainment can begin on chutes and spillways. The American Society of Civil Engineers Task Committee on Air Entrainment in Open Channels [5] has summarized tests in which air entrainment is generated by the boundary layer on the side walls of chutes. They also reported tests in which air entrainment was observed downstream of piers on overflow spillways. This latter case is the result of the flow rolling over on itself as it expands after passing through the opening between the piers. Levi [49] reported on longitudinal vortices on spillway faces. These vortices can entrain air if they intersect the water surface. All of these forms of air entrainment are apparent in figure 1.

Air entrainment implies a process by which air enters into a body of water. Normally, the appearance of "white water" is considered to be synonymous with entrainment. This is not always true. For instance, if the water surface is rough enough and moving at a sufficiently high velocity, the surface will appear to be white even though the water volume contains no air. The whiteness of the water is caused by the large number of reflections coming from different angles off the rapidly moving highly irregular surface (refer to frontispiece). For high water velocities, one’s eye does not respond rapidly enough to observe each individual reflection. Instead, these individual reflections blur into a fuzzy mass which appears white. High speed photography of "white water" demonstrates this effect very well. This leads one to the obvious conclusion that a flow could conceivably appear frothy but actually does not entrain any air! With air in the water, reflections also come from the surface of the bubbles. These reflections produce the same impression...
of "white water" as the water surface reflections.

Experiments have shown that flow in channels with mild slopes do not entrain air even though the boundary layer intersects the water surface. Thus, some degree of turbulence must be exceeded for the entrainment process to begin.

The turbulence causes the water surface to become irregular enough to trap bubbles of air. These bubbles of air are then diffused downward into the body of water if the vertical water velocities induced by the turbulence in the flow are larger than the terminal velocities of the bubbles. The terminal velocity $V_t$ of a bubble is defined as the rate of rise of a bubble in a liquid in which the effects of turbulence, walls, other bubbles, and acceleration are negligible. The interaction of the terminal velocities of bubbles and the turbulence are considered in the following section.

**BUBBLE DYNAMICS**

**Terminal Velocity of a Single Bubble in Still Water.**

In still water, surface tension is the predominant effect on the shape of very small bubbles. Therefore, small bubbles tend to be perfect spheres. The motion of these bubbles through a fluid can be described by a balance between the buoyant forces and the viscous forces. However, as the bubbles become larger, surface tension effects become small with respect to shear forces. The shape of these larger bubbles can be approximated by a spherical cap having an included angle of about 100 degrees and an unstable relatively flat base (fig. 2). Because different effects predominate at different bubble diameters, one can expect the correlation between the bubble size and its terminal velocity to vary as the bubble diameter varies. Haberman and Morton [26] have experimentally determined the terminal velocity as a function of the bubble size (fig. 3).

Assuming the bubble to be a rigid sphere, the terminal velocity $V_t$ of small bubbles can be written as

$$V_t = \frac{2}{9} \left( \frac{R^2 g \left[ 1 - \frac{\rho_g}{\rho_w} \right]}{\nu_f} \right)$$

where

- $g$ = gravitational constant (acceleration), 9.81 m/s²
- $R$ = bubble radius, mm
- $\nu_f$ = water kinematic viscosity,
  - $1.5 \times 10^{-6}$ m²/s at 10 °C
- $\rho_w$ = water density, 1000 kg/m³ at 10 °C
- $\rho_g$ = air density, 1.29 kg/m³ at 10 °C

This also is known as Stokes' solution.³ Substituting the respective values for air, water, and gravity into equation 1 gives the terminal velocity of a small air bubble in meters per second as

$$V_t = 1.45 R^2$$

Theoretically, this relation is valid only for bubble radii $R$ smaller than 0.068 mm.

For bubble radii between 0.068 and 0.40 mm, the empirical relation

$$V_t = 0.625 R^2$$

fits the data.

With bubble radii between 0.40 and 10 mm, the terminal velocity is about equal to 0.25 m/s. As the bubble diameter increases from 0.4 to

³G.G. Stokes was the first investigator to analytically determine the drag on a slowly moving sphere in a viscous fluid falling as a result of its mass relative to the fluid mass.
OPEN CHANNEL FLOW

Figure 1.—Forms of air entrainment on a spillway—Canyon Ferry Dam, Montana. P801-D-79276
10 mm, its form changes from a sphere to a spherical segment. When the diameter is about 2 mm, an instability in the bubble path can be observed. This instability gives the bubbles an irregular or spiral trajectory. Comolet [17] argues that in this region both buoyant and surface tension forces are significant with respect to inertial forces, and proposes the equation

$$V_t = [0.01 R_b + (0.079/R_b)]^{1/2} \quad (5)$$

For bubbles larger than 10 mm, the terminal velocity is only a function of the ratio between the buoyant and inertial forces. Davies and Taylor [18] show the terminal velocity is

$$V_t = \frac{2}{3} (g R_c)^{1/2} \quad (6)$$

where $R_c$ is the radius of curvature of the bubble cap.

Using this relation and the spherical segment geometry of figure 2, the terminal velocity in terms of an equivalent radius can be shown to be equal to

$$V_t = (g R_b)^{1/2} \quad (7)$$

or

$$V_t = 0.10 R_b^{1/2} \quad (8)$$

for $R_b$ in millimeters and $V_t$ in meters per second.

This equation approaches Comolet's relation (eq. 4) asymptotically when the bubble radius becomes large.

### Bubble Size in Shear Flows

The mean bubble size in flowing water or in mechanically agitated systems is determined primarily by the shearing stresses within the fluid. This effect can be visualized by examining the two extreme conditions. Assume that very small bubbles are introduced into a turbulent flow. As the bubbles rise, they tend to form into a mass or agglomerate because of entrainment in each other's wake. As the individual bubbles touch they coalesce to form a
larger bubble. This process continues until larger and larger bubbles are formed.

At the other extreme, assume that a very large bubble is introduced into a turbulent flow. The turbulence of the flow field introduces shear stresses which tends to tear or fracture the bubble into smaller and smaller bubbles.

Due to the simultaneous action of agglomeration and fracture, it can be inferred that some critical bubble size is reached which represents a balance between surface tension forces and fluid stresses. This relation is expressed through a suitably defined Weber number $W$. The only equation available for estimating the critical bubble size was developed by Hinze [35]. The equation is

$$d_{0.5} = 0.725 \left[ \frac{\sigma}{\rho_w} \left( \frac{P}{M} \right) \right]^{1/3}$$  \hspace{1cm} (9)

where

- $d_{0.5}$ = bubble diameter for which 95 percent of the air, by volume, is contained in bubbles of this diameter or smaller
- $P/M$ = rate of energy dissipation per unit mass
- $\rho_w$ = fluid density
- $\sigma$ = interfacial surface tension
The rate of energy dissipation per unit mass for flow in pipes can be estimated in the following manner; Rouse [59] showed that the rate of energy dissipation in a length of conduit \( L \) is given by

\[
P = Q \gamma h_l
\]

where

\( Q = \) discharge
\( h_l = \) head loss through a length of conduit \( L \)
\( \gamma = \) specific force of fluid

The unit mass is given by

\[
M = q_w A_L = \frac{Q L}{V}
\]

where

\( A = \) cross sectional area of conduit
\( V = \) mean flow velocity

Therefore, for flow in conduits, the rate of energy dissipation per unit mass is given by

\[
\frac{P}{M} = \frac{gh_l V}{L} = \gamma S_f V
\]

where \( S_f = \) slope of energy grade line = \( h_l / L \)

Substitution of the rate of energy dissipation per unit mass of equation 12, into 9 gives

\[
d_{ts} = 0.725 \left[ \frac{a}{Q_w} \right]^3 \left( \frac{1}{\gamma S_f V} \right)^{2/3}
\]

In dimensionless terms equation 13 can be written as

\[
\frac{d_{ts}}{D} = 0.658 \left[ \left( \frac{a}{\gamma D^2} \right)^3 \left( \frac{D^3 g}{Q^2} \right) \left( \frac{1}{S_f} \right)^{2/3} \right]
\]

where \( D = \) conduit diameter

The first dimensionless ratio \( \gamma D^3 / a \) is known as the Bond, Eotvos or Laplace number. The second ratio \( D^3 g / Q^3 \) is another form of the Froude number \( V / (g D)^{1/2} \). Additional information concerning the mean concentration distribution of the bubbles and the direction of their motion in nonhorizontal flows can be found in this chapter (Design Parameters —Air distribution in the mixing zone) and in the following chapter (Flow in Partially Filled Conduits—Analytic Estimates).

**Terminal Velocity of Bubbles in Turbulent Flow**

Even though a bubble diameter can be determined for turbulent flow from equation 14, the terminal velocity of these bubbles cannot be determined simply from figure 3. The figure can be used only to estimate the terminal velocity of single bubbles in still water. Both turbulence and the presence of other bubbles modify the terminal velocity shown on figure 3.

As with sediment particles, turbulence tends to keep the air bubbles in suspension. Thus, the effect of turbulence is to reduce the terminal velocity of the bubbles. If \( V_t \) is the terminal velocity of bubbles in still water, then \( V_f \) is the terminal velocity in a turbulent flow. Their relation can be expressed as

\[
V_f = a V_t
\]

where \( a \) is an empirically determined variable.

Haindl [28] determined the relation between the variable \( a \) and a form of the Froude number \( F \) applicable to annular jumps. By making appropriate assumptions, the Froude numbers were converted to equivalent quantities of the dimensionless discharge parameter. The relation between the variable \( a \) and the dimensionless discharge parameter is given on figure 4.
The effect of turbulence on the bubbles can be visualized by considering the buoyant and the turbulent diffusion forces acting on the bubbles. The bubbles tend to move upward because of buoyancy. Whereas turbulence tends to move bubbles from areas of high concentration into areas of low concentration. The balance between mass flow rates caused by these two forces is given—normal to the channel bed by

\[ CV_f = \varepsilon \frac{dC}{dy} \]  

where
- \( C \) = local air concentration
- \( V_f \) = terminal velocity of bubbles in turbulent flow
- \( y \) = vertical direction
- \( \varepsilon \) = mass transfer coefficient of bubbles

If a functional relation could be obtained for \( V_f \) and \( \varepsilon \), then the air concentration as a function of depth could be determined.

References could not be found that indicate the magnitude of the effect of air concentration on the terminal velocity of a bubble. Further
vertically divided into four zones, Killen [41] and Killen and Anderson ([42], fig. 5). These are:

1. An upper zone of flying drops of water,
2. A mixing zone where the water surface is continuous,
3. An underlying zone where air bubbles are diffused within the water body, and
4. An air free zone.

The upper zone consists of water particles that have been ejected from the mixing zone. These particles can rise a considerable distance above the mean water surface. Normally, this region is neglected in engineering considerations since its mass is small.

The mixing zone consists of a region of surface waves having random amplitudes and frequencies. A knowledge of the characteristics in the mixing zone is extremely important since all air ingested into the main body of the water or released from the flow must pass through this zone. Also, the maximum wave heights that occur in the mixing zone determine the height of the open channel sidewalls if overtopping is to be prevented.

The underlying zone is a region into which the waves do not penetrate. The air concentration at any depth in this zone is determined by the number of air bubbles and their size. The primary factor influencing the air concentration distribution is the turbulence intensity distribution throughout the flow. Using turbulent boundary layer theories, it has been possible to develop correlations for the air concentration distribution in this zone. However, the problem has not been solved completely since the air bubbles tend to inhibit the turbulence. The interrelations between air concentrations, bubble size distribution, and turbulence intensities have not been determined yet.

An air-free zone exists only in that section of the channel where aeration is still developing. In most practical applications, the boundary between the air free zone and the underlying zone cannot be determined accurately. At the interface, the air concentration has a very small value and the rate of change in concentration with depth is small. Halbron et al., [31] noted extremely fine bubbles which could not be detected near the bottom of their channel by the air concentration measuring apparatus; this indicates that the location of the interface may in fact be a function of the sensitivity of the measuring instrument.

In addition to defining the flow structure in a vertical plane, it also is possible to identify flow regimes in a longitudinal direction for flow in a wide channel. Here, a wide channel is defined as one in which the channel width is greater than five times the flow depth. Borman [11] identifies three distinct regions in self-aerating flows in wide channels. They are:
OPEN CHANNEL FLOW

1. A regime of no air entrainment where the turbulent boundary layer has not reached the water surface,
2. A regime of developing air entrainment in which the air concentration profiles are not constant with distance, and
3. A regime of fully developed air entrainment in which the air concentration profiles are constant with distance.

Keller, Lai, and Wood [39] divide Bormann's middle regime into two sections. The first is a region where the aeration is developing, but the air has not reached the bottom of the chute. The second is a region where the air has reached the bottom of the chute, but the air concentration profile continues to vary with distance (fig. 6).

![Diagram of air entraining flow regimes in open channel flow.](image)
DESIGN PARAMETERS

From an engineering viewpoint, the significant parameters in the design of a conveyance structure are:

- Distance to the beginning of aeration
- Distance to fully develop aerated flow
- Mean air concentration in the flow
- Flow depth of the aerated flow
- Water and air velocities in the aerated flow

The results of investigations concerning these parameters are presented in the following sections.

Location of Beginning of Aeration

The point at which "white water" begins in a wide channel generally is accepted to be the location where the turbulence effects generated at the channel floor first reach the water surface. Many different investigators have proposed equations for the location of this so-called "critical point." Many of the early predictions were rather poor. However, as the understanding of boundary layer growth over smooth and rough surfaces has improved, predictions of the "critical point" location also have improved. Although some questions concerning the theory still exist, present methods yield results that are sufficiently accurate for engineering purposes.

Typical examples of early correlations for the boundary layer thickness $\delta$ are those by Annemuller [4] who gave

$$\frac{\delta}{x} = 0.01$$

(17)

where $x$ = distance from start of boundary layer growth,

Hickox [34] who gave

$$L_c = 14.7 q^{0.53}$$

(18)

where

$L_c$ = distance to start of self-aeration
$q$ = unit discharge, cubic feet per second per foot of width,

and Beta et. al., [10] who recommended values of

$$\frac{\delta}{x} \text{ between } 0.016 \text{ and } 0.01.$$

Schlichting [63] applied the results of measurements made in a pipe directly to a flat plate and found that the boundary layer thickness was given by

$$\frac{\delta}{x} = 0.37 \left( \frac{U x}{\nu_f} \right)^{0.2}$$

(19)

where

$x$ = distance measured from the chute entrance
$U$ = free stream velocity
$\delta$ = distance from the boundary at which the velocity equals 99 percent of $U$
$\nu_f$ = kinematic viscosity of water

The free stream velocity is

$$U = \sqrt{2g(A z + H_o - d \cos \alpha)}$$

(20)

where

$d$ = water depth of computational point
$g$ = gravitational constant (acceleration)
$H_o$ = total head on crest
$\Delta z$ = difference in elevation from crest to computational point
$\alpha$ = angle channel makes with horizontal

Equation 19 is also the expression used by Rouse [59] for flow over smooth surface.
analyzing the results of Bauer [8], Halbronn
[30] showed that the value of the coefficient for
open channel flow should be equal to 0.16
instead of 0.37.

Equation 19 neglects two important con-
siderations:

1. The surface can be hydraulically rough, and

2. Intermittent turbulence is present at a
distance of up to 1.2d from the channel
floor.

A hydraulically rough surface is one in which

\[ \frac{u_* k_s}{v_f} \geq 70 \] (21)

where

- \( k_s \) = equivalent sand grain roughness
- \( u_* \) = shear velocity
- \( \tau_0 \) = wall shear stress
- \( \rho_w \) = water density
- \( T_* \) = wall shear stress

If the surface is rough, then the effect of the
roughness height must be included in the com-
putations.

The second consideration means that “white
water” generally will occur before the boundary
layer, as previously defined, actually reaches
this difficulty by redefining the boundary layer
thickness. Keller and Rastogi [40] recognized
the same problem. Their point of incipient air
entrainment also occurs at a location which
lies somewhat above the previously defined
boundary layer thickness. Thus, these methods
account for the intermittent turbulence that
occurs outside the conventionally defined
boundary layer thickness.

Bormann [11] used a rather novel method of
determining the “critical point.” Bormann’s
method involves the simultaneous solution of
equations relating local loss coefficient \( C_f \),
boundary layer thickness \( \delta \), and the distance
Reynolds number \( R_s \). In Bormann’s scheme
the local loss coefficient can be viewed as a
parametric term through which the other
parameters are related. The appealing aspect of
this approach is that it can be compared with
the presently available equations for flow over
smooth and rough boundaries.

The correlations relating the boundary layer
thickness \( \delta \) and the local loss coefficient \( C_f \) are

\[ \frac{1}{C_f^{1/2}} = 3.85 \log \left( \frac{\delta U}{v C_f} \right) + 3.67 \] (22)

for hydraulically smooth surfaces, and

\[ \frac{1}{C_f^{1/2}} = 3.85 \log \left( \frac{\delta}{k_s} \right) + 6.45 \] (23)

for hydraulically rough surfaces.

The correlation between distance Reynolds
number and the local loss coefficient was de-
termined empirically from

\[ C_f = \frac{b_s}{(\log R_s)^{0.33}} \] (24)

where \( R_s \) = distance Reynolds number = \( \frac{U x}{v} \)

The \( b_s \) value (fig. 7) can be approximated by

\[ b_s = 0.32 + 8.15 k_s^{0.47} \] (25)

where

- \( b_s \) = empirical coefficient accounting for
sand grain roughness
- \( k_s \) = equivalent sand grain roughness,
mm

Alternatively, the value of \( b_s \) can be approx-
imated by

\[ b_s = 0.66 k_s^{0.33} \] (26)

For this approximation, the sand grain
roughness must be equal to or greater than
0.01 mm; any smaller values represent a
smooth surface.
Figure 7.—Experimentally determined local loss coefficient $C_f$, Bormann [11].
To solve these equations it is necessary to use the following trial and error procedure.

a. Determine the energy grade line elevation $H_o$ where $y_k$ equals the critical depth from

$$H_o = 1.5 \left( \frac{y_k}{g} \right)^{1/2} = 1.5 \left( \frac{y_k^2}{g} \right)^{1/3} \quad (27)$$

b. Estimate the water depth $d$ at a given distance $x$ from the entrance

c. Calculate the freestream velocity $U$ from

$$U = \left[ 2g (H_o x + \sin a - d \cos a) \right]^{1/2} \quad (29)$$

d. Determine the local loss coefficient $C_f$ as a function of the boundary layer thickness $\delta$ using equations 22 or 23 as appropriate. In these equations the flow depth assumed in b. is used for the boundary layer thickness.

e. Calculate the local loss coefficient as a function of distance Reynolds number from equation 24 where the distance Reynolds number is defined as

$$R_x = \frac{U_x}{v} = \frac{U^3}{2g \nu \sin a} \quad (30)$$

f. Repeat steps b. through e. with revised values of depth until the same values are obtained for the local loss coefficients in equations 22 or 23 and 24.

As an example, this procedure was applied to a surface having a sand grain roughness $k_s$ which corresponds to a float finish concrete surface. This finish is equivalent to a Manning’s roughness coefficient of 0.013. The results are presented in a set of design curves shown on figure 8. Similar curves can be prepared for other values of Manning’s coefficients.

Finally, the latest development in calculating the distance to the point of air entrainment is by Keller and Rastogi [40]. Their method involves integrating the equations of motion using a finite element scheme. The calculated velocity profiles agree very well with the experimentally determined values. However, their studies have not been put into a form that is useful for designers.

**Location of Fully Aerated Flow**

The location at which the flow becomes fully aerated has not been studied. Straub and Lamb [67] show that a constant air concentration distribution can be achieved, but the distance required for its development is not specified. Bormann [11] measured the location of the air-water interface, but not the concentration profile. For Bormann’s results one can imply that the length of the developing aeration regime is of the same order as the length of the developing boundary layer regime.

Keller, Lai, and Wood [39] indicate that the length of the developing aeration region can be defined in terms of the air concentration. In their definition, the length of the developing region is at least as long as the distance from the “critical point” to a location where a 5-percent air concentration has reached middepth of the total flow depth. The data by Straub and Lamb [67] show that this criterion is inadequate.

The length of the developing aeration regime cannot be determined analytically. This is an area in which additional research could be pursued gainfully.

**Air Concentration Profiles**

**Definition of concentration.**—The conventional definition of concentration is the quantity—usually measured by volume—of a material $A$ either dissolved or suspended in another material $B$. Thus, if material $A$ is air and material $B$ is water, air concentration would be the volume of air in a given volume of water.
However, when the amount of suspended material becomes large, the reference volume is the sum of the volume of material B and the volume of material A. In this case the average air concentration \( \bar{C} \) is given by

\[
\bar{C} = \frac{\omega_a}{\omega_a + \omega_w}
\]  

(31)

where 
\( \omega_a = \) volume of air \\
\( \omega_w = \) volume of water

This latter definition of concentration is used throughout this report.

The concentration also can be expressed in terms of volumetric flow rates as

\[
\bar{C} = \frac{Q_a}{Q_a + Q_w} = \frac{\beta}{\beta + 1}
\]  

(32)

where 
\( Q = \) discharge \\
\( \beta = \frac{Q_a}{Q_w} \)

The early method of measuring the concentration of air-water flows used a pitot-tube-type sampler developed by Viparelli [72]. This type of device gives accurate results in the underlying zone (fig. 4). However, in the mixing zone, the sampler records not only air in water, but also the air between the waves. Therefore, the measurements indicate air concentrations that are too large in the mixing zone. It should be noted that in some cases the total air discharge and not air concentration is desired. For instance, in closed conduit flow, the total air discharge is required for proper vent sizing. In this case, the pitot tube sampler would yield the desired air flow quantities in all four air flow zones.
Killen and Anderson [42] showed that the air concentration—as indicated by a pitot tube sampler—is related to the actual air concentration in the water by

\[
C_a = \frac{C_m - 1 + P_w}{P_w} \tag{33}
\]

where
- \(C_a\) = actual air concentration in percent of volume
- \(C_m\) = air concentration measured by a pitot tube sampler
- \(P_w\) = probability that the water surface is equal to or greater than the given elevation (refer to the following sec.—Air distribution in the mixing zone)

The water surface probability in equation 33 can be measured with electrical probes described by Killen [41]. He also reported on an electrical device that measures the actual air concentration directly.

Almost all references to air concentration in the bibliography include values for the amount of air between the waves. Therefore, values for the amount of air in flowing water generally are excessive. Unless noted otherwise, the term “air concentration” refers only to the amount of air actually entrained in water.

**Air distribution in the mixing zone.**—Many investigators have observed that measured air concentration distribution obey one law in the mixing zone and another in the underlying zone. Apparently, these two laws were discovered by Anderson [2] who examined the results of a large number of experiments.

Investigators reasoned that in the mixing zone the air concentration should follow a Gaussian law of normal probability, since this is also the law for the wave height distribution. They found the relation is given by

\[
\frac{dC_m}{dy} = \frac{2(1-C_t)}{h^{1/2}} \exp \left[-\left(\frac{y'}{h}\right)^2\right] \tag{34}
\]

where
- \(C_m\) = air concentration (including the air between the waves) at any elevation in percent of total volume
- \(C_t\) = air concentration at the bottom of the mixing zone
- \(h\) = mean wave height = \(2^{1/2}s\)
- \(s\) = standard deviation of the wave height distribution
- \(y\) = distance normal to the channel bottom
- \(y'\) = distance normal to the bottom of the mixing zone, figure 5

The integral form of the equation is

\[
\frac{1-C_m}{1-C_t} = \frac{2}{\sqrt{\pi}} \int_{y'}^{\infty} \exp \left[-\left(\frac{y'}{h}\right)^2\right] d\left(\frac{y'}{h}\right) \tag{35}
\]

This equation is almost identical to the cumulative Gaussian or normal distribution function

\[
P_y = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\eta} \exp \left[-\frac{\eta^2}{2}\right] d\eta \tag{36}
\]

where \(\eta = y'/s\)

The air distribution equation, equation 35, includes a factor of 2 because the relation applies to positive values of \(y'\); whereas equation 36 applies to both positive and negative values.
of $y'$. By reversing the limits of integration in equation 36, it can be shown that

$$\frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \exp \left( -\frac{\eta^2}{2} \right) d\eta = 1 - P_g$$

Combining equation 35 with equation 37 gives

$$\frac{1 - C_m}{1 - C_t} = 2(1 - P_g)$$ (38)

Values of the normal distribution function $P_\eta$ can be found in statistics texts. A graphical representation of the normal distribution function and the air concentration distribution is given in figure 9.

The actual air concentration in the waves is a function of the wave height probability and the air concentration at the bottom of the mixing zone. The relation is given by

$$C_a = \frac{P_w - 2(1 - C_t)(1 - P_g)}{P_w}$$ (39)

For the wave height distribution given in this chapter (Design Parameters—Water Surface Location), a family of curves can be derived for the actual air concentration distribution in the mixing zone (fig. 10). The maximum upper limit of these curves is discussed in the following subsection Mean air concentration.

These distributions are valid for both the developing and the fully developed aerated flow regimes. They can be used for the determination of the air concentration at any elevation if the standard deviation of wave height distribution and the air concentration at the bottom of the mixing zone are known. Unfortunately, these parameters can neither be predicted in the developing aerated flow regime nor in the fully developed aerated flow regime. Therefore, the equations only can be used to fit experimental data. Additional research is needed in this area.

---

**Air distribution in the underlying zone.**—The underlying zone consists of a region into which waves do not penetrate. Straub and Anderson [66] were successful in deriving an equation that describes air distribution in this zone (also refer to Streeter [68]).

They defined the bubble mass transfer coefficient $\epsilon$ as

$$\epsilon = \zeta \frac{(\tau_o/\eta)^{1/2}}{d_t} \left( \frac{d_i - y}{d_t} \right) y$$ (40)
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where

\( d_t = \) total depth of underlying and air free zones
\( k = \) Von Karman universal constant equal to 0.4
\( \zeta = \) air concentration distribution constant
\( \rho_l = \) liquid density
\( \tau_o = \) wall shear stress

For the mass transfer coefficient \( \varepsilon \) into equation 16—and assuming the terminal velocity of bubbles is constant—upon integration one obtains

\[
C = C_i \left( \frac{y}{d_t - y} \right)^m
\]

where

\[
m = \frac{V_f}{\xi k \left( \frac{\tau_o}{Q} \right)^{1/2}} = \frac{V_f}{\xi k \nu_u}
\]

\( C = \) local air concentration
\( C_i = \) air concentration at \( d_t/2 \)
\( \nu_u = \) shear velocity—\( \left( \frac{\tau_o}{Q} \right)^{1/3} \)

This equation fits the data well for fully developed aerated flows by properly selecting \( C_i, d_t, \) \( d_t = \) and \( m \) (fig. 11). The measured air concentration distribution in the mixing zone given by equation 38 also will give good results. The satisfactory fit of the data with these two equations implies that turbulent diffusion and buoyancy are the two most important factors affecting the determination of the air concentration distribution in the fully developed aerated flow zone.

In the developing aerated flow regime, equation 41 is not valid. In the developing aeration regime two conditions must be considered. The first condition is where air is being insufflated into the flow. For this case, the governing equation is

\[
q_a = \varepsilon \frac{dC}{dy} \quad CV_f
\]

where \( q_a = \) insufflation rate of air per unit surface area

The other condition is that of excess air being present in the water. In this condition, air is released from the flow until an equilibrium state is reached. Equation 42 also is valid for this condition except the sign \( q_a \) will be negative which indicates that air is leaving the water.
Killen [41] measured air concentration distributions for the first condition of air insufflation. However, no one has developed equations for the observed distribution. This is another area needing additional research.

**Mean air concentration.**—The mean air concentration for the entire flow stream is defined by

$$\overline{C} = \frac{1}{d_m} \int_0^{d_m} C_a dy$$

(43)

The depth $d_m$ represents some upper bound for the water surface. Straub and Anderson [66] define the depth as representing that point where the measured air concentration is 0.99. A more reasonable reference depth would be the depth which is exceeded by only 1 percent of the waves.

The air concentration $C_a$ in equation 43 is the actual air concentration and does not include air moving between the waves (in this chap., Design Parameters—Definition of concentration). However, all the published data were measured with a pitot-tube-type sampler and, thus, include the interwave air motions in the mixing zone. The mean air concentration $\overline{C}$ based on the actual air concentration $C_a$ distribution can be estimated by the following
reasoning. The upper limit of the air concentration can be approximated by assuming the bubbles to be spherical. If they retain a spherical shape, Gardner [23] showed that the air concentration for the bubbles packed in their most dense configuration is about 75 percent. The upper limit for the Straub and Anderson data [66] is about 84 percent. Thus their data are roughly 12 percent too high.

Since the air concentration is obtained from an integration of the local air concentration distribution, buoyancy and turbulent diffusion must be the most significant parameters which influence the mean concentration for any flow. The buoyant forces are governed primarily by the bubble size. With respect to turbulence, Hinze [35] indicates that bubbles tend to be broken up by both viscous shear forces and turbulent shear forces. This tendency to be broken up is resisted by interfacial tensile forces. For high enough degrees of turbulence, the viscous shear forces are insignificant with respect to the turbulent shear forces. With solid boundaries the significant parameter defining the turbulent shear forces is the wall shear stress, \( \tau_0 \).

The following presents the development of an empirical equation to predict the mean air concentration in the fully developed aerated flow zone. The development is based upon the classical methods of dimensional analysis.

Assuming that buoyancy and turbulent diffusion are significant, it is possible to express the volume flow of air \( Q_a \) as a function of:

- Gravity
- Turbulent shear stress
- Interfacial forces
- Fluid properties of both air and water
- Characteristic flow dimensions

The expression is

\[
Q_a = f(V, b, d, g, \mu_a, \mu_w, q_a, q_w, \sigma, \tau_0) \tag{44}
\]

where

- \( Q_a \) = volume flow of air, \( m^3/s \)
- \( V \) = water velocity, \( m/s \)
- \( b \) = width of flow channel, \( m \)
- \( d \) = water depth, \( m \)
- \( g \) = gravitational constant (acceleration) = 9.81 \( m/s^2 \)
- \( \mu \) = dynamic viscosity, \( N\cdot s/m^2 \)
- \( \rho \) = density, \( kg/m^3 \)
- \( \sigma \) = interfacial surface tension, \( N/m \)
- \( \tau_0 \) = wall shear stress, \( Pa \)

\( f(\cdot) \) denotes "a function of"

Subscripts \( a \) and \( w \) refer to air and water, respectively.

Using \( V, d, \) and \( q_w \) as the repeating variables results in the following dimensionless parameters.

\[
\frac{Q_a}{V^2} = \frac{Q_a}{V^2d^2} \tag{45}
\]

By examining the magnitude of these dimensionless terms and transforming some of them, it is possible to develop parameters that should be used for correlating the available model and prototype data. For instance, the dependent parameter can be written as

\[
\frac{Q_a}{V^2} = \frac{Q_a}{V^2d^2} \left( \frac{b}{d} \right) = \frac{Q_a}{Q_w} \left( \frac{b}{d} \right) \tag{46}
\]

Since

\[
\beta = \frac{Q_a}{Q_w} \quad \text{and} \quad \bar{C} = \frac{\beta}{\beta + 1}
\]

by definition, the mean concentration is also a function of the same dimensionless terms given above.
The first independent parameters in the parenthesis of equation 47 can be written as

$$\frac{1}{F^2} = \frac{\tau_o}{V^2 \rho_w}$$

where $P =$ Prandtl velocity ratio $= \frac{V}{(\tau_o/\rho_w)^{1/2}}$

The Prandtl velocity ratio represents the ratio of the inertial force to the wall shear force.

For the condition of uniform flow on a wide, steep chute, the wall shear stress $\tau_o$ is given by

$$\tau_o = \gamma d \sin \alpha$$ \hspace{1cm} (47)

where

- $d =$ flow depth
- $\alpha =$ angle chute invert makes with horizontal
- $\gamma =$ specific force of fluid

substituting this into the Prandtl velocity ratio $P$ gives

$$\frac{\tau_o}{V^2 \rho_w} = \frac{\gamma d \sin \alpha}{V^2} = \frac{gd \sin \alpha}{V^2} = \sin \alpha = \frac{F^2}{F}$$ \hspace{1cm} (48)

Thus for uniform flow on wide, steep chutes, the parameters $\sin \alpha/F^2$ represents the ratio of turbulent shear forces to the inertial forces.

The tensile force parameter, $\sigma/(V^2 d \rho_w)$, is simply the reciprocal of the Weber number squared

$$\frac{\sigma}{V^2 d \rho_w} = \frac{1}{W^2}$$ \hspace{1cm} (49)

where $W =$ Weber number $= \frac{V}{(\sigma/\rho_w d)^{1/2}}$

The Weber number represents the ratio of the inertial forces to the interfacial tensile forces. The surface tension is only a function of temperature (fig. 12)

The bubble size is determined by the ratio of turbulent shear forces to the interfacial tensile forces. In terms of the previously developed parameters, the ratio is

$$\frac{gd \sin \alpha}{V^2} = \frac{\left(\frac{V^2 d \rho_w}{\sigma}\right)}{F}$$ \hspace{1cm} (50)

Reducing this ratio to the first power of $V$ and generalizing results in

$$\frac{(\sin \alpha)^{1/2} W}{\sigma}$$ \hspace{1cm} (51)

The ratio of viscous forces to inertial forces can be written as

$$\frac{\mu_w}{V d \rho_w} = \frac{V}{V d} = \frac{1}{R}$$ \hspace{1cm} (52)
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where $R = \text{Reynolds number}$

The use of channel width $b$ and flow depth $d$ in the list of variables implies the use of a rectangular cross section. More generality can be obtained by the use of the hydraulic radius as the significant length parameter. The hydraulic radius is defined by

$$H = \frac{A}{W}$$

where

$A = \text{cross sectional area of water prism}$
$W = \text{wetted perimeter}$

With the exception of the Froude number, the hydraulic radius seems to be the most appropriate characteristic length to correlate air entrainment for any cross sectional shape.

The effective depth is the characteristic length to be used with the Froude number. The effective depth $y_e$ is defined as

$$y_e = \frac{A}{B}$$

where

$A = \text{cross sectional area of water prism}$
$B = \text{top width of water prism}$

By using the above transformations, the mean air concentration can be written as

$$\bar{C} = f \left( \frac{(\sin \alpha)^{1/2} W}{F}, W, \frac{Q_a}{Q_w}, \frac{\mu_a}{\mu_w}, R, F \right)$$

Equation 54 represents a seven-dimensional surface. Determining relations between all the variables is practically impossible. However, the problem can be made simpler by neglecting those independent variables which have either a small range of values or a negligible effect on the mean air concentration. For example, the tensile force varies less than 10 percent over the temperature range normally existing in the field and laboratory (fig. 12). Therefore, the tensile force parameters, $W$, of equation 49 can be neglected.

Neglecting the tensile force parameter does not mean that the ratio of the turbulent shear to interfacial tensile force parameter also can be neglected. This latter ratio, which governs the bubble diameter, must be retained.

For flows that are turbulent enough to entrain air naturally, the dynamic pressure forces determine the size of the largest air bubbles. These dynamic forces are a result of changes in velocity over distances that are about the same scale as the diameter of the bubble. For typical flows on spillways and in steep chutes the dynamic forces predominate over the viscous forces. Therefore, the dimensionless terms involving viscosity are not significant with respect to the magnitude of the other terms.

Finally, the dimensionless density ratio varies almost linearly from 0.0012 at 4 °C to 0.0011 at 30 °C. Therefore, this term also can be considered as unimportant for the temperature range that is typical in hydraulic structures.

The reduced form of equation 54 is

$$\bar{C} = f \left( \frac{(\sin \alpha)^{1/2} W}{F}, F \right)$$

This equation represents a three-dimensional surface which can be defined from experimental and field investigations. The Straub and Anderson [66] data from model studies and data from prototype studies of Michels and Lovely [54] as well as Thorsky et. al., [70] were used to determine the functional relations between the mean air concentration $\bar{C}$, Froude number $F$, and turbulent-interfacial tension force ratio.

The variation of the mean concentration as a function of Froude number was determined...
from the model data. For this correlation the turbulent-interfacial tension force ratio was approximately constant. With mean air concentrations less than 0.6, the correlation had the form

$$\bar{C} = a_0 + a_1 F$$  \hspace{1cm} (56)

where $a_0$ is a function of the turbulent to interfacial tension force ratio.

The values of $a_1$ for turbulent to interfacial tension force ratios of 0.166, 0.114, and 0.085 are 0.0469, 0.0436, and 0.0556, respectively. Although the value of $a_1$ apparently increases as the turbulent to interfacial tension force ratio decreases, the data are insufficient to support the conclusion. Therefore, $a_1$ was taken to be equal to the mean of the values or 0.050.

The function $a_0$ was determined from both the model data and tests on prototype chutes and spillways. The prototype values correspond with the initiation of air entrainment. The values of $a_0$ were determined from the equation

$$a_0 = \bar{C} - 0.05F$$  \hspace{1cm} (57)

The curve

$$a_0 = -\frac{(\sin \alpha)^{1/2}W}{63F}$$  \hspace{1cm} (58)

approximately fits the data (fig. 13).

Therefore, the mean air concentration correlation is given by

$$\bar{C} = 0.05F - \frac{(\sin \alpha)^{1/2}W}{63F}$$  \hspace{1cm} (59)

for

$$0 \leq \bar{C} \leq 0.6$$

If $\bar{C}$ is greater than 0.6, the air concentration values must be taken from the empirical curves on figure 14.

With the exception of the spillway data, the Froude and Weber numbers were calculated using the relation for normal depth without aeration. The model tests correspond with fully developed aerated flow.

In the absence of any better information, these curves also can be used to estimate the mean air concentration in developing flow. For this case, the Froude and Weber numbers are calculated from the depth and velocity values that result from the gradually varied flow computations without considering aeration of the flow.

The use of these curves are illustrated by an example.

Example

Given a rectangular chute 15 meters wide, calculate the development of the air entrainment along the length of the chute. The discharge is 20 m$^3$/s. The channel is built on a 1:3 slope and has a Manning’s $n$ value of 0.0100.

The computer program HFWS (app. II) was used to compute the water surface profile and the mean air concentrations (fig. 15). The program can be used with either drawdown or backwater curves for rectangular, trapezoidal, circular, and transitional cross sections.

Water Surface Location

One of the first concepts that developed in self-entraining flows was bulking. With this concept, the air was considered to be evenly distributed throughout the flow. As a result, the depth of the mixture was increased by a predictable amount (fig. 16). The relation is given by

$$\frac{d_b}{d} = \frac{1}{1 - \bar{C}}$$  \hspace{1cm} (60)

where

- $d$ = flow depth (without aeration)
- $d_b$ = bulked flow depth
TURBULENT TO INTERFACIAL TENSION FORCE RATIO

\[ \frac{F}{(\sin \alpha)^{1/2} W} \]

Figure 13.—Air entrainment coefficient.
$g$ = gravitational constant
$V$ = mean flow velocity
$W$ = weber number = $V/\left(\sigma/\rho y_e\right)^{1/2}$
$y_e$ = effective depth
$\alpha$ = angle chute invert makes with horizontal
$\rho$ = density of water
$\sigma$ = interfacial surface tension

\[ F = \frac{V}{\sqrt{g y_e}} \]

**Figure 14.** Air entrainment in open channel flow.
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Discharge \( Q = 20 \text{ m}^3/\text{s} \)
Chute width \( B = 15 \text{ m} \)
Manning's \( n = 0.010 \)

**Figure 15.**—Example of air entrainment in chutes.
Air-water flow in hydraulic structures

Nonaerated flow

Bulked flow

Actual flow condition

True mean depth

Mean air concentration \( \bar{c} \), percent

Figure 16.—Definitions of aerated flow depth.
For chutes having rectangular cross sections, equation 60 can be written as

\[ \bar{C} = 1 - \frac{VdB}{VdB} \]  

(61)

where

- \( R \) = width of rectangular chute
- \( V \) = velocity of air-water mixture

Thomas [69] used the bulked depth concept indirectly to determine the air content on the Kittitas chute*. He measured the bulked depth (with some difficulty), total waterflow and the velocity of the air-water mixture. The velocity of the air-water mixture was determined by the salt velocity method. Thomas measured \( d_b, V, \) and \( Q \). He calculated the air concentrations using equation 61 by setting

\[ Q = VdB \]  

(62)

The determination of airflow rates by this method raises two questions that need to be considered in some detail. These are:

1. Does the air content influence the flow velocity and hence, the true mean depth?
2. How accurate are the measurements of the bulked flow depth?

Straub and Anderson [66] found that the depth of the flow actually decreases as the air concentration increases above a value of 25 percent (fig. 17). In other words, as the air concentration increases, the flow velocity increases. For instance, at the 74-percent air concentration, the flow velocity is about 1.8 times the nonaerated flow velocity. Therefore, the Standard Step Method and Manning’s equation can be used to estimate the true depth up to air concentrations of about 25 percent. For higher concentrations the values must be adjusted by figure 16.

The bulked flow depth virtually is impossible to measure accurately because the surface is highly turbulent (note frontispiece). Thomas [69] noted the following:

“The choppy water surface and the large amount of spray rendered it difficult to determine where the point of the gage should be to give a reading that would be indicative of the actual depth of flow. The surface conditions also made it difficult to observe the point of the gage. The depth of flow was considered to be at the base of the loosely flying spray and drops of water. The top of the main portion of the flow included numerous small waves or rollers. The vibration of the point gage was relied upon more than visual observation to insure that the point was at relatively the same position in the flow for successive readings.”

The practical difficulties of determining accurate measurements indicate a need to reexamine the actual flow conditions (fig. 16). A careful consideration of actual flow conditions, such as the frontispiece and the preceding note, reveals that the entire concept of a bulked flow depth is a poor representation of reality. Thus, other means of describing the location of the water surface should be considered.

Water surface fluctuations that occur in open channels can be described by several methods. Longuet-Higgins [50], in studies of ocean waves, based his analysis on a probability distribution of wave heights. He defined a wave height as the difference in elevation between a crest (maximum) and its succeeding trough (minimum). Relatively good correlations were

*The Kittitas chute is the common name given to describe the Main Canal - Sta. 1146+30 (feet) Wasteway in the Kittitas Division, Yakima Project - Washington. The care with which the tests were conceived and executed have made them a valuable reference source—even up to the present time.
obtained between the data and a Rayleigh probability density function. This function can be written as:

\[ P_h = \eta \exp\left(-\eta^2/2\right) \tag{63} \]

where

- \( \eta \) = normalized wave height = \( M_o/s \)
- \( M_o \) = wave height amplitude, (half crest to trough distance)
- \( s \) = root-mean-square value of the wave height amplitudes
- \( P_h \) = probability that the wave height is equal to the given height

Cartwright and Longuet-Higgins [13] used another description of the fluctuating water surface. They based their analysis on the maximum difference on elevation between a wave crest and the mean level of the water surface for a given wave period. These differences can be normalized by dividing them by the root-mean-square value of the water surface fluctuation about the mean water surface elevation. Since a crest can occur at an elevation below the mean water surface, the dimensionless crest elevations can have negative values. The specific shape of the distribution depends upon the relative width of the frequency spectrum \( E \). This parameter defines a series of probability distributions that range from a Rayleigh to a Gaussian distribution (fig. 18). The value of the parameter \( E \) for ocean waves is about 0.6
A third method of describing the water surface fluctuations is not based upon a definition of wave heights. Instead, the difference between the instantaneous water surface and the mean value of the water surface is used. This probabilistic description of the water surface is probably the most useful in hydraulic structures since designers require a knowledge of the mean flow depth and a measure of how frequently the water surface exceeds some specified elevation above the mean depth.

Unfortunately, very little research has been concerned with a probabilistic description of the water surface in open channel flow. Preliminary investigations in the laboratory indicate that the distribution of the water surface in fully developed, rough, open-channel flow is nearly Gaussian. If the distribution is Gaussian, a plot of the cumulative probability distribution versus flow depth will be a straight line on arithmetic probability paper (fig. 19). A wave depth probe of the type described by
Killen [41] or the circuitry given in appendix I can be used to measure the cumulative probability directly. The root-mean-square value of the water surface fluctuation can be read from the plot. The root-mean-square value is the difference between the depth having an 84.13-percent probability and the mean depth (50% probability). Alternatively, the mean depth and the 15.87-percent probable depth can be used.

Effect of Air Entrained Flow on Stilling Basin Performance

The effect of air entrained flow on the performance of stilling basins has been considered for many years. Gumensky [25] concluded that for all practical purposes the conjugate depth of the hydraulic jump could be determined from nonaerated flow equations. Rajaratnam [57] followed with some investigations that indicated a significant effect might be produced by the air entrainment. However, later studies by Herbrand [33] showed that the coefficients used in Rajaratnam’s equations produced too large an effect on the result and that they were not necessary. Therefore, in rectangular channels the conjugate depth \( y_c \) can be calculated with sufficient accuracy from

\[
y_c = 0.5y_t (\sqrt{8F^2 + 1} - 1)
\]

where the depths and Froude numbers are calculated with the appropriate bulked flow depths (fig. 16).
Closed Conduit Flow

CLASSIFICATION OF FLOW

The conventional term for the concurrent flow of air and water is two-phase flow. Here, phase refers to one of the states of matter (gas, liquid, or solid). Technically the term two-phase flow should be reserved to describe the motion of a substance which is present in two of its phases, such as a flow of ice and water. The word multicomponent is a better description of flows which do not consist of the same chemical substance, such as air and water. If both components move in the same direction, the flow is termed concurrent flow. If the components move in opposite directions, the flow is countercurrent.

Closed conduit flow can be classified according to the type of pattern that develops. The flow patterns which develop depend upon the airflow rate relative to the water flow rate and the slope of the conduit. For example, the flow patterns in horizontal conduits have been defined by Baker [7], (fig. 20). The correlation can be applied to other gases and liquids by substituting appropriate quantities into the following parameters:

- $G_g =$ mass velocity of gas, kg/(m$^2 \cdot$ s)
- $G_l =$ mass velocity of liquid, kg/(m$^2 \cdot$ s)
- $\lambda = [(\rho_g/\rho_a)(\rho_l/\rho_w)]^{1/2}$
- $\mu =$ dynamic viscosity, Pa $\cdot$ s
- $\rho_g =$ gas density, kg/m$^3$
- $\rho_a =$ air density (at 101.3 kPa and 20 °C) = 1.20 kg/m$^3$
- $\rho_l =$ liquid density, kg/m$^3$
- $\rho_w =$ water (at 101.3 kPa and 20 °C) = 988 kg/m$^3$
- $\sigma =$ interfacial surface tension, N/m
- $\sigma_{sw} =$ air-water surface tension (at 101.3 kPa and 20 °C) = 0.0728 N/m
- $\psi = (\rho_w/\rho_l)[\mu(\rho_w/\rho_l)^2]^{1/3}$, Pa$^{1/3} \cdot$ s$^{1/3}$
These various flow patterns were described by Alves [1] according to the physical appearance of the flow as follows (fig. 21):

- **Bubble flow.**—The air forms in bubbles at the upper surface of the pipe. The bubble and water velocities are about equal. If the bubbles are dispersed through the water, the flow is called "froth flow."
- **Plug flow.**—For increased airflow rates the air bubbles coalesce with plugs of air and water alternately flowing along the top of the pipe.
- **Stratified flow.**—A distinct horizontal interface separates the air and water flows.
- **Wave flow.**—As the airflow rate is increased, surface waves appear on the stratified flow interface.
- **Slug flow.**—Wave amplitudes are large enough to seal the conduit. The wave forms a frothy slug where it touches the roof of the conduit. This slug travels with a higher velocity than the average liquid velocity.
- **Annular flow.**—For greater airflow rates the water flows as a film on the wall of the pipe, while the air flows in a high-speed core down the axis of the pipe.
- **Spray flow.**—For very great airflow rates the annular film is stripped from the pipe walls and is carried in the air as entrained droplets.

A similar set of flow pattern descriptions exist for vertical flows. They are:

- **Bubble flow.**—The air is distributed in the water as spherical or spherical cap bubbles which are small with respect to the conduit diameter.
**Slug flow.**—As the air flow increases, alternate slugs of air and water move up the pipe. The transition from bubble flow to slug flow is shown on Figure 22. This transition occurs when the bubble diameter is about one-half the conduit diameter.

If the vertical conduit is rectangular instead of cylindrical, the appropriate relation for slug flow is given by Wallis [73] as

\[
\frac{V_s}{V_t} = (0.325 + 0.184 \left( \frac{D_e}{D_b} \right)(\frac{D_e}{D_b})^{1/2}
\]

where

- \(D_s\) = larger dimension of a rectangular conduit
- \(D_b\) = smaller dimension of a rectangular conduit
- \(D_e\) = bubble diameter
- \(V_s\) = terminal velocity of air bubbles in slug flow
- \(V_t\) = terminal velocity of air bubbles in still water

With respect to the flow quantities, Martin [52] found that the transition from bubbly to slug flow occurs at a void fraction somewhere between 19 and 23 percent.

The void fraction \(\theta\) is the average volumetric concentration in a length of pipe (assuming uniform flow) and expressed as

\[
\theta = \frac{\omega_w}{AL}
\]

It is not clear whether the term slug refers to a slug of air or a slug of water. The air bubble could be called a slug due to its bullet or slug shaped form. The water could be called a slug due to its similarity in form to the terrestrial gastropod in horizontal flows or due to its impact properties in vertical flow. The author prefers the reference to slugs of air.
Froth flow.—As the airflow increases, the slugs break up into a turbulent disordered pattern of air and water. The annular and spray flow patterns are identical in both vertical and horizontal pipes. In hydraulic structures, the conduits may also be placed on a slope. The additional complexities in the flow patterns caused by slope will be discussed later.

From a designer’s viewpoint, air-water flows in closed conduits can be classified into four general categories. Each category may contain only one or a combination of the flow patterns enumerated previously. These categories are:

1. Flow in partially filled conduits,
2. Flow having a hydraulic jump that fills the conduit,
3. Flow from control devices, and
4. Falling water surface.

Each category listed above is considered in detail in the following subsections.

In addition to the four categories of flow, two others are considered separately. These are:

- Flow in pipelines and siphons
- Flow in vertical shafts

The pipelines and siphons require special consideration because of their length. Vertical shafts present special problems because of the various types of flow which can exist in the shaft.
FLOW IN PARTIALLY FILLED CONDUITS

Model Predictions

Flow in a partially filled conduit can be thought of as open-channel flow in a closed conduit. The air flows through the passage which is formed above the water surface.

The total volume flow of air, which enters at the upstream end of the air passage, equals the sum of the air that is insufflated into the flow and that which flows above the water surface as a result of the air-water shear forces. The quantity of air insufflated into the flow can be estimated from equation 59. The quantity of air that flows above the water surface is a function of the waterflow properties and the pressure drop in the air vent. This can be expressed as

\[ Q_a = f(L, V, g, p, \gamma_e, \rho_w) \]  

(67)

where

- \( A \) = cross sectional area of water prism
- \( g \) = gravitational constant (acceleration)
- \( L \) = conduit length
- \( p \) = pressure intensity
- \( Q_a \) = total airflow rate
- \( T \) = top width of flow passage
- \( V \) = mean water velocity
- \( \gamma_e \) = effective depth = \( A / T \)
- \( \rho_w \) = water density

Applying dimensional analysis to equation 67 with \( \gamma_e, V, \) and \( \rho_w \) as the repeating variables gives

\[ \frac{Q_a}{Q_w} = f\left(\frac{L}{\gamma_e}, \frac{1}{\gamma_e}, \frac{p}{\gamma'}, \frac{\rho V^2}{\gamma'} \right) \]  

(68)

where

- \( F \) = Froude number
- \( Q_w \) = waterflow rate
- \( \gamma' \) = specific force of water

The interrelation between these parameters can be found for a specific geometry through the use of model studies.

There are many literature references that indicate model predictions often underestimate in the quantity of air which actually flows in prototype structures. However, very careful model tests in which all air- and waterflow passages were modeled in their entirety have shown good agreement between model and prototype measurements.

For instance, Sikora [65] showed that the airflow rates could be accurately predicted from model studies. His tests were with three geometrically similar models having scales of 1:1, 1:2, and 1:4 (fig. 23). The pressure values on the figure refer to the difference between atmospheric pressure and the air pressure at the upstream end of the waterflow passage.
existed in the tunnel for all discharges. A scale effect was not detectable in their investigations.

These studies clearly indicate that for estimating airflow rates using models, it is necessary to accurately reproduce the entire airflow passage above the water surface. In those cases where air enters the water conduit through a vent, two options are available for measuring the airflow rates. The options depend upon whether or not the air vent has been designed.

Air vent not designed. — If the air vent design has not been determined, it is necessary to measure the airflow rate while controlling the air pressure at the upstream end of the water conduit. These tests must be performed for a series of flow depths and flow rates in the water conduit.

The upstream air pressures can be controlled by incorporating an air pump into the airflow measuring device. To be applicable for all possible designs, the pressure should be varied over the maximum possible range. The lowest end of the range corresponds with the condition of no airflow through the vent. The upper end of the range is achieved when the upstream air pressure is equal to the atmospheric pressure.

A good example of this procedure is the work by Sikora [65] who developed a set of curves for the airflow in the horizontal leg of morning-glory spillway (fig. 24).

Once the family of curves for the airflow rates has been experimentally determined it is possible to investigate the effect of adding various size air vents to the structure. This is done by first developing an expression for the air vent characteristics in terms of the dimensionless parameters on figure 24.

For air velocities less than 100 m/s and values of $fL/4H \geq 4$, the volume flowrate $Q_a$ through a vent can be expressed as

$$Q_a = A_v \left( \frac{2g}{\pi} \left[ \frac{Q_w/Q_a}{\sqrt{\frac{p_{atm}/\gamma}{2g} - \frac{p_1/\gamma + \Delta z (Q_w/Q_w)}}} \right] \right)^{1/2}$$

where

$A_v = \text{cross sectional area of vent}$

$f = \text{Darcy-Weisbach friction factor}$

$g = \text{gravitational constant (acceleration)}$

$H = \text{hydraulic radius of prototype air vent}$

$K_v = \text{entrance loss}$

$K_s = \text{singular (form) loss in vent, the greatest of which is the entrance loss}$

$L = \text{vent length}$

$p_1 = \text{pressure at vent exit}$

$p_{atm} = \text{atmospheric pressure}$

$\Delta z = \text{difference between vent intake and vent exit elevations}$

$\gamma = \text{specific force of water}$

$\rho_a = \text{air density}$

$\rho_w = \text{water density}$

Volume flowrate of water can be expressed as

$$Q_w = A \left( \frac{2g}{\pi} \left[ \frac{V^2}{2g} \right] \right)^{1/2}$$

where

$A = \text{cross sectional area of water prism}$

$V = \text{mean waterflow velocity in conduit}$

Using these two expressions, the dimensionless airflow rate $\beta$ can be expressed as

$$\beta = \frac{Q_a}{Q_w} = \frac{A_v}{A} \left[ \sum K_s + fL/4H \right] \left[ \frac{(p_{atm}/\gamma) - (p_1/\gamma + \Delta z (Q_w/Q_w))}{V^2/2g} \right]^{1/2}$$

where $\Delta z \frac{Q_a}{Q_w}$ is negligible.

The first ratio inside the brackets is a function of the fluid properties, the singular losses, and the flow geometry. The second ratio is in the form of a pressure factor or Euler number. By using this equation, the characteristics of a given vent can be plotted on the dimensionless airflow curves (fig. 24). The intersection points
CLOSED CONDUIT FLOW

A = cross sectional area of water prism
A_d = cross sectional area of conduit
d_e = deflector height
F = Froude number = \frac{V}{\sqrt{gYe}}
\rho = air density
\gamma = gravitational constant
p = pressure at end of air vent
\Delta p = pressure drop across vent
Q_a = volume flowrate of air
Q_w = volume flowrate of water
V = mean flow velocity
Ye = effective depth

---

Spillway
Air vent
Deflector

Hydraulic jump with submerged flow
Outlet submerged
Free surface flow

\beta = 0.0066 (F - 1)^{1.4}
\beta_{max} = \frac{A_d - A}{A} = 0.245

L/D = 6.7
d_e / D = 1/4

Figure 24. — Model tests on a spillway, Sikora [65].
of the two sets of curves gives the pressures and airflow rates for a given set of air vent parameters. If the resulting values are not satisfactory, another set of vent characteristics is chosen and the process repeated.

**Air vent designed.**—For some studies the design of the air vent is available. In these cases it is necessary to calculate the total loss for the vent and to simulate this loss in the model air vent. The loss for the prototype and the model must include both frictional and form losses. Normally, the air vent velocities are kept low enough so that incompressible loss coefficients are valid. The model air vent is simulated correctly when the loss coefficients in the model and prototype vents are made equal. If devices such as nozzles or orifices are installed into the model air vent for flow measurement purposes, the loss across them must be included in computing the total model air vent loss coefficient. In the case of an orifice, its loss coefficient often constitutes the entire loss for the model air vent. It is possible to express the required orifice size as

\[
A_o = \frac{A_v}{C_o L_r^2 \left(1 + \sum K_s + fL/4H\right)^{1/2}} \tag{72}
\]

where

- \(A_o\) = orifice area
- \(A_v\) = prototype air vent area
- \(C_o\) = orifice discharge coefficient
- \(f\) = Darcy-Weisbach factor for prototype air vent
- \(H\) = hydraulic radius of prototype air vent
- \(K_s\) = singular losses (including entrance, bends, and changes in area)
- \(L\) = length of prototype air vent
- \(L_r\) = prototype to model scale ratio

If the orifice is placed on the end of the model air vent pipe, its discharge coefficient is obtained from figure 25.

**Analytic Estimates**

In many instances, model tests for predicting the airflow rates have not been performed. For these cases, the airflow rates often can be estimated closely enough by an approximate method. For this estimation three rather gross assumptions must be made, namely:

1. The amount of air flowing through the vent is a function of only the air insufflated into the flow and the air that is induced to flow by the moving water boundary,
2. The amount of air insufflated into the flow can be predicted by open channel flow equations, and
3. The air motion above the water surface is determined solely by the boundary layer \(\delta\) thickness at the most downstream conduit location.

These assumptions neglect the fact that air actually can enter from the downstream end of the conduit. Schlichting [63] showed that with Couette-Poiseuille\(^6\) flow in the laminar region, a flow reversal occurs when

\[
P_o = \frac{h_a^2}{2\mu V_o} \left(\frac{dp}{dx}\right) < -1 \tag{73}
\]

\(^6\)The dimensionless parameter \(P_o\) is known as the Poiseuille number. Its primary use is in the laminar fluid friction field. For example, in a round circular pipe, the Poiseuille number is equal to 32. In this case the pipe diameter is substituted for the height of the airflow passage in equation 73. Couette flow exists between two parallel walls when one wall is moving and the other is stationary. The motion is due solely to the shear field created by the relative movement of the two walls. Couette flow has no pressure gradient in the direction of flow. Couette-Poiseuille flow describes a Couette type flow having a longitudinal pressure gradient. Turbulent Couette-Poiseuille flow should describe the air motion above a moving water surface in a closed conduit.
CLOSED CONDUIT FLOW

Closed conduit flow equations:

- $A_d =$ conduit area
- $A_o =$ orifice area
- $H_m =$ head across orifice
  \[ H_m = h_m(\rho_m/\rho_a) \]
- $Q =$ volume flowrate of air
- $\rho_a =$ density of air
- $\rho_m =$ density of manometer fluid

Vena contracta

\[ \frac{A_o}{A_d} = \frac{d_o^2}{D^2} \]

\[ C_o = \frac{Q}{A_o \sqrt{2g H_m}} \]

\[ C_o = \frac{Q}{A_d \sqrt{2g H_m}} \]

**Figure 25.**—Discharge coefficients for orifice at end of pipe.
where

\[ h_a = \text{height of airflow passage} \]
\[ \frac{dp}{dx} = \text{pressure gradient in the air} \]
\[ V_o = \text{maximum water surface velocity} \]
\[ \mu = \text{dynamic viscosity of air} \]

Leutheusser and Chu [48] have investigated Couette flow in the turbulent region. Insufficient tests have been made to determine the magnitude of the dimensionless parameter \( P_o \) for the turbulent Couette-Poiseuille flow. However, some laboratory tests indicate that with turbulence, reverse flow begins when

\[ P_o \approx -1000 \]

The amount of air flowing above the water surface can be visualized by considering a boundary layer which increases in thickness from a value of zero at a gate, to a maximum value at the end of the conduit (fig. 26). The growth of a turbulent boundary layer that is induced by a moving rough boundary has not been studied. As a first approximation it is assumed that

\[ \delta = 0.01x \]

where

\[ \delta = \text{boundary layer thickness} \]
\[ x = \text{distance from gate} \]

The velocity distribution within the boundary layer is assumed to obey a power law of the order:

\[ u = V_o \left( \frac{y_a}{\delta} \right)^{1/n_v} \]

where

\[ n_v = \text{velocity distribution power law coefficient} \]

The value of the coefficient \( n_v \) varies between 10 for flow over smooth surfaces to 5.4 for flow over rough surfaces when the Reynolds number is about \( 10^6 \). Normally \( n_v \) is assumed to be equal to 7. This approach is similar to that used by Campbell and Guyton [12] except they assumed the boundary layer always coincided with the roof of the conduit.

The boundary layer entrains the maximum amount of air at the extreme downstream location in the conduit. To maintain continuity, flow at upstream locations consists of boundary layer flow plus some mean flow (fig. 26). The air velocity at the water surface must be equal to the water velocity. Therefore, at the upstream locations, the air velocity above the water surface may have a larger magnitude than that at the water surface. Careful laboratory experiments by Ghetti [24] of the Vaiont Dam (Italy) gated outlets show that the maximum air velocity near the water surface at the vent can be as much as four times the water velocity.

For some flow conditions the boundary layer will reach the roof of the conduit. When this happens the roof will begin to retard the flow. If the water surface and the roof of the conduit had equal roughness values, the maximum flow rate would be given by turbulent plane Couette flow. For this case the maximum airflow rate \( Q_m \) is

\[ Q_m = \frac{A_s V_o}{2} \]

where

\[ A_s = \text{cross sectional area of airflow passage (rectangular)} \]
\[ V_o = \text{maximum water surface velocity} \]
Volume flowrate of air, $Q_a$

Boundary layer

A. Profile sketch

Mean flow required by continuity

Superposition of mean flow with boundary layer flow

At $1$

$\frac{U}{V_0} = \left(\frac{Q_a}{\delta}\right)^{1/n_V}$

$\delta$

Boundary layer depth greater than flow passage depth

At $2$

B. Velocity distribution

FIGURE 26.—Airflow above water surface.
Actually the roughness of the water surface is greater than that of the conduit roof. This increased roughness will produce higher air velocities near the water surface which result in airflow rates greater than those given by equation 77. Sikora [65] reasoned that the mean air velocity could not exceed the mean water velocity. This leads to the expression for the maximum possible airflow rate in a closed conduit, which is

\[
\left( \frac{Q_a}{Q_w} \right)_{\text{max}} = \frac{A_d}{A} - 1 \tag{78}
\]

where

- \(A_d\) = cross sectional area of conduit
- \(A\) = maximum cross sectional area of water prism

Application of equation 78 without regard to the boundary layer thickness will result in excessively large values of the airflow rates. However, for design purposes, this approach may be satisfactory since the resulting air vent will be oversized.

FLOW HAVING A HYDRAULIC JUMP THAT FILLS THE CONDUIT

Kalinske and Robertson [38] studied the special case of two-layer flow in which a hydraulic jump fills the conduit. From dimensional analysis and model studies, they determined that the amount of air entrained by the jump is given by

\[
Q_a = 0.0066 (F-1)^{1.4} \tag{79}
\]

where \(F\) = Froude number upstream of the hydraulic jump.

In a circular pipe the Froude number can be calculated conveniently from the flow depth \(y\) using

\[
F = \frac{V}{(gy_e)^{1/2}} \tag{80}
\]

where

- \(A\) = cross sectional area of water prism
- \(D\) = conduit diameter
- \(T\) = top width of flow passage
- \(g\) = gravitational constant (acceleration)
- \(V\) = mean flow velocity
- \(y_e\) = effective depth = \(A/T\)
- \(y\) = flow depth

Equation 79 is good only if all air entrained is passed downstream. Prototype tests— for which a hydraulic jump formed in the conduit and in which the conduit velocities were large enough to convey all the entrained air out of the conduit—confirm the experimentally derived curve (fig. 27).

If the conduit is horizontal or sloping upward in the direction of flow then all the entrained air will move with the flow. However, if the conduit slopes downward in the direction of flow air bubbles can either move upstream or downstream relative to the pipe wall.

The direction of movement taken by the bubbles can be examined by considering the relative magnitudes of the buoyant and drag forces upon a stationary bubble in the flow (fig. 28). For example, the bubble will move perpendicular to the pipe axis only when the upstream component of the buoyant force vector equals the drag force component. This can be written as

\[
(q_w - q_e) \frac{\pi D_e^2}{6} (gS_o) = C_b \frac{\pi D_e^2}{4} \left( \frac{q_w V^2}{2} \right) \tag{81}
\]

where

- \(C_b\) = drag coefficient on bubble
- \(D_e\) = equivalent bubble diameter
- \(S_o\) = pipe slope = \(\sin \alpha\)
CLOSED CONDUIT FLOW

FROUDE NUMBER OF FLOW, $F-1$

RELATIVE AIRFLOW RATE $\beta = Q_a/Q_w$

$F=\frac{V}{VgY_e}$

$V$ = mean velocity of water
$g$ = gravitational constant
$Y_e$ = effective depth

Kalinske & Robertson tests, [38] (model)

- Ikari Dam, Murakawa et al., [53]
- Navajo Dam, WPRS, (not published)
- Pine Flat Dam, USCE, [70] (prototype)

Figure 27.—Air entrainment with hydraulic jump closing conduit.
Rearranging terms and dividing by the conduit diameter gives

$$\frac{V^2}{gD} = \frac{4}{3} \left[ 1 - \left( \frac{q_e}{q_w} \right) \right] \frac{D_e}{D} \left( \frac{S_o}{C_b} \right)$$

or

$$\frac{Q_c^2}{gD^5} = \frac{\pi^2}{12} \left[ 1 - \left( \frac{q_e}{q_w} \right) \right] \frac{D_e}{D} \left( \frac{S_o}{C_b} \right)$$

where

- $Q_c =$ critical discharge needed to carry bubbles with the flow
- $D =$ conduit diameter

This relation shows that the critical discharge for bubble motion is a function of the effective bubble diameter $D_e$, the densities, $q$, the drag coefficient $C_d$ of the bubble, and the pipe slope $S_o$. Unfortunately, the drag coefficient and effective bubble diameter can not be predicted for flow in a pipe. Therefore, the techniques of dimensional analysis must be used to determine the significant parameters for correlations.

As was shown under Design Parameters—Mean air concentration, the effective bubble diameter is a function of the interfacial surface tension and the friction slope. In terms of dimensionless parameters, the critical discharge required to move the bubbles can be expressed as

$$\frac{Q_c^2}{gD^5} = f \left( \frac{\gamma D^2}{\sigma}, S_f, S_o, C_b \right)$$

The parameter $\gamma D^2/\sigma$ is designated frequently as the Eötvös number $E$.

Kalinske and Bliss [37] found relatively good correlations for the initiation of bubble movement by using only the pipe slope $S_o$ and the Eötvös number. Data by Colgate [15] also fits their curves relatively well (fig. 29).

Additional studies are required to define the bubble motion curve (fig. 29) for slopes greater than 45 degrees. Martin [52] showed that a stationary air pocket forms when the dimensionless discharge $Q_w^2/gD^5$ is equal to 0.30 for vertically downward flow. Therefore, the increasing trend of the curve in figure 29 probably does not continue past the 45-degree slope.

As the bubbles travel downstream in sloping conduits, they tend to rise to the top of the conduit and form large pockets of air. Runge and Wallis [61] discovered that the rise velocity of these pockets is greater in sloping conduits than it is in vertical conduits (fig. 30). For a specific range of discharge, a flow condition can exist whereby bubbles will move downstream and form into pockets that move against the flow in an upstream direction.

Sailer [62] investigated prototype cases in which large air pockets moved against the flow with sufficient violence to completely destroy reinforced concrete platforms. The reverse flow region has been delineated on figure 29 using the data of Colgate [15] and the slug-flow curve of figure 30. The five structures pointed out by Sailer as having experienced blowbacks are indicated by crosses on figure 29. It is noted that
two of these structures lie within the blowback zone at design discharge. The other three must pass through the blowback zone in coming up to the design discharge. For pipe slopes less than 0.1, the width of the blowback zone is so small that problems normally are not experienced.

**FLOWS FROM CONTROL DEVICES**

Flows from control devices refer to cases in which the primary cause of the air demand is due to the workflow conditions at a control device. Two types of flow control devices that will be considered are gates and valves. These devices also induce air movement in open channel flows. However, in unconfined flows the water movement does not cause low pressures which must be relieved by air vents.

A distinction is made in the field of hydraulic machinery between valves and gates even though both serve as flow control in a closed conduit. A valve is a device in which the controlling element is located within the flow (fig. 31). A gate is a device in which the controlling element is out of the flow when it is not controlling and which moves transverse to the flow when controlling (fig. 31). The jets from gates are different than those from valves: therefore, the two cases are considered separately.

**FIGURE 29.** Bubble motion in closed conduits flowing full.
Flows From Valves

Around the beginning of the 20th century, many outlet valves were placed on or near the upstream faces of the dams. Nearly all were severely damaged by cavitation erosion. Since a satisfactory method could not be found to reduce or eliminate the damage at all gate positions, the operating ranges of these valves were severely restricted. Because of this limitation, the location of the throttling valves was shifted to the downstream side of the dam. Present practice is to avoid placement of throttling valves within the conduit. Nevertheless, from time to time it is necessary to place the valves within the conduits. This is especially true when the downstream conduit is a tunnel—when spray could cause icing problems—and when a flow control station is placed in a pipeline.

If stratified or wave flow exists downstream of the valve, air is induced to move by a relatively low water velocity acting over a large surface area. However, if the flow from the valve impinges on the downstream conduit walls, the airflow is induced by high velocity waterjet acting over a relatively small surface area. In this case, the significant airflow parameters are the:

- Kinetic energy of the waterflow,
- Gate opening, and
- Air pressure at some characteristic location.

Parameters such as length of conduit downstream of the valve and the Froude numbers of the downstream flows are obviously of lesser importance.
### Closed Conduit Flow

#### Service Classification

<table>
<thead>
<tr>
<th>Throttling Valves</th>
<th>Fixed-Cone Valve</th>
<th>Hollow-Jet Valve</th>
<th>Needle Valve</th>
<th>Tube Valve</th>
<th>Sleeve Valve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Fixed-Cone Valve</td>
<td>Hollow-Jet Valve</td>
<td>Needle Valve</td>
<td>Tube Valve</td>
<td>Sleeve Valve</td>
</tr>
<tr>
<td>Maximum head (approx.)</td>
<td>300 m</td>
<td>300 m</td>
<td>300 m</td>
<td>90 m</td>
<td>75 m</td>
</tr>
<tr>
<td>Discharge coefficient</td>
<td>0.85</td>
<td>0.70</td>
<td>0.45 To 0.60</td>
<td>0.30 To 0.55</td>
<td>0.80</td>
</tr>
<tr>
<td>Submerged operation</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Throttling limitations</td>
<td>None</td>
<td>Avoid very small discharge</td>
<td>None</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Spray</td>
<td>Very heavy</td>
<td>Moderate</td>
<td>Small</td>
<td>Moderate</td>
<td>None</td>
</tr>
<tr>
<td>Leakage</td>
<td>No</td>
<td>None</td>
<td>None</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Nominal size range, diameter (b)</td>
<td>200- to 2740-mm</td>
<td>200- to 2740-mm</td>
<td>200- to 2440-mm</td>
<td>910- to 2440-mm</td>
<td>310- to 610-mm</td>
</tr>
<tr>
<td>Availability</td>
<td>Commercial standard</td>
<td>Special design</td>
<td>Special design</td>
<td>Special design</td>
<td>Special design</td>
</tr>
<tr>
<td>Maintenance required</td>
<td>Paint</td>
<td>Paint</td>
<td>Paint</td>
<td>Paint</td>
<td>Paint</td>
</tr>
</tbody>
</table>

**Comments and Notes:**
- Coefficients are approximate and may vary somewhat with specific designs.
- Valve sizes shown are representative, and are not limiting.
- Air-venting required.
- Valve is designed for use only in fully submerged conditions.
- Larger sizes seem feasible and will probably be developed.

#### Throttling Gates

<table>
<thead>
<tr>
<th>Unbonneted Slide Gate</th>
<th>Bonneted Slide Gates</th>
<th>Jet-Flow Gate</th>
<th>Top-Shaft Radial Gate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Unbonneted Slide Gate</td>
<td>&quot;High Pressure&quot; Type</td>
<td>Streamlined Type</td>
</tr>
<tr>
<td>Maximum head (approx.)</td>
<td>25 m</td>
<td>60 m</td>
<td>60 m</td>
</tr>
<tr>
<td>Discharge coefficient</td>
<td>0.6 To 0.8</td>
<td>0.95</td>
<td>0.97</td>
</tr>
<tr>
<td>Submerged operation</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Throttling limitations</td>
<td>Avoid very small discharge</td>
<td>Avoid very small discharge</td>
<td>None</td>
</tr>
<tr>
<td>Spray</td>
<td>Minimum</td>
<td>Minimum</td>
<td>Minimum</td>
</tr>
<tr>
<td>Leakage</td>
<td>Small</td>
<td>Small</td>
<td>Small</td>
</tr>
<tr>
<td>Nominal size range (b)</td>
<td>3660- wide d 3600-mm high</td>
<td>1830- to 3660-mm high</td>
<td>250- to 3650-mm dia.</td>
</tr>
<tr>
<td>Availability</td>
<td>Commercial standard</td>
<td>Special design</td>
<td>Special design</td>
</tr>
<tr>
<td>Maintenance required</td>
<td>Paint</td>
<td>Paint</td>
<td>Paint</td>
</tr>
</tbody>
</table>

**Comments and Notes:**
- Coefficients are approximate and may vary somewhat with specific designs.
- Valve sizes shown are representative, and are not limiting.
- Air vents required.
- Use of stainless steel surfaces will reduce maintenance requirements and cavitation damage hazard.
- Seal replacement in 5-15 years is probable depending on design and use.

**Figure 31.** Valve and gate data, Kohler [44].
Colgate [14] made model studies of airflows in valves having a fixed-cone. His results were given in terms of gate opening and discharge. Transforming these values into the appropriate dimensionless parameters results in good correlations for all conditions that were tested (fig. 32). In this case, the kinetic energy of the flow is proportional to the total upstream head. Thus

\[
Q_a = f \left( G, \frac{\Delta p}{Y} \right)
\]

(85)

where

- \(G\) = gate opening in percent
- \(H_t\) = total potential and kinetic energy (upstream)
- \(\Delta p / Y\) = differential between atmospheric pressure and air pressure at end of vent
- \(Y\) = specific force of water

Once curves like those presented in figure 32 are developed, it is possible to determine the airflow rates through any air vent that is connected to the structure by using equation [71]. To perform the determination, equation 71 is plotted on figure 31. The intersections of the two sets of curves give the airflow rates for any particular vent.

**Flows From Gates**

At small gate openings, a considerable amount of spray is produced by flow which impinges in gate slots. This spray induces considerably more air movement than that produced by stratified or wave flow. In a sense, the effect of spray in producing air movement is similar to that of flow from valves. However, with spray the jet does not impinge on the walls of the downstream conduit; therefore, a seal does not form.

The significant parameters for flow with spray are the same as those for flow from valves; i.e.,
- Gate opening,
- A reference air pressure, and
- The total upstream energy.

Model studies can be used to obtain estimates of the airflow rates which can be expected when spray is present.

As the gate opening increases, the amount of spray decreases. Typically, spray is not significant for openings greater than 10 or 20 percent. The exact percentage depends upon the design of the gate. For the larger openings, the airflow rate is controlled by the two-layer flow relations. That is, the significant parameters are:
- Length of conduit,
- Froude number of the flow, and
- Air pressure at some reference location.

For jet-flow gates a point is reached—as the gate opening increases above some value—where the flow impinges on the downstream conduit. Typically this occurs at a 50- to 60-percent opening. With impinging flow, the airflow rate is correlated with the parameters used for flow from fixed-cone valves. For this type of flow, the length to diameter ratio of the conduit is significant only if the downstream conduit length is less than the distance to the impingement point or if the adverse pressure gradient is large.

**FALLING WATER SURFACE**

A falling water surface in a closed conduit induces airflow in the conduit. This flow is analogous to that induced by a piston in a cylinder; the water corresponds to the piston. A typical example of this type of flow occurs during an emergency closure of the intake gate to a penstock (fig. 33). As the gate closes, water flowing into the penstock from the reservoir is
CLOSED CONDUIT FLOW

\[ g = \text{gravitational constant} \]
\[ H_t = \text{total potential and kinetic energy} \]
\[ h_m = \text{head across manometer} \]
\[ p = \text{pressure intensity} \]
\[ P_{\text{in}} = \text{internal pressure} \]
\[ Q_a = \text{volume flowrate of air} \]
\[ Q_w = \text{volume flowrate of water} \]
\[ V = \text{mean flow velocity} \]
\[ \gamma = \text{specific force of water} \]

**Figure 32.** Airflow rate for two 137.5-mm fixed-cone (Howell-Bunger) valves.
a. ENERGY AND PIEZOMETRIC GRADE LINES
IN PENSTOCK AND DRAFT TUBE

- Loss across gate
- Loss due to flow entering from gate chamber

b. ENERGY AND PIEZOMETRIC GRADE LINES
AT INTAKE STRUCTURE

FIGURE 33.—Falling water surface.
gradually stopped. However, water in the penstock continues to flow through the turbine in the powerplant. Eventually the gate becomes fully closed. For water to continue flowing from the penstock, air must be allowed to enter the system through a vent located just downstream from the intake gate.

The airflow and waterflow relations—through the penstock and gate chamber—can be simulated analytically by the appropriate mathematical model, Falvey [22]. This model, based upon momentum and continuity equations, yields the airflow rates, etc., as a function of time.

With relatively long penstocks; i.e., length to diameter ratios exceeding 30, the maximum airflow rate occurs slightly after the emergency gate closes completely. The magnitude of the airflow rate is equal approximately to the penstock discharge prior to the start of the gate closure. These observations provide “rules of thumb” which can be used for the design of the air vent structures on dams. The computer program presented in appendix III should be run if a time history of the air-water flow relation is required or if shorter penstocks are being analyzed. This program is a generalized version of the original program and includes typical turbine characteristics.

Good correlations have been found between the computer model calculations and prototype measurements (fig. 34).

AIR VENT DESIGN CRITERIA FOR CLOSED CONDUITS

Purpose

The design of air vents for closed conduits requires careful consideration. The preliminary step is to decide the purpose that the vent is to perform. For instance, air vents can permit air to enter a structure to prevent collapse or to prevent the formation of low pressures within the flowing water which could lead to cavitation and its possible attendant damage. Conversely, air vents can permit air to escape from a structure. In this case the purpose is to bleed air from a conduit prior to operation.

Location

The next step is to locate the vent properly. General rules cannot be delineated for all cases other than the vent usually is placed where the pressure in the conduit is the lowest. For instance, in gates the appropriate location is immediately downstream of the gate (fig. 31B). For valves the air vent is upstream from the point where the water jet impinges on the conduit walls (fig. 32). In some cases the location must be determined by intuition or carefully conducted model studies.

Maximum Airflow Rate

After the vent is located, the maximum airflow rate through the vent must be estimated. This estimate should be based upon a consideration of the various types of flow which are possible in the water conduit. The previous sections have presented in detail some methods of estimating the maximum airflow rates for specific types of closed conduit flows.

Structural Considerations

The pressure drop across the air vent causes a reduced pressure in the penstock and gate structure. Each part of the structure which is subjected to reduced pressure should be analyzed to determine if it will withstand the imposed loads.

Physiological Effects

The effects of noise produced by high air velocities as well as the structural integrity must
FIGURE 34.—Comparison of field data with computer prediction.
be considered in the design of air vents. The limiting air velocity—with respect to noise—in a vent has been established (by the Water and Power Resources Service) to be about 30 m/s. Above this velocity an objectionable whistling sound occurs. The intensity of the sound and not its mere presence is the governing factor. For instance, ear protection is required for exposure times greater than eight hours and pressure levels above 85 dB (decibels) Beranek and Miller [9]. For pressure levels above 135 dB, ear protection is required for any exposure time.

Field measurements 5 meters from an air vent having an 80-m/s velocity produced sound level intensities of 105 dB. With this sound intensity, ear protection is required for exposure times exceeding 7 minutes. Since sound level intensities increase by the 6th to 8th power of velocity Davies and Williams [19], a 200-m/s air velocity would have produced a sound level intensity between 128 and 136 dB which is damaging to the ears for any exposure time. Based upon this limited result, a 90-m/s flow velocity appears to be a good value to use as a design criterion for air vents that operate for a short duration. If the air will flow through the vent for extended periods, the upper limit on the air velocity should be restricted to the 30-m/s value.

Safety of Personnel

Another design consideration concerns the safety of personnel in the vicinity of the vent when it is operating. Generally, personnel barriers should be placed around vents at locations where the air velocities exceed 15 m/s. This will prevent personnel and loose objects from being swept either through the air vent or held on the air vent louvers.

Freeze Protection

In areas where the vents operate in cold weather for prolonged periods, the vents should be protected from freezing. Icing occurs when supercooled air passes through the louvers and screens at the vent intake. In some cases ice buildup was sufficient to completely block the flow area (fig. 35). Icing protection includes using heating elements on the louvers, rerouting the vent to place the intake in a warm portion of the structure, or redesign of the intake to eliminate ice buildup areas.

Cavitation Damage

The pressure downstream of gates discharging into conduits should be prevented from becoming too low. If the pressure does drop excessively, cavitation damage may result during prolonged periods of operation. Unfortunately, general guidelines concerning minimum acceptable pressures cannot be given. Each gate or valve design has its own particular characteristics. Some designs are more susceptible to cavitation damage than others. Research studies are needed to define minimum pressure values for the different classes of gates and valves.

Water Column Separation

If the pressure in the water column reaches vapor pressure of the water a possibility exists
that the column will separate. Depending upon the geometry of the conduit, the separation can occur at either one location or at several locations. If water column separation is indicated, special waterhammer computations should be performed to determine the overpressures when the water columns rejoin.

AIR VENT DESIGN CRITERIA FOR PIPELINES

Introduction

Flow in long pipelines presents a separate class of considerations from those already discussed. One of the reasons for the new set of considerations is the fact that the pipeline profile normally follows the ground surface topography very closely. This causes intermediate high locations which provide an opportunity for the collection of air pockets. To assure trouble-free pipeline operation, details of alinement, location, and sizing of vent structures must be considered.

There are essentially four main categories of pipelines. They are:

1. Gravity pipelines in which the water flows from a higher elevation to a lower one through the effect of gravity (fig. 36A).
2. Sagpipes (inverted siphons)9 in which the flow from one canal to another is passed under a road or across a valley (fig. 36B).
3. Pump lifts in which the water flows from a low elevation to a higher one through pump action (fig. 36C).
4. Siphons in which some portion of the pipe is designed to operate at subatmospheric pressures (fig. 36D). This type of structure is used frequently to prevent water from the upper reservoir from passing back through the pump if a loss of electrical power occurs.

9See footnote 1.
Gravity systems, figures 36A and B, normally have different alignment problems than pumping systems (fig. 36C and D); therefore, the two are considered separately.

Gravity Systems

A vertical section through a typical gravity system is shown on figure 37. The same type of layout also applies to sag pipes if the open vent structures are replaced by canal sections. Two types of summit configurations are depicted. In one case the intermediate summit is above the downstream vent structure. This forms a pool upstream of the summit at the no-flow condition. In the other case, the intermediate summit is below the downstream vent structure. Therefore, it is submerged by the pool which forms at the no-flow condition.

To prevent difficulties during startup operations, certain criteria should be followed regarding both the vertical and horizontal alignment at the upstream vent structure and at intermediate summits whose elevations lie above the downstream open vent structure.

**Vertical alignment criteria.**—The pipe invert should be placed on a uniform slope between the vent or summit and the adjacent downstream pool. If this cannot be achieved then the pipe should be placed on continually steeper slopes so that during filling the flow continues to accelerate to the pool level. If the flow were allowed to decelerate, the water depth in a circular pipe could gradually increase until the pipe was about 82 percent full. At this depth the flow could become unstable, alternating between full conduit flow and the 82-percent depth.

At less than design discharge, the flow downstream of nonsubmerged summits passes from free-surface to closed-conduit flow. An air-entraining hydraulic jump always forms when the flow makes this transition. The entrained air can form large air pockets under certain circumstances which move against the direction of flow. This condition is commonly referred to as blowback (refer to previous section—Flow Having a Hydraulic Jump That Fills the Conduit).

If the alignment cannot be planned to avoid either operating in or passing through the blowback region delineated in figure 29, then the pipe diameter should be altered to avoid the region.

Some attempts have been made to collect the large air bubbles which form on the crown of the pipe and lead them away from the pipeline (fig. 38). In the example, the flow conditions never entered the blowback flow region. Therefore, the complicated air collection
system was not needed. If flow had entered the blowback region, this structure probably would not have worked. Colgate [15] found that an unsteady flow condition develops when large air bubbles are bled from a pipeline with too small a vent. To minimize the unsteady flow it is necessary for the vent diameter to equal the pipeline diameter. The design of antiblowback structures like the type shown on figure 38 should not be attempted without hydraulic model studies.

**Horizontal alignment criteria.**—At the open vent structures and at the intermediate summits higher than the downstream vent, the pipe should not contain bends for 10 pipe diameters upstream of the location. In addition bends should be avoided in the section between the vent on the summit and the adjacent downstream pool. These criteria prevent transverse waves from being formed on the free water surface which can exist downstream of the vent or summit at partial flows. These transverse waves could roll over with enough amplitude to intermittently seal the pipeline.

**Vent location.**—The type of air release structure to be used at a summit is determined by the...
FIGURE 38.—Vent structure, 244-D-799
distance from the pipe invert to the hydraulic grade line at the summit. For summits higher than the downstream vent, an open vent is desirable. The maximum allowable vent height is determined from topographic, aesthetic, and economic considerations. Normally, open vents at intermediate summits are not feasible if the distance to the hydraulic grade line \( H_i \) exceeds 6 to 10 meters.

For summits lower than the downstream vent, the type of air release structure is more difficult to determine. If the distance to the hydraulic grade line \( H_2 \) is less than about 6 meters, an open vent should be used. However, if the distance exceeds 6 meters an air valve installation should be used (fig. 39). Since mechanical air valves tend to chatter and spit water if their operating pressures are too low, the top of the air valve should be set at least 3 meters below the pool level.

To provide desirable operating characteristics at all discharges, vents also are required at locations other than at the intermediate summits. If the water velocities are of sufficient magnitude to carry air bubbles with the flow, then vents are needed downstream of changes from negative to positive pipe slopes. Without the vents the air slugs, which collect on the crown of the pipe, will attain very high velocities in areas with large positive slopes. These slugs can damage the vent structures at intermediate locations, at downstream connecting canals, and can cause slamming of air valves. These vents should be located less than 30 meters downstream from the negative to positive pipe slope change. If the distance from the intersection of the pool with the negative slope and the proposed vent exceeds 20\( D \), where \( D \) is the conduit diameter, then the vent should be placed at the greater of the two distances (fig. 40). The criterion for the vent type is the same as for vents placed at intermediate summits below the downstream vent structure. If the distance between the upstream and downstream vent structures is very great,
Lescovich [47] recommends that air valves be placed every 500 to 1000 meters along descending, horizontal, or ascending stretches that have no intermediate summits.

**Pumping Systems**

All intermediate summits are potential locations for the collection of air pockets. If these pockets begin to develop, the hydraulic gradient downstream of the summit will equal approximately the pipe slope in the area where the air pocket has formed. For a pipe slope greater than the full-flow hydraulic gradient, the air pocket will require a greater head differential to produce a given discharge. Conversely, for a constant head differential, the presence of the air pocket will result in decreased discharges. The limiting condition is a complete blockage of flow. In pumping systems this blockage is known as *air binding* [58]. With air binding the shutoff head of the pump will have been reached (fig. 41). One obvious solution to the problem of air collection at summits is to provide air release valves or vent structures at these locations. Another solution is to align the pipeline so that all intermediate summits are eliminated.

**Vent Structure Design Considerations**

Vent structures have three primary purposes:
1. Evacuation of air during filling,
2. Removal of air during operation, and
3. Prevent pipe collapse during draining.
Each is considered in detail. The size of the vent and the piping connecting the vent to the pipeline is determined by the purpose for which the vent is installed.

*Evacuation of air during filling.*—The filling rate of pipelines usually is set at 5 to 15 percent
of the design discharge. The actual rate is governed by the maximum waterhammer pressures that the pipeline and valves can withstand. These pressures are generated when the water column in the penstock reaches the air release valve. Based on waterhammer considerations the filling rate of pipelines can be computed from

$$Q_a = \frac{gA_p \Delta h_{sw}}{c}$$  \hspace{1cm} (86)$$

where

- $Q_a$ = penstock filling rate equals airflow rate through vent
- $A_p$ = cross sectional area of penstock
- $c$ = celerity of waterhammer wave in penstock
- $g$ = gravitational constant (acceleration)
- $\Delta h_{sw}$ = allowable head rise in penstock due to waterhammer pressures

Lescovich [47] indicated that large orifice air valves should be used to permit air escape during filling (fig. 42). In this case a large orifice refers to diameters greater than 25 millimeters. This type of air valve is designed to remain closed after the pipeline is filled. Thus, they cannot be used to release small amounts of air that accumulate during operation. These valves will open immediately when the pipeline pressure drops below atmospheric. This allows air to reenter the pipeline and prevents a vacuum from forming.

Normally, air velocities discharging from an air valve should not exceed 30 m/s. The primary reason for limiting the velocity is to prevent the air valve from being blown shut. Some air valves are designed to eliminate this problem.

With the 30-m/s velocity limitation, the air can be considered to be incompressible. The equation for the airflow rate is

$$Q_a = A_o C_o \left( \frac{2 \Delta P}{ho_a} \right)^{1/2}$$  \hspace{1cm} (87)$$

where

- $A_o =$ orifice area, m²
- $C_o =$ orifice coefficient ≈ 0.6
- $\Delta P =$ pressure differential across the orifice, kPa
- $\rho_a =$ air density (at 20 °C and a pressure of 101.3 kPa, $\rho_a = 1.204$ kg/m³)

From this equation, performance curves for large-orifice air valves can be derived (fig. 43).

If the desired capacity cannot be achieved with a single air valve, the valves can be placed in clusters—up to four valves—on a single vent pipe from the pipeline.

**Removal of air during operation.**—Two types of structures are used to remove air during operation. These are open-vent structures and small-orifice air valves. In either case the connection to the pipeline must be large enough
CLOSED CONDUIT FLOW

A. Lowered position - Float allows air to flow into or out of pipeline

B. Raised position - Air cannot enter or leave pipeline

Figure 42.—Large-orifice air valve.

to collect the slugs and bubbles of air which are traveling on the crown of the pipeline.

Colgate [15] investigated the sizing criteria for open-vent structures. He found that if the collection port was too small, portions of large air slugs would pass by the vent. To trap all the air it was necessary for the diameter of the collection port to be equal to the pipe diameter. Additional tests were made to investigate the size of the vent structure itself. It was found that if the air vent diameter was less than the pipeline diameter, an unsteady flow was established in the vent as large air bubbles exited from the vent. This unsteady flow pumped air back into the pipeline. To minimize pumping it was necessary to make the vent diameter equal to the pipeline diameter.

Colgate [15] concluded that the collection and evacuation of air from a pipeline can be best accomplished by a vertical air vent which is connected directly to the pipeline. The diameter of the vent should be at least equal to the diameter of the pipeline. From access considerations, the minimum vent diameter usually is set at about 1 meter. Removal of air is promoted if the pipe slope immediately downstream of the vent is made steep enough to cause the air bubbles to return upstream. Figure 29 can be used to determine the required slope for a given discharge.

For the case in which the hydraulic grade line is too far above the pipeline to economically install an open vent, air valves are used to remove the air. Investigations concerning the design of
Air at 20 °C and 101.3 kPa

Figure 43.—Performance curves for large-orifice air release valves.

A collector have not been performed. Based upon the design of open vents it can be assumed that the diameter of the collector should be at least equal to that of the pipeline. The height of the collector also should be one pipeline diameter. In many cases, manholes in the pipeline can serve as collectors.

To release air from pipelines under high pressures, small-diameter orifice installations are used (fig. 44). The small orifice assures that the opening force of the float is not exceeded by the closing force whose magnitude is equal to the internal gage pipe pressures times the orifice area. The volume flow of air relation through an orifice with a back pressure is given by

\[ Q_a = 460A_o[p_{in}/p_{atm}]^{0.2857} - 1]^{1/2} \quad (88) \]

for

\[ \frac{p_{atm}}{p_{in}} > 0.53 \]

and

\[ Q_a = 11.8A_o[p_{in}/p_{atm}]^{0.7143}]^{1/2} \quad (89) \]

for

\[ \frac{p_{atm}}{p_{in}} \leq 0.53 \]

These equations are presented as performance curves (fig. 45).

To prevent the air valves from freezing, frequently they are placed in concrete structures located below the frost line (fig. 46). In this case it is necessary to provide adequate ventilation into or out of the structure. The required ventilation area is based upon a 2.5-m/s maximum air velocity through the gross area of a fixed louver vent. If wire mesh screen is used, the maximum air velocity is 6.6 m/s through the gross area of the screen.
Prevent pipe collapse during draining.— The venting criteria discussed thus far are based upon the need to remove air from the pipeline. In several instances above-ground steel pipelines have collapsed because vacuum formed during rapid draining operations or because of breaks in the pipeline. Parmakian [56] developed criteria for the size and location of air valves to be placed in steel pipelines to protect them against collapse.

On steel pipes, the collapse pressure can be estimated from (Parmakian [56])

\[
p_c = 3.5 \times 10^8 \left(\frac{t}{D}\right)^3
\]

\[
= p_{atm} - (p_{in})_{abs} - (p_{in})_{gage}
\]

where

- \(D\) = conduit diameter, mm
- \(p_{atm}\) = atmospheric pressure, kPa
- \(p_c\) = collapse pressure, kPa
- \(p_{in}\) = internal absolute or gage pressure, kPa
- \(t\) = pipewall thickness, mm

With stiffener rings, the appropriate equation is

\[
p_c = \frac{5.1 \times 10^8 (t/D)^2.5}{(L_s/D)}
\]

where \(L_s\) = distance between stiffener rings.

These two equations are presented graphically in figure 47.

Applying a safety factor \(N\) to the internal collapse pressure \(p_c\) gives the allowable internal pressure \(p_a\) as

\[
p_a = p_{atm} - \frac{p_c}{N}
\]

If the ratio of the internal to atmospheric pressure is greater than 0.53 then the volume
flow of air into the pipeline through an orifice is given by

\[ Q_a = C_d A_o \left( \frac{P_{in}}{P_{atm}} \right)^{1/2} \left[ \frac{2P_{atm}}{Q_{atm}} \left( \frac{x}{x+1} \right) \right]^{1/2} \]  
\[ (93) \]

If the ratio is equal to or less than 0.53 then the airflow rate into the pipeline through an orifice is given by

\[ Q_a = C_d A_o \left( \frac{2}{x+1} \right)^{1/2} \left[ \frac{2P_{atm}}{Q_{atm}} \left( \frac{x}{1+x} \right) \right]^{1/2} \]  
\[ (94) \]

Using

\( C_d = 0.6 \)

\( P_{atm} = 101.3 \text{ kPa} \)

\( x = 1.4 \)

\( Q_{atm} = 1.20 \text{ kg/m}^3 \)

in equations 93 and 94 results in

\[ Q_a = 460A_o \left( \frac{P_{in}}{P_{atm}} \right)^{0.715} \left[ 1 - \left( \frac{P_{in}}{P_{atm}} \right)^{0.286} \right]^{1/2} \]  
\[ (95) \]

for

\( \frac{P_{in}}{P_{atm}} > 0.53 \)

and

\[ Q_a = 119A_o \]  
\[ (96) \]

for

\( \frac{P_{in}}{P_{atm}} \leq 0.53 \)

These equations are presented as performance curves for various size vacuum relief valves (fig. 48).

Parmakian presented an alternate method of determining the required air vent size in terms of a dimensionless ratio. The ratio is in the form of an Euler number and is given by

\[ \left( \frac{\Delta V^2}{C_o^2 P_{atm} \nu_{atm}} \right)^{1/4} = \frac{1}{C_o^{1/2} E_u^{1/4}} \]  
\[ (97) \]

where

\( C_o = \text{orifice discharge coefficient} \)

\( E_u = \text{Euler number} = \frac{P_{atm}}{Q_a \nu^2} \)

\( P_{atm} = \text{atmospheric pressure} \)

\( \Delta V = \text{change in water velocity approaching and leaving air vent} \)

\( Q_a = \text{density of air at standard atmospheric pressure} \)

\( \nu_{atm} = \text{specific volume of air at atmospheric pressure} \)

The pressure and specific volume of the atmosphere are both functions of elevation (fig. 49). This alternate method results in the required air vent orifice diameter as a function of the pipeline diameter (fig. 50).

Normally, air valves are placed at the crests in the pipeline profile and at locations where the pipeline begins a steep downward slope.
Figure 45.—Performance curves for small-orifice air release valves.
Surface to be horizontal

Air vent, see Vent Detail
Insulated cover not shown

C Air valve, air vent and manhole

E Pipe well

PLAN-AIR VALVE

2:1

Original ground surface

Precast concrete cover required but not shown

Insulated cover, see Section E-E
Pipe well, reinf. concrete pressure pipe

Locate manhole on upslope side of structure

E Pipe

Pipe joint

Miter cut

SECTION C-C

Air valve
Gate valve
Steel riser pipe

Short radius elbow

Adhere galvanized sheet metal to one side of rigid styrofoam insulation for cover and drop in panels

Flexible foam insulation, adhere to concrete pipe

Bend sheet metal as shown

Cut insulation to fit around vent pipe

INSULATED COVER DETAIL
SECTION E-E
(Valve not shown)

FIGURE 46.—Typical frost protection installation.
CLOSED CONDUIT FLOW

FIGURE 47.—Collapsing pressure of a steel pipe with stiffener rings.
FIGURE 48.—Performance curves for large-orifice vacuum relief valves.
Figure 49.—Specific volume and barometric pressure of air as a function of elevation.
$P_c = \text{collapse pressure}$

$P_{\text{atm}} = \text{atmospheric pressure}$

$d_o = \text{orifice diameter in air valve.}$

$C_o = \text{orifice discharge coefficient} = 0.5$

$\Delta V = V_2 - V_1$

$p_2 = p_{\text{atm}} - \frac{p_c}{N}$

$N = \text{safety factor}$

$k = \text{gas constant} = 1.4$

Figure 50.—Required air relief orifice diameter to prevent collapse of steel pipelines.
CLOSED CONDUIT FLOW

FLOWS IN VERTICAL SHAFTS

Classification of Airflows

Three types of hydraulic structures that use a vertical shaft to convey water from one elevation to another are:

- Spillways
- Intakes
- Drop shafts

The air entrainment properties of these structures are important since at certain flowrates explosive air blowbacks are possible (fig. 51). Often extensive studies are necessary to design vent structures to remove the air which is entrained in the vertical shaft Anderson [3] and Babb [6].

The amount of air entrained in the shaft is strongly dependent upon the type of flow into the shaft and upon the water level in the shaft. The inlet flow can vary from radial to tangential with flow entering around the circumference of the shaft. Typical types of inlet structures (fig. 52) are:

- Circular weirs
- Vortex inlets
- Radius elbows

The effect of water surface (reservoir) elevation at the entrance to a shaft can be examined by considering the discharge characteristics of a vertical shaft spillway (fig. 53). For low water surface levels the discharge is proportional to the three-halves power of the total head on the crest. The flow in the shaft clings to the walls in

Figure 51.—Observed air blowback in morning glory spillway at Owyhee Dam, Oregon. P801-D-79280
Figure 52.—Typical types of vertical shaft inlet structures.

Figure 53.—Vertical shaft spillway discharge characteristics.
a relatively thin sheet. The volume flow rate of air is determined primarily by the shear action of the air-water interface and by entrainment into the mass of the water. This type of flow has been designated as region I on figure 53. As the water discharge increases with increasing reservoir elevation, a point is reached when the sheet of water is sufficiently thick to completely fill the upper end of the conduit. This water discharge separates region I from region II type flows.

Region II type flows are characterized by an annular hydraulic jump. Further increases in reservoir elevation merely cause the location of the jump to move upward in the vertical shaft. When the jump reaches a point near the top of the shaft, the flow is said to become submerged.

For reservoir elevations in excess of that required to produce the submerged water flow, all inflow of air to the shaft ceases. The discharge for this flow range is proportional to the one-half power of total energy over the crest.

If the bottom of the shaft is always submerged, then a region I type flow will not develop. Instead, the air motion will be described by a region II type flow up to the point when the vertical shaft is submerged.

The airflow rates discussed above should not be confused with those that are present in the portions of the structure downstream of the vertical shaft. The methods discussed in this chapter—Flow in Partially Filled Conduits—should be used to analyze the flow of air in the horizontal sections of vertical shaft spillways and similar structures. Mussalli and Carstens [55] studied surging problems that develop as the horizontal conduit seals [fig. 21 (5)]. However, they did not develop any air entrainment criteria for the vertical shaft.

**Region I Airflow Rates**

The airflow rate down the vertical shaft can be calculated by assuming:

a. The water flow on the shaft walls is similar to open channel flow, and
b. The lower end of the shaft is open to the atmosphere.

If the inlet is not designed to keep the water flow attached to the wall, the airflow rate cannot be calculated.

Several methods are available to estimate the airflow rate when the water forms in a sheet on the walls. For instance, the air insufflated into the flow can be estimated from equation 59 using open channel flow relations. The amount of air flowing on the core of the pipe can be determined from

$$Q_a = V_o A_c$$  \hspace{1cm} (98)

where

- $A_c$ = cross sectional of air in core
- $V_o$ = maximum water velocity in vertical shaft

Hack [27] recommends that the total airflow be determined from

$$Q_s = 0.35 + 16.1 \overline{C}^{2.88}$$  \hspace{1cm} (99)

where $\overline{C}$ = mean air concentration

The mean air concentration is estimated from

$$\overline{C} = \left(1 + \left[4(1 - e^{k_r(F_o^{-4/3} - F^{-4/3})})\right]^{-1}\right)^{-1}$$  \hspace{1cm} (100)

where

- $D$ = conduit diameter
- $F$ = Froude number at end of shaft
- $F_o$ = Froude number at point where boundary layer intersects water surface
- $k_r = 1.8 r_s + 0.0108$
- $k_s$ = equivalent sand grain roughness
- $r_s$ = relative roughness = $k_s/D$

The point where the boundary layer intersects the water surface is found through the application of equations 27 through 30.


Region II Airflow Rates

Various investigators have studied the entrainment of air by an annular jet.

Haindl [29] found that the air entrainment obeys a law very similar to that found by Kalinske and Robertson [38] for a hydraulic jump in a conduit. The relation is

\[ \beta = \frac{Q_a}{Q_w} = 0.02 (F - 1)^{0.86} \]  

(101)

where \( F = \text{Froude number} \)

\[ F = \frac{Q_w}{R_j D \left[ 1 - (R_j/D) \right][gR_j]^{1/2}} \]  

(102)

\( D = \text{outside jet diameter (conduit diameter)} \)
\( g = \text{gravitational constant (acceleration)} \)
\( Q_a = \text{volume flowrate of air} \)
\( Q_w = \text{volume flowrate of water} \)
\( R_j = \text{thickness of annular jet} \)

Kleinschroth [43] found a correlation for flows in vertical shafts having a vortex inlet. The relation is

\[ \beta = 0.022 \left( \frac{h_f}{D} \right)^{3/5} \]  

(103)

where
\( h_f = \text{distance from the inlet to the water level in the vertical shaft} \)
\( D = \text{shaft diameter} \)

Reverse Airflow in a Vertical Shaft

All the preceding relations assume that the waterflow rates are sufficient to remove all the entrained air from the system. Martin [51] showed that slug flow begins when the dimensionless airflow \( \beta \) exceeds 0.223. It was shown earlier that these slugs move up the shaft for

\[ \frac{Q^2}{gD^2} < 0.3 \]  

(104)

Therefore, for dimensionless water flow ratios less than 0.3, the airflow quantities given by equations 101 and 103 are too large. In addition, it is possible that blowback will occur in the shaft.

Submergence

The water depth which causes a vertical shaft to flow submerged has been determined only for the case of radial inflow. Jain, Raju, and Garde [36] determined that the submergence at which airflow down the shaft ceases is given by

\[ \frac{S}{D} = 0.47 F^{1/2} \]  

(105)

where
\( D = \text{shaft diameter} \)
\( F = V/(gD)^{1/2} \)
\( g = \text{gravitational constant (acceleration)} \)
\( S = \text{submergence depth} \)
\( V = \text{mean water velocity in shaft flowing full} \)

For a vortex inlet or for approach flow having some circulation, the required submergence would be greater than that given by equation 105.
Free Falling Water Jets

Free falling water jets have important aeration effects in the case of unconfined flow discharging from gates and valves. Three main areas of concern are:

1. Jet characteristics,
2. Airflow around the jet, and
3. Air entraining characteristics as a falling jet enters a pool of water.

Each of these subtopics will be considered in detail.

Jet Characteristics

Dodu [20] and Rouse et. al., [60] have shown that the jet characteristics are a function of the conduit geometry and flow dynamics upstream from the point where the jet begins. For instance, a laminar jet exiting from a carefully shaped orifice connected to a large tank of quiescent water can have such a smooth surface that the jet appears to be made of glass. If the water surface in the tank is disturbed, however, waves will form on the surface of the jet. For turbulent flow, the jet always disintegrates somewhere along its length if it is allowed to travel far enough. However, the distance to the point at which the breakup occurs is controlled by the turbulent intensity within the jet. By changing the flow geometry upstream of the jet, its turbulent intensity is varied.

It should be emphasize that the breakup of the jet is caused primarily by turbulence internal to the jet and only secondarily by the action of the air into which the jet discharges [60]. Tests by Schuster [64] in which a jet discharged into a vacuum show exactly the same jet texture and breakup characteristics as observed by Dodu [20] of a jet discharging into air. From this, one can conclude that physical models should accurately predict the spread and energy content of prototype jets if the turbulent intensity in the model is similar to that in the prototype.
When comparing model and prototype jets near their origin, the prototype jet apparently is surrounded by much more spray than the model (fig. 54A). This difference is due partially to the time scale relations between the two jets. The prototype represents, in essence, a high-speed photograph of the model. The many small drops in the model (fig. 54A) appear as a frothy spray when photographed at a much faster camera speed.

The effect of the air on the jet becomes significant only after the jet atomizes into individual drops.

In the region far from the origin of the jets, their trajectory is affected by air resistance and a large portion of the stream falls vertically downward as spray (fig. 54B).

Hinze [35] studied the breakup of individual falling drops. His results have been replotted in the form of Weber and Reynolds number relations (fig. 55). Here the Weber number is defined as

\[ W = \frac{U_d^2 D_d}{\sigma/\rho_f} \]  

(106)

where

\( D_d = \) diameter of water drop  
\( U_d = \) velocity of water drop relative to air velocity  
\( \rho_f = \) water (drop) density  
\( \sigma = \) interfacial surface tension

The Reynolds number is defined as

\[ R = \frac{U_d D_d}{v} \]  

(107)

where \( v = \) kinematic viscosity of the drop

From figure 55 and fall velocity equation for a rigid sphere, it is possible to estimate the maximum stable drop diameter as about 0.4 mm.

The fall velocity of spheres can be determined from a form of the Stokes equation (eq. 1). The relationship is

\[ V = \left\{ \frac{4}{3} \left( \frac{\rho_w}{\rho_a} - 1 \right) \frac{\rho D_d}{C_a} \right\}^{1/2} \]  

(108)

where \( C_a = \) drag coefficient of a sphere

The drag coefficients of spheres can be found in fluid mechanics texts [59].

The maximum stable drop diameter usually is not observed in nature because the larger free falling drops have some survival time associated with them. As small drops atomize they pass from a spherical shape to a torus with an attached hollow bag-shaped film. As this bag bursts, the entire torus breaks up. A certain time is required for this process to take place. Komabayasi et al., [45] found that drops of 7- and 5-millimeter diameters took 10 and 200 seconds, respectively, to break up. Thus, a 7-mm-diameter-drop would have to fall more than 85 meters at a terminal velocity of 8.5 m/s to breakup into smaller drops. A distance of more than 1300 meters would be required for the 5-m-diameter drop to breakup at a terminal fall velocity of 6.7 m/s.

Airflow Around the Jet

The airflow around a jet depends primarily upon the velocity of the jet and roughness of the jet. Dodu [20] found that the velocity distribution in the air around the jet was approximately logarithmic up until the point where the jet breaks up. The velocity distribution should follow a law expressed by

\[ \frac{U_j - u}{u_a} = f \left( \frac{\gamma_s - r}{\delta} \right) \]  

(109)
where
- $r =$ water jet radius
- $U_j =$ water jet velocity
- $u =$ air velocity at a point located $y_a$ distance from the jet centerline
- $y_a =$ distance from water surface
- $\delta =$ boundary layer thickness
- $u_* =$ shear velocity $= (\tau_j/\rho_a)^{1/2}$
- $\rho_a =$ air density
- $\tau_j =$ shear stress at water jet

The functional relation should be very similar to that for flow over a flat plate as studied by Bormann [11] (fig. 56).

Unfortunately, data are not presently available that will allow the computation of the shear velocity and boundary layer thickness in the air surrounding a jet for a given jet geometry and flow rate.

Air Entraining Characteristics as a Falling Jet Enters a Pool

Ervine and Elsawy [21] studied the air entrained by a rectangular jet falling into an open pool. They developed an empirical relation that predicts the relative quantity of air taken into the water by the jet. The relation is

$$\beta = \frac{Q_a}{Q_w} = 0.26 \frac{b_n (H_f)^{0.446}}{p_n (d_n)} \left(1 - \frac{V_m}{V_i}\right)$$  \hspace{1cm} (110)$$

where
- $b_n =$ nappe width
- $d_n =$ nappe thickness
- $H_f =$ fall height of a waterjet
- $p_n =$ nappe perimeter
- $Q_a =$ volume flow of air
- $Q_w =$ volume flow of water
- $V_i =$ nappe velocity at impact
- $V_m =$ minimum velocity required to entrain air $= 1.1 \text{ m/s}$
Model

Prototype

Prototype

FIGURE 54.—Breakup of a water jet from a hollow-jet valve. P801-D-79281
FREE FALLING WATER JETS

Figure 55.—Water drop breakup.
Test configuration with water

Corresponding configuration in air

Figure 56.—Velocity distribution for flow over a flat plate, Bormann [11].


[38] Kleinschroth, A., “Stromungsvorgange im Wirbelfallschacht,” Institut fur Hydraulik und Gewasserkunde, Technische Universitat Munchen, Mitteilungen Heft Nr. 8, Germany, (Flow


Appendix

I - Probability Depth Probe

II - Mean Air Concentration, Free Surface Flow, Computer Program

III - Air Demand, Falling Water Surface, Computer Program
Appendix I - Probability Depth Probe

A water surface probe was developed by Killen [41] at the St. Anthony Falls Hydraulic laboratory (University of Minnesota) which permits a direct determination of the probability that the water surface is greater than or equal to a given elevation. The original probe circuitry has been modernized to function with operational amplifiers (fig. I-1).

Experiments showed that a probe consisting of two parallel wires separated by a short distance exhibited a temperature drift when the probe was removed from the water. By connecting one wire to a metal point gage and the other to an electrode in the body of water, the drift was eliminated (fig. I-2).

The electronics, battery, and controls are conveniently mounted in a utility box (fig. I-3). The following steps are necessary to put the unit in operation:

1. Zero integrating voltmeter with zero control on box (within about 5 millivolts acceptable)
2. Set gain control on box for 10 volts when probe is shorted (in water). Note reading on digital voltmeter.
3. Repeat steps one and two (if necessary repeat twice).
4. Take reading. Voltage read should indicate percentage of the probe shorted (i.e., in water).
AIR-WATER FLOW IN HYDRAULIC STRUCTURES

\( \mu A747 \) FAIRCHILD IC

![Electronics schematic](image)

**Figure 1-1.** Electronics schematic.

![Probe schematic](image)

**Figure 1-2.** Probe schematic.

![Controls in utility box](image)

**Figure 1-3.** Controls in utility box.
Appendix II - Mean Air Concentration, Free Surface Flow, Computer Program

A detailed description of the computer program input is given in the program listing. The program was written to read the input data from a file called HSPWY.

An example of the input format and its output is presented at the end of the computer listing.
AIR-WATER FLOW IN HYDRAULIC STRUCTURES

PROGRAM HFWS

PROGRAM HFWS(HSPWY=/S0,OUTPUT,TAPE2=HSPWY,TAPE3=OUTPUT)
DIMENSION TITL(10)

C THIS PROGRAM COMPUTES THE AMOUNT OF AIR INSUFFLATED INTO OPEN
C CHANNEL FLOW. THE CHANNEL CAN HAVE A TRAPEZOIDAL, CIRCULAR, OR
C A TRANSITIONAL CROSS SECTION. THE PROGRAM IS GOOD FOR RESERVOIR,
C DRAWDOWN, AND BACKWATER CURVES. IF A HYDRAULIC JUMP FORMS IN
C THE CIRCULAR OR TRANSITION SECTIONS, THE AIR CONTENT OF THE
C WATER IS COMPUTED WITH EQUATIONS GIVEN BY KALINSKI AND
C ROBERTSON. A KINETIC ENERGY CORRECTION FACTOR OF 1.1 IS APPLIED
C TO EVERY STATION.

C THE PROGRAM ALSO CHECKS FOR THE FORMATION OF DAMAGING
C CAVITATION. IF A POTENTIAL FOR DAMAGE EXISTS, THE
C PROGRAM INDICATES THE SITE AND THE HEIGHT OF THE
C OFFSET. IT IS ASSUMED THAT THE OFFSETS ARE INTO THE
C FLOW.

C THE REQUIRED INPUT IS;
C 1) DISCHARGE, INITIAL DEPTH, RUGOSITY, DIRECTION OF
C COMPUTATION, METRIC, INITIAL BOTTOM SLOPE (DISTANCE VERTICAL
C TO DISTANCE HORIZONTAL).

C TYPICAL RUGOSITIES
---------------------
CONCRETE 0.3-3.0 MM
STEEL 0.05 MM
TUBING 0.0015 MM

C DIRECTION OF COMPUTATION
--------------------------
UPSTREAM 0
DOWNSTREAM 1

C DIMENSIONAL UNITS
---------------------
METRIC 0
ENGLISH 1

C 2) THE TITLE, CENTERED IN A FIELD OF 60 CHARACTERS

C 3) THE NUMBER OF STATIONS

C 4) THREE DESCRIPTION CARDS ARE REQUIRED FOR EACH STATION.
THESE CONSIST OF THE FOLLOWING;
CARD 1
------
SHAPE OF CROSS SECTION
I=0-RECTANGULAR OR TRAPEZOIDAL
I=1-CIRCULAR
I=2-TRANSITION

SLOPE OF CROSS SECTION AT TRANSITION
IS=1-SECTION VERTICAL
IS=2-SECTION NORMAL TO FLOOR
APPENDIX II

PROGRAM HFWS

CARD 2. IS ONE OF THE FOLLOWING:

A) RECTANGULAR OR TRAPEZOIDAL
STATION, INVERT ELEVATION, BOTTOM WIDTH,
CHANNEL SIDE SLOPE (DISTANCE HORIZONTAL TO UNITY VERTICAL).

B) CIRCULAR SECTION
STATION, INVERT ELEVATION, RADIUS OF CIRCULAR SECTION.

C) TRANSITION SECTION
THE TRANSITION SECTION IS ESSENTIALLY A SQUARE WITH CIRCULAR FILLETS IN THE FOUR CORNERS AND A CENTER SUPPORT WALL. THE DATA TO DESCRIBE THIS SECTION INCLUDES;

THE STATION, INVERT ELEVATION,
WIDTH OF SECTION, RADIUS OF UPPER FILLETS,
(IF NOT CLOSED CONDUIT FLOW, SET EQUAL TO ZERO),
ELEVATION OF CENTERLINE OF UPPER RADIUS POINT,
(IF NOT CLOSED CONDUIT FLOW, SET EQUAL TO ZERO),
RADIUS OF LOWER FILLETS, THICKNESS OF SUPPORT WALL.

CARD 3


THE HEAD LOSS DUE TO TRANSITIONS IS EQUAL TO THE TRANSITION LOSS FACTOR TIMES THE DIFFERENCE IN UPSTREAM AND DOWNSTREAM VELOCITY HEADS.

THE HEAD LOSS DUE TO VERTICAL BENDS IS EQUAL TO THE BEND LOSS FACTOR TIMES ONE HALF THE SUM OF THE UPSTREAM AND DOWNSTREAM KINETIC ENERGIES.

THE BEND ANGLE IS EQUAL TO THE INCLUDED ANGLE OF THE VERTICAL CURVE, IN RADIANS, BETWEEN STATIONS.

IF THE INVERT MOVES AWAY FROM THE FLOW, THE SIGN OF THE VERTICAL RADIUS OF CURVATURE IS NEGATIVE. IF THE SECTION BETWEEN STATIONS IS STRAIGHT, SET THE VERTICAL RADIUS OF CURVATURE EQUAL TO 0.

THE INPUT IS READ WITH A FREE FORMAT. THIS MEANS THAT THE DATA FOR EACH CARD MUST BE ON A SINGLE LINE AND SEPARATED BY ONE OR MORE BLANKS, OR BY A COMMA OR A SLASH, EITHER OF WHICH MAY BE PRECEDED OR FOLLOWED BY ANY NUMBER OF BLANKS. BLANKS ARE NOT ALLOWED AS SUBSTITUTIVE FOR ZERO. A DECIMAL POINT OMITTED FROM A REAL CONSTANT IS ASSUMED TO OCCUR TO THE RIGHT OF THE RIGHTMOST DIGIT OF THE MANTISSA. EXTRANEOUS DATA ON A CARD WILL BE READ ON
AIR-WATER FLOW IN HYDRAULIC STRUCTURES

PROGRAM HFWS

SUBSEQUENT READ COMMANDS. THIS WILL RESULT IN *ERROR DATA INPUT*DIAGNOSTICS.

THE INITIAL DEPTH MUST BE AT LEAST 0.5 PERCENT LARGER OR SMALLER THAN THE CRITICAL DEPTH TO INITIATE THE COMPUTATIONS.

THE AIR CONTENT IS COMPUTED FROM

\[ C = 0.05 \cdot \frac{F - \sqrt{\sin(\alpha)}}{W/(61\cdot F)} \]

WHERE

\[ F = \frac{V}{\sqrt{G \cdot Y(Eff)}} \]
\[ W = \frac{V}{\sqrt{\sigma/\rho \cdot H_R}} \]
\[ H_R = \text{HYDRAULIC RADIUS} \]
\[ \sigma = \text{INTERFACIAL TENSION} \]
\[ \rho = \text{DENSITY OF WATER} \]
\[ G = \text{ACCELERATION OF GRAVITY} \]
\[ Y(Eff) = \text{EFFECTIVE DEPTH} \]

\[ \text{AIR RATIO IS DEFINED AS } \beta = \frac{Q(\text{AIR})}{Q(\text{WATER})} = \frac{C}{1-C} \]

IF A HYDRAULIC JUMP FORMS IN THE CONDUIT,

\[ \beta = 0.0066 \cdot (F - 1.)^{1.4} \]

INITIALIZATION OF DATA

\[ S = -0.1 \]
\[ V = 0.0 \]
\[ H_V = 0. \]
\[ E L I N V = 0. \]
\[ S A T = 0. \]
\[ E T = 0. \]
\[ F = 0. \]
\[ T O T A L = 0.0 \]
\[ B E T A = 0.0 \]
\[ P I = 3.1415926 \]
\[ J I C = 0 \]
\[ K = 0 \]
\[ N L = 1 \]
\[ D I A = 0. \]
\[ D T = 0. \]

INPUT OF FLOW DATA

READ (2,*) Q,DN,RUG,NCURV,MST,SB
IF(MET.EQ.0)RUG=RUG/1000.
IF(MET.EQ.2)2,3
2 CALL EXIT
3 G= 9.807
IF(MET.BQ.1)G= 32.2
VIS= 1.3E-05
IF(MET.BQ.0)VIS= 1.3E-06
DNO= DN
READ(2,4)(TITL(I),I=1,10)
4 FORMAT(10A6)
READ (2,*) NS

LOOP WHICH INCREASES THE STATIONS
APPENDIX II

PROGRAM HFWS

195
CHECK = EOL
DO 68 NT = 1, NS
SAVE = STA
STORE = S
STORE1 = ELINV
RGL = CHECK
HIDEL = HV
IF(NM NE 2) GO TO 14

COMPUTATION OF HYDRAULIC PROPERTIES OF FIRST STATION

DO 68 NT = 1, NS
SAVE = STA
STORE = S
STORE1 = ELINV
EGL = CHECK
HIDEL = HV
IF(NM NE 2) GO TO 14

IF(HE.MEK.EQ.U.) GO TO 5

HYDROSTATIC DEPTH WITH BEND
D = DN/SQRT(1.0 + SB*SB) + 2.0*HV*DN/BENDR
GO TO 6

HYDROSTATIC DEPTH WITHOUT BEND
5 D = DN/SQRT(1.0 + SB*SB)
6 TOTAL = ELINV + D + HV

WRITE STATEMENTS FOR TITLES
WRITE(3,7)(TITL(I), I = 1, 10)
7 FORMAT(1H1, 28X, 10A6/28X, 20H---------------------- ,)

IF(MET.EQ.1) WRITE(3,8) Q, DN, RUG, CN
8 FORMAT(19X, 3HQ =, F7.1, 4H CMS, 3X, 10HINITIAL DEPTH =, F6.2, 3X, 15HDEPT
H VEL
+ 42HOCITY PIEZ GRADE LINE Q AIR/WATER,
+ 27H PROFILE NORMAL CRITICAL /
+ 4X, 2HPT, 9X, 2HPT, 20X, 2HPT, 6X, 6HPT/SEC, 6X, 2HPT, 9X, 2HPT,
+ 32X, 2HPT, 7X, 2HPT/)
IF(MET.EQ.0) RUG = RUG/100.

IF(MET.EQ.1) WRITE(3,9) Q, DN, RUG, CN
9 FORMAT(66X, 6HENRY, 33X, 5HDEPTH/
+ 47H STATION INVERT ELEV SLOPE DEPTH VEL
+ 42HOCITY PIEZ GRADE LINE Q AIR/WATER,
+ 27H PROFILE NORMAL CRITICAL /
+ 4X, 2HPT, 9X, 2HPT, 20X, 2HPT, 6X, 6HPT/SEC, 6X, 2HPT, 9X, 2HPT,
+ 32X, 2HPT, 7X, 2HPT/)
IF(MET.EQ.0) RUG = RUG/100.

IF(MET.EQ.1) WRITE(3,10) Q, DN, RUG, CN
10 FORMAT(19X, 3HQ =, F7.1, 4H CMS, 3X, 10HINITIAL DEPTH =, F6.2, 3X, 15HDEPT
H VEL
+ 42HOCITY PIEZ GRADE LINE Q AIR/WATER,
+ 27H PROFILE NORMAL CRITICAL /
+ 4X, 2HPT, 9X, 2HPT, 20X, 2HPT, 6X, 6HPT/SEC, 6X, 2HPT, 9X, 2HPT,
+ 32X, 2HPT, 7X, 2HPT/)
IF(MET.EQ.0) WRITE(3,11) Q, DN, RUG, CN
11 FORMAT(66X, 6HENRY, 33X, 5HDEPTH/
+ 47H STATION INVERT ELEV SLOPE DEPTH VEL
+ 42HOCITY PIEZ GRADE LINE Q AIR/WATER,
+ 27H PROFILE NORMAL CRITICAL /
+ 4X, 2HPT, 9X, 2HPT, 20X, 2HPT, 6X, 6HPT/SEC, 6X, 2HPT, 9X, 2HPT,
+ 32X, 2HPT, 7X, 2HPT/)
IF(MET.EQ.1) WRITE(3,12) Q, DN, RUG, CN
12 FORMAT(19X, 3HQ =, F7.1, 4H CMS, 3X, 10HINITIAL DEPTH =, F6.2, 3X, 15HDEPT
H VEL
+ 42HOCITY PIEZ GRADE LINE Q AIR/WATER,
+ 27H PROFILE NORMAL CRITICAL /
+ 4X, 2HPT, 9X, 2HPT, 20X, 2HPT, 6X, 6HPT/SEC, 6X, 2HPT, 9X, 2HPT,
+ 32X, 2HPT, 7X, 2HPT/)
IF(MET.EQ.0) WRITE(3,13) Q, DN, RUG, CN
13 FORMAT(19X, 3HQ =, F7.1, 4H CMS, 3X, 10HINITIAL DEPTH =, F6.2, 3X, 15HDEPT
H VEL
+ 42HOCITY PIEZ GRADE LINE Q AIR/WATER,
+ 27H PROFILE NORMAL CRITICAL /
+ 4X, 2HPT, 9X, 2HPT, 20X, 2HPT, 6X, 6HPT/SEC, 6X, 2HPT, 9X, 2HPT,
+ 32X, 2HPT, 7X, 2HPT/)
IF(MET.EQ.1) WRITE(3,14) Q, DN, RUG, CN
14 FORMAT(19X, 3HQ =, F7.1, 4H CMS, 3X, 10HINITIAL DEPTH =, F6.2, 3X, 15HDEPT
H VEL
+ 42HOCITY PIEZ GRADE LINE Q AIR/WATER,
+ 27H PROFILE NORMAL CRITICAL /
+ 4X, 2HPT, 9X, 2HPT, 20X, 2HPT, 6X, 6HPT/SEC, 6X, 2HPT, 9X, 2HPT,
+ 32X, 2HPT, 7X, 2HPT/)
PROGRAM HFWS

IF (I-1) 17,18,19

REMOVING TRAPEZOIDAL CHANNELS
READ (2,*) STA,ELINV,W,SR
READ (2,*) TLF,BLF,BENDR,BENDA
CALL TRAP(DN)
GO TO 22

CIRCULAR CONDUITS
READ (2,*) STA,ELINV,R
READ (2,*) TLF,BLF,BENDR,BENDA
DIA= 1.99999'R
CALL CIRC(DIA)
GO TO 22

CIRCULAR TO RECTANGULAR TRANSITIONS
READ (2,*) STA,ELINV,W,R,ELC,Rr,T
READ (2,*) TLF,BLF,BENDR,BENDA
IF(ELC.LT.ELINV)GO TO 20
SB= (ELINV-STORR~)/ABS(SAVE-STA)
IF(NCURV.EQ.1)SB= -SB
WMIN= (W-T)/2.
IF(R.GE.WMIN.AND.IS.EQ.1)DT= 0.99999*(ELC-ELINV+SQRT( R*(W-T)-0.25*(W-T)*(W-T)))/2.
IF(R.GE.WMIN.AND.IS.EQ.2)DT= 0.99999*((ELC-ELINV)*SQRT(SB*SB+1.)+SQRT(R*(W-T)-0.25*(W-T)*(W-T)))
IF(R.LT.WMIN.AND.IS.EQ.1)DT= 0.99999*((ELC-ELINV)*R)
IF(R.LT.WMIN.AND.IS.EQ.2)DT= 0.99999*((ELC-ELINV)*SQRT(SB*SB+1.))+R
GO TO 21

DT= DN
CALL TRANSCDT)

USING EQ 4-13 FROM OPEN CHANNEL FLOW BY HENDERSON,
MACMILLAN, 1970. THIS IS THE COLEBROOK EQUATION.

REY= 4.0*HR*(Q/A)/VIS
FRICT= 0.
DO 23 N=1,20
FRICT= -2.*ALOG10(RUG/(12.*HR)+2.5*FRICT/REY)
IF(ABS(1.-FRICT/FRICT).LE.0.01)GO TO 24
FRICT= FRICT
CONTINUE

IF(MET.EQ.0)CN=(HR**0.166667/8.*G)*FRICT
IF(MET.EQ.1)CN=(HR**0.166667)*1.49/((SQRT(8.*G)*FRICT)

COMPUTATION OF BOTTOM SLOPE

IF(NT.EQ.1)GO TO 25
SB= (ELINV-STOREL)/ABS(SAVE-STA)
IF(NCURV.EQ.1)SB= -SB

CHECK FOR MAXIMUM DISCHARGE IN CLOSED CONDUITS

IF(I.EQ.0)GO TO 26
QMAX= A*SQRT(ABE(SB))*HR**0.66667/CN
IF(MET.EQ.1)QMAX= 1.49*QMAX

COMPUTATION OF CRITICAL DEPTH BY NEWTONS METHOD

DYN= DN/2.
YC= DN
DO 31 NN=1,25
IF(I-1)27,28,29
CALL TRAP(YC)

310
315
APPENDIX II

PROGRAM HFWS

325   DDBY= 2.*SS
GO TO 30
28    CALL CIRC(YC)
DDBY= (DIA-2.*YC)/SQRT(YC*DIA-YC*YC)
GO TO 30
316   CALL TRANS(YC)
DDBY= 0.
30    VC= Q/A
HVC= VC*YC/(2.*VC)
HVCK= A/(2.*B)
DDBY= DDBY*HVCK/B-0.5*(1.+2.*B*VC/A)
YYC= YC
335   YC= YC-DHVC/DFDY
CONTINUE
340   IF(YC.LE.0.)YC=ABS(YC)
341   IF(I.EQ.1.AND.YC.GT.DIA)YC= (YCO+DIA)/Z.
342   IF(I.EQ.Z.AND.YC.GT.DT)YC= (YCO+DT)/Z.
345   COMputation OF NORMAL DEPTH BY NEWTONS Method

41   IF(YN.LT.YC)GO TO 42
355   IF(I.EQ.O.AND.SB.GE.0.)GO TO 33
350   IF(SB.GT.0. AND QMAX.GT.Q)GO TO 33
330   YN= 1000.
350   IF(SB.LT.0.)YN= -1000.
GO TO 41
330   DY= -DN/Z.
YN= ABS(DN)
DO 40 NN=1,25
340   IF(I-1)34,35,36
345   CALL TRAP(YN)
GO TO 37
350   CALL CIRC(YN)
GO TO 37
360   CALL TRANS(YN)
345   USING EQ 4-13 FROM OPEN CHANNEL FLOW BY HENDERSON,
MACMILLAN, 1910. THIS IS THE COLEBROOK EQUATION.

365   REY= 4.*HR*(Q/A)/VIS
FRICO= 0.
DO 38 N=1,20
FRIC= -2.*ALOG10(RUG/(12.*HR)+2.5*FRICO/REY)
IF(ABS(1.-FRICO/FRIC)<0.01)GO TO 39
370   CONTINUE
380   IF(MET.EQ.0)CN=-(HR**0.146667)/(SQRT(8.*G)*FRICO)
375   IF(MET.EQ.1)CN=-(HR**0.146667)*1.49/(SQRT(8.*G)*FRICO)
QM= A*SQRT(8.)*HR**0.66667/CN
DHDY= HR/YN-2.*HR/(3+2.*YN)
FUN= Q-QM
IF(ABS(FUN/Q).LE.0.01)GO TO 41
DFDY= -HR/YN/2.*QM*DHDY/(3.*HR)
380   CONTINUE
390   YNO= YN
YN= YN-FUN/DFDY
IF(YN.LE.0.)YN=ABS(DY)
IF(I.EQ.1.AND.YN.GT.DIA)YN= (DIA+YNO)/Z.
385   IF(I.EQ.Z.AND.YN.GT.DT)YN= (DT+YNO)/Z.
CONTINUE

40   DETERMINATION OF PROFILE TYPE

41   IF(YN.GT.YC)GO TO 42
PROGRAM HFWS

IF(YN.EQ.YC)GO TO 43
IF(SB.BQ.O.)GO TO 44
IF(SB.LT.O.)GO TO 45

MILD SLOPE
AN= 1HM
IF(DN.GE.YN)M=1
IF(DN.LT.YN.AND.DN.GT.YC)M=2
IF(DN.LE.YC)M=3
GO TO 46

STEEP SLOPE
AN= 1HC
IF(DN.GE.YC)M=1
IF(DN.GT.YN.AND.DN.LT.YC)M=2
IF(DN.LE.YN)M=3
GO TO 46

CRITICAL SLOPE
AN= 1HC
IF(DN.GE.YN)M=1
IF(DN.LT.YN)M=3
GO TO 46

HORIZONTAL
AN= 1HH
IF(DN.GE.YC)M=1
IF(DN.LT.YC)M=3
GO TO 46

ADVERSE
AN= 1HA
IF(YN.GE.YC)M=1
IF(YN.LT.YC)M=3
GO TO 46

430 COMPUTATIONAL LOOP TO DETERMINE WATER DEPTH
DO 60 J=1,100

435 COMPUTATION OF HYDRAULIC PROPERTIES
IF (I-1) 47,48,49

440 HYDRAULIC PROPERTIES FOR RECTANGULAR SECTION
CALL TRAP(DN)
FRUD= F

445 NOTE THE FROUDE NUMBER IS NOT CORRECTED FOR SLOPE
AND ENERGY CORRECTION FACTOR
F=(Q/A)/((SQRT(G*A/B))
IF(WT.EQ.1)GO TO 50
CKFRUD= (FRUD-1.)/(F-1.)
GO TO 50

450 HYDRAULIC PROPERTIES FOR CIRCULAR SECTION
CALL CIRC(DN)

455
APPENDIX II

PROGRAM HFWS

464 IF(JIC.EQ.1)GO TO 69
FRUD= F
F=(Q/A)/(SQR(G*A/B))
IF(NT.EQ.1)GO TO 50
CKFRUD= (FRUD-1.)/(F-1.)
GO TO 50

C

C

C

HYDRAULIC PROPERTIES IN TRANSITION

465 CALL TRANS(DN)
IF(JIC.EQ.1)GO TO 69
FRUD= F
F=(Q/A)/(SQR(G*A/B))
IF(NT.EQ.1)GO TO 50
CKFRUD= (FRUD-1.)/(F-1.)

C

C

C

COMPUTATION OF DEPTH BY STANDARD STEP METHOD

470 V=Q/A
HV= 1.1*V*V/(Z.*G)

C

C

C

COMPUTATION OF MANNINGS N VALUE FROM THE RUGOSITY

475 USING EQ 4-13 FROM OPEN CHANNEL FLOW BY HENDERSON,
MACMILLAN, 1970. THIS IS THE COLEBROOK EQUATION.

C

C

C

REY= 4.*HR*(Q/A)/VIS
PRICO= 0.
DO 51 N=1,20
FRICT= -2.*ALOG10(RUG/(12.*HR)) - 2.5*PRICO/REY
IF(ABS(1. - FRICO/FRICT).LE.0.01)GO TO 52
CONTINUE

C

C

C

1.1 IS THE KINETIC ENERGY CORRECTION FACTOR

480 HT= TLF*ABS(HV-HIDEl)
HB=BLF*(HV-HIDEl)/2.0
S=(CN*CN*V*V)/(HR**1.3333)
D= UN/SUNY(1.0+SB*SB)
IF(MET.EQ.1)S=S/2.208

C

C

C

NT= 1 IS THE CONDITION FOR THE FIRST STATION

490 IF(NT.EQ.1)DYDX= (SB-S)/(1. - F*F)
IF(NT.EQ.1)CHECK= ELINV+HV+DN/SQRT(1.+SB*SB)
IF(MET.EQ.1)GO TO 63
AVGS=(STORE+S)/Z.O
IF(BENDR.NE.0.)GO TO 53

C

C

C

COMPUTATION OF HEAD LOSS WITHOUT BENDS

500 RUN= SQRT((STA-SAVE)*(STA-SAVE)+(STOREL-ELINV)*(STOREL-ELINV))

C

C

C

HF=RUN*AVGS
SUM=HF+HT+HB
GO TO 54

C

C

C

COMPUTATION OF HEAD LOSS WITH BENDS

510 RUN= BENDA*ABS(BENDR)
HF= RUN*AVGS
SUM= HF+HT+HB
D= DN / SQRT(1.0+SB*SB)+2.0*HV*DN / BENDR
**PROGRAM HFWS**

520 54 COMputation of energy grade line

`TOTAL=BLINV+D+HV`
```
CHECK = EGL+SUM
```
```
IF(NCURV.EQ.1)CHECK = EGL-SUM
```

530 DETERMINATION OF FLOW DEPTH USING NEWTONS METHOD

```
EGLCB = TOTAL-CHECK
DSDY = 1./SQRT(B+SB*SB)-2.*HM*B/A
DSDY = 2.*HF*B*HT)*B/A+4.*HF*B*B/
(3.*HR*(B+2.*DN)*(B+2.*DN))
```
```
IF(NCURV.EQ.1)DCDY = -DCDY
```
```
DSDY = DSDY+DCDY
```
```
DN = ABS(DN-YN)*DINC/(ABS(DINC)*2.)
```
```
DN = DN-DINC
```
```
IF(DN.LE.O.)DN = (DN+DINC)/2.
```

550 CHECK ON THE ACCURACY OF COMPUTATIONS

```
PCTEL = lOO.**(TOTAL-CHECK)/(ABS(TOTAL)-D-HV))
```
```
IF(CFCFRU.LT.O.O)GO TO 73
```
```
IF(ABS(PCTEL).LT.O.O1)GO TO 59
```
```
GO TO 60
```

565 COMPUTATION OF AIR CONTENT WITH OPEN CHANNEL FLOW

```
IF(SB.LE.O.)C = 0.
```
```
IF(SB.LE.O.)GO TO 61
```
```
ALPHA = ATAN(SB)
```
```
W = 100.**(V/SQRT(0.74/HR))
```
```
IF(MET.EQ.1)W = V/SQRT(0.00257/HR)
```
```
C = 0.05*F-SQRT(SIN(ALPHA)*W/(63.*F))
```
```
IF(C.LT.O.)C = 0.
```
```
IF(C.GE.0.74)C = 0.74
```
```
BETA = C/(1.-C)
```
```
GO TO 61
```
```
CONTINUE
```
```
IF(J.GE.100)GO TO 73
```

580 WRITE (3,12) STA, BLINV, SB, DN, V, D, CHECK, BETA, AN, M, YN, YC

590 NL = NL+1

595 CHECK ON SPACING OF STATIONS

```
(C THIS KEEPS ERROR IN DEPTH TO LESS THAN 1-PERCENT)
```

600 CONTINUE

610 IF(J.GE.100)GO TO 73

580 WRITE (3,12) STA, BLINV, SB, DN, V, D, CHECK, BETA, AN, M, YN, YC

590 NL = NL+1

600 CHECK ON SPACING OF STATIONS

```
(C THIS KEEPS ERROR IN DEPTH TO LESS THAN 1-PERCENT)
PROGRAM HFWS

585  \[ \frac{\Delta y}{\Delta x} = \frac{(S - B)}{(L - F')/F} \]

IF(\[\frac{\Delta y}{\Delta x}\] = 1.E-10) GO TO 63

590  \[ E_{xy} = 100 \times 0.5 \times \frac{(RUN/DNO)}{(RUN/DNO)^2} \frac{ABS}{(1. - FRUDO \times FRUDO) \times (1. - FRUDO \times FRUDO)} \]

IF(E_{xy} < 1.0) GO TO 63

595  \[ DX = RUN/2 \]

WRITE(3, 62) ERRY, DX

62  \[ \text{ERROR IN DEPTH EXCEEDS } 5.0 \text{ PERCENT, PLEASE ADD INTERMEDIATE STATIONS WITH } DX = 6.0 \]

63  \[ DNO = DN \]

600  \[ FRUDO = F \]

IF(MET.EQ.0) DNO = 10.33 \times (1. - ELINV/4300) ** 5.255 - 0.13

605  \[ SIG = 2.5 \times \frac{(PABS - D)}{(V * V)} \]

IF(MET.EQ.1) SIG = SIGR/2.206

610  \[ CAVITATION OF BOUNDARY ROUGHNESS \]

IF(SIG1.GT.SIGR.AND.SIG1.GT.SIG) GO TO 68

WRITE(3, 64) HO, NSL

64  \[ \text{CAVITATION WILL OCCUR FOR OFFSETS GREATER THAN } 1.0 \frac{F}{H} \text{CHAMFERS REQUIRED} / \]

65  \[ HO = 10800. \times (SIG1**2.91) \times (V*V/DN)**0.196 \]

66  \[ SIG = 32.4 \times \frac{CN \times CN}{HR} \times 0.3333 \]

IF(MET.EQ.1) SIG = SIGR/2.206

67  \[ CAVITATION OF BOUNDARY ROUGHNESS \]

C COMPUTATION OF REQUIRED CHAMFER

\[ \alpha = 18.762 \times \frac{SIG}{HO} \]

68  \[ \text{CONTINUE} \]

C NORMAL TERMINATION OF PROGRAM

GO TO 1

C ABNORMAL TERMINATION OF PROGRAM

C COMputation of Air CONTENT with a Hydraulic JUMP

69  \[ IF(\text{F.GT.0.}) \] \text{beta} = 0.0065 \times (F-1.)**1.4
SUBROUTINE TRAP

SUBROUTINE CIRC

SUBROUTINE TRAP(DN)

SUBROUTINE CIRC(DN)

HYDRAULIC PROPERTIES OF TRAPEZOIDAL SECTIONS

HYDRAULIC PROPERTIES OF CIRCULAR SECTIONS

COMMON A,B,HR,W,SS,R,IS,PI,ELINV,SB,ELC,R1,T,G,JIC

A=W*DN+SS*DN**DN
B=W+2.*SS*DN
HE=A/(W+2.*SQRT(DN**DN+((SS*DN)**SS*DN)))
RETURN

COMMON A,B,HR,W,SS,R,IS,PI,ELINV,SB,ELC,R1,T,G,JIC
IF (DN-R) 1,2,3

LESS THAN HALF FULL
SUBROUTINE CIRC

10 1 ROOT = SQRT(R*R - (R-DN)*(R-DN))
TERM = ROOT/(R-DN)
ANGLE = ATAN(TERM)
A = ((R*R)*ANGLE) - ((R-DN)*ROOT)
HR = A/(R*2.0*ANGLE)
15 B = 2.*ROOT
RETURN

C EXACTLY HALF FULL
C

20 2 A = (PI*R*R)/2.0
B = 2.*R
HR = R/2.0
RETURN

C GREATER THAN HALF FULL
C

25 3 IF(DN.GT.2.*R)GO TO 5
IF(DN.EQ.2.*R)GO TO 4
ROOT = SQRT(R*R - (DN-R)*(DN-R))
TERM = ROOT/(DN-R)
ANGLE = ATAN(TERM)
A = ((R*R)*ANGLE) - ((DN-R)*ROOT)
B = 2.*ROOT
HR = A/((2.0*PI*R) - (R*2.0*ANGLE))
35 RETURN

C EXACTLY FULL

40 4 A = PI*R*R
B = 0.
HR = R/2.
RETURN

C FLOW FILLS CONDUIT

45 5 JIC = 1
RETURN
END

SUBROUTINE TRANS

SUBROUTINE TRANS(DN)

HYDRAULIC PROPERTIES OF CIRCULAR TO RECTANGULAR
SECTIONS WITH AND WITHOUT DIVIDER WALLS

5 COMMON A, B, HR, W, SS, R, IS, PI, ELINV, SB, ELC, R1, T, G, JIC
SEC = (1. - PI/4.)
IF(ELC.GE.ELINV)GO TO 3
DELV = (ELC-ELINV)
IF(IS.EQ.1)GO TO 1
10 SEC = (ELC-ELINV)*SQRT(SS*SB-1.)
DEDV = (ELC-ELINV)*SQRT(SS*SB-1.)
15 DIFF1 = DN-DELV
GO TO 2

SECTION NORMAL TO FLOOR
C

DELV = (ELC-ELINV)*SQRT(SS*SB-1.)
DIFF1 = DN-DELV
GO TO 2

SECTION VERTICAL
C
SUBROUTINE TRANS

1 DN = DN*SQRT(SB*SB+1.0)
DIFF1 = DN - DELV
2 IF (DIFF1.GT.0.) GO TO 7

20

C C
C EQUAL TO OR LESS THAN THE DEPTH OF THE RECTANGULAR SECTION
C

25

3 IF (DN.LT.R1) GO TO 5
A = DN*(W-T)-2.*SEC*R1*R1
B = W-T
IF (T.GT.0.) GO TO 4
HE = A/(W+2.*DN-4.*SEC*R1)
RETURN

C C
C WITH DIVIDER WALL
C

35

4 HE = A/(W-T-4.*DN-4.*SEC*R1)
RETURN

C C
C EQUAL TO OR LESS THAN DEPTH OF RADIUS IN THE BOTTOM CORNER OF C C
C TRANSITION
C

45

5 ROOT = SQRT(2.*DN*R1-DN*DN)
TERM = ROOT/(R1-DN)
ANGLE = ATAN (TERM)
A = R1*R1*ANGLE-ROOT*(R1-DN)+(W-2.*R1-T)*DN
B = W-T-2.*R1+2.*SQRT(R1*R1-(R1-DN)*(R1-DN))
IF (T.GT.0.) GO TO 6
HR = A/(W-2.0*R1+2.0*R1*ANGLE)
RETURN

C C
C WITH DIVIDER WALL
C

60

6 HR = A/(W-2.0*R1+2.0*R1*ANGLE-T+2.0*DN)
RETURN

C C
C GREATER THAN THE DEPTH OF THE RECTANGULAR SECTION
C

55

7 IF (R-DIFF1) 12, 9, 8
8 ROOT = SQRT(R*R-DIFF1*DIFF1)
ARG2 = DIFF1/ROOT
PH1 = ATAN (ARG2)
TW = W-2.0*R
IF (TW.GE.T) GO TO 10

C C
C CHECK WITH UPPER RADIUS GREATER THAN HALF WIDTH OF SECTION
C OR WITH RADIUS POINT INSIDE CENTER PIER
C

65

9 PH2 = PI/2.
ROOT = 0.

C C
C COMPUTATION OF AREA AND TOP WIDTH OF WATER SURFACE
C

75

10 A = DN*(W-T)-DIFF1*(W-TW)-2.*SEC*R1*R1+R*R*PH2+
+ROOT*DIFF1
B = TW-2.*ROOT
IF (T.EQ.0.) GO TO 11
HR = A/(W-4.*SEC*R1-T+2.*(DN-DIFF1)+2.*DN +
+2.*R*PH1)
RETURN

80

11 HR = A/(W-4.*SEC*R1-T+2.0*(DN-DIFF1)+2.0*R*PH1)
APPENDIX II

SUBROUTINE TRANS

RETURNS

FLOW FILLS CONDUIT

12 JIC = 1

A = DN*(W-T)-DEPTH*(W-TW)-2.*SEC*R1*R1+R*R*PHI1+ROOT*DEPTH
B=0.

IF(T.EQ.0.)GO TO 13

RETURN

13 HR = A/(W-4.*SEC*R1-T+W*(DN-DEPTH)+2.*DN+2.*R*PHI1)

RETURN

END

DATAFILE-HS

20., 0.563, 0.01, 1, 0, 0, 33333

Q = 20,000 CMS INITIAL DEPTH = .563 M SURFACE = 1,7000 MM W = .0142

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<th>SLOPE</th>
<th>DEPTH M</th>
<th>VELOCITY M/SEC</th>
<th>PHN M</th>
<th>ENERGY GRADE LIMIT</th>
<th>Q AIR/W WATER PROFILE</th>
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DATA FILE - HS WPY

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0, 100., 15., 0.
0, 0, 0, 0.
0, 1
0.720, 99.76, 15., 0.
0, 0, 0, 0.
0, 1
0.765, 99.594, 15., 0.
0, 0, 0, 0.
0, 1
2.610, 99.127, 15., 0.
0, 0, 0, 0.
0, 1
5.332, 99.289, 15., 0.
0, 0, 0, 0.

0, 0, 0, 0.
0, 0, 0, 0.
0, 0, 0, 0.
0, 0, 0, 0.
0, 0, 0, 0.
0, 0, 0, 0.
0, 0, 0, 0.
0, 0, 0, 0.
0, 0, 0, 0.
0, 0, 0, 0.
INTRODUCTION

The original version of the falling water surface computer program contained simplifications that could have led to significant errors in the flow simulation. The present program was modified to represent more realistically the flow conditions at the gate and to include a better representation of the hydraulic turbine characteristics. The complicated prototype geometry is input into the program through the use of a few parameters which approximate the actual geometry.

JUNCTION ENERGY EQUATIONS

The original study assumed that the relation between the flow from the reservoir and gate chamber could be described by the junction flow equations for a pipe tee. A more realistic flow description—considering the varying area created by the closing gate—would be the formation of a submerged hydraulic jump downstream from the gate (fig. III-1).

Writing the momentum equation in the horizontal direction on the prism of water between points $V$ and $P$ gives:

$$

\frac{P_V}{\gamma} = \frac{2A_G}{(A_P + A_G)} \left[ \left( \frac{P_P}{\gamma} \right)^2 + \frac{A_G + A_P}{2A_G} \right] - \frac{2A_P V_P^2}{A_G} - \frac{2}{C_c G_P A_G^2} \frac{A_P^2 V_P^2}{2g}

$$

where

$$

V_V = \frac{Q_R}{G_P A_G C_c} \quad \text{and} \quad V_P = \frac{Q_P}{A_P}, \text{figure III-1}

$$

The contraction coefficient $C_c$ is related to the discharge coefficient $C_d$ through [59]

$$

C_c = \frac{(C_d/G_P)^2}{2} \left( \frac{G}{H_c} \right)

+ \frac{(C_d/G_P)^4}{2} \left( \frac{G}{H_c} \right)^2 + 4(C_d/G_P)^2

$$
\[ p_{GC} = \frac{p_V}{\gamma} \cdot \Delta P \]  
\[ V_{GC} = \frac{Q_R}{(C_c G_o W)} = \frac{Q_R}{(C_c G_p A_G)} \]

When all water has drained from the gate chamber, but the gate is still submerged, the pressure at point \( P \) is given by
APPENDIX III

TURBINE CHARACTERISTICS

The head loss coefficient across the turbine was arbitrarily assumed constant in the original study.\(^1\) The validity of the assumption is investigated in the following paragraphs.

The energy loss across the turbine is given by

\[
h_d = K_t \frac{V_P^2}{2g}
\]

(6)

The loss coefficient \(K_t\) can be written

\[
K_t = \frac{2g h_d}{V_P^2}
\]

(7a)

or

\[
K_t = \frac{\pi^2 g}{8} \left( \frac{D_P}{D_3} \right) \left( \frac{D_3^{3/2} h_d}{Q} \right)^2
\]

(7b)

where \(D_3\) is a characteristic dimension of the turbine runner.

The quantity \(\frac{Q}{D_3^{5/2} h_d}\) is a basic parameter used in describing the turbine characteristics. The quantity is known as the specific discharge and usually is written \(Q_{11}\). The subscripts signify the discharge from a one-meter-diameter runner having a one-meter head across the turbine runner. Thus, the loss coefficient also can be written:

\[
K_t = \frac{\pi^2 g}{8} \left( \frac{D_P}{D_3} \right)^4 \left( \frac{1}{Q_{11}} \right)^2
\]

(8)

The discharge through a turbine is dependent upon the head across the unit, runner speed, and wicket gate opening. The runner speed and head across the unit are described by a dimensionless parameter known as \(\phi\) (phi) defined by

\[
\phi = \frac{\pi D_3 n}{60 \sqrt{2gh_d}}
\]

(9)

where \(n\) is the rotational speed in revolutions per minute.

Typical turbine characteristic curves for a runner having a specific speed\(^3\) of about 230 show that the specific discharge is not significantly affected by changes in the rotational speed (fig. III-2). The maximum change in specific discharge between the maximum efficiency and the runaway condition at a constant gate opening amounts to about 3 percent. The corresponding change in the loss coefficient is about 6 percent.

Loss coefficient curves can be prepared for any turbine runner. These curves will be a function of the wicket gates opening and the phi value (fig. III-3).

If the generator unit remains connected to the electrical system during a closure of the emergency gate, the turbine will operate at a constant speed. The condition with the turbine at synchronous speed but with no flow is called motoring.\(^4\) During motoring the turbine will develop a head in the penstock. The magnitude of this head must be input into the computer program. For reaction turbines the magnitude

\[^1\]Ibid.

\[^2\]The specific speed of a turbine is given by

\[
n_s = \frac{n (P_d)^{1/3}}{(h_d)^{1/2}}
\]

where

- \(h_d\) = turbine head, m
- \(n\) = rotational speed, r/min
- \(P_d\) = turbine power output, kW

\[^4\]Power requirements for motoring can be minimized by depressing the water surface in the draft tube. This is used for either power factor adjustment or for spinning reserve. During an emergency, depression of the water surface in the draft tube cannot be assumed to occur.
$Q = \text{discharge, m}^3/\text{s}$

$n = \text{rotational speed, r/min}$

$g = \text{gravitational constant, m/s}^2$

$h_d = \text{head across turbine, m}$

$D_3 = \text{discharge diameter of runner, m}$

**Figure III.2.** Typical turbine characteristics of runner specific speed 230 in m-kW units.
$D_r = \text{discharge diameter of runner, m}$
$g = \text{gravitational constant (acceleration), m/s}^2$
$h_d = \text{head across turbine, m}$
$n = \text{rotational speed, r/min}$
$n_s = \text{specific speed} = \frac{n(P_d)^{1/2}}{(h_d)^{5/4}} = 230$
$P_d = \text{turbine power output, kW}$
$V_p = \text{penstock water velocity, m/s}$
$\phi = \text{speed coefficient} = \frac{nnD_r}{60\sqrt{2gh_d}}$

**Figure III-3.—Turbine loss coefficient.**
of the motoring head is about 20-to 25-percent of the turbine net head.

**GEOMETRY**

The two most significant geometric effects which must be calculated for free surface flow in the penstock are the air volume in the penstock and the water surface area as a function of water surface elevation.

The actual variation of the air volume in the penstock is a complicated function due to the circular section of the penstock and vertical bends. The air volume must be computed as a function of elevation for enough points to define the prototype curve. Two straight lines are fitted through the prototype values (fig. III-4). The coefficients of these two straight lines are input into the program.

A similar procedure is used for the water surface area as a function of elevation. However, the prototype curve is fitted with a sine wave and a vertical line instead of two straight lines (fig. III-5).
PROGRAM HFVENT

C

COMPUTATION OF AIR FLOW INTO GATE CHAMBER DURING AN EMERGENCY
C CLOSURE OF GATES FOR THE PENSTOCK INTAKE STRUCTURE

REAL MACHA,MACHGA,NVENTS
COMMON CD,CKV,ABSPGC,PM,PGC,WSREF,HCOL,WS,OR,DP,GRAY,PTOHD,
+UP,LT,UT,PL,PR,TLOGS,AR,AD,AREA,AG,EK,GC,PG0,DFIRST,WFLA(50)
+MACHA(50),MACHGA(50),AVOL,PIN,CVEL,T(50),JCK,GCPL,RES,2P,DP,TH,
+UGCI,GC2,HYDIA,VL0C1,ELCI,VL0C2,ELCE,CF,P3,RE,AC,HCLAB,
MACHA~5O~,MACHGA~5O!,AVOL,PIN,CVEL,DELX,DELY

C

INPUT DATA

2 FORMAT(1H1,/) READ (3,1)

C

COMPUTATION CONTROLS

C

SYSTEM OF UNITS
C C = 1 METRIC, SET METRIC = 1
C C = 0 ENGLISH, SET METRIC = 0
METRIC = 1

C

INITIAL TIME
T(1) = 0.

C

TIME INTERVAL BETWEEN CALCULATIONS (SEC)
DELT = 1.

C

MAXIMUM TIME WHICH IS TO BE CALCULATED (SEC)
TMAX = 40.

C

THE TITLE AND AIR VENT DESCRIPTION ARE INPUT IN THE
C FORMAT STATEMENTS. THEY CAN CONTAIN UP TO 60 CHARACTERS
C EACH. BOTH DESCRIPTIONS MUST BE CENTERED IN THE FIELD.
2 FORMAT(GX,GOH MORROW POINT DAM
C +1
C 3 FORMAT(GX,GOH ONE 840MM BY 915MM AIR VENT
C +1

C

WATER FLOW QUANTITIES

C

TURBINE LOSS COEFFICIENT
TLOSS = 111.2

C

TYPE OF TURBINE OPERATION DURING CLOSURE
C C IF UNIT RUNS AT CONSTANT SPEED, SET NSPEED = 1.
C C IF THE CIRCUIT BREAKERS ARE OPEN SO THAT
C C THE SPEED CAN VARY, SET NSPEED = 0. BOTH
C C OF THESE TYPES OF OPERATION ASSUME A BLOCKED
GATE OPERATION.)
NSPEED = 1

C

HEAD DEVELOPED BY TURBINE DURING MOTORING
C (SPEED NO LOAD)
HMOTOR = 27.5
AIR-WATER FLOW IN HYDRAULIC STRUCTURES

PROGRAM HFVENT

C GATE CLOSING RATE (PERCENT/SEC)
VGR= 0.3991

C DARCY-WIESBACH FRICTION FACTOR IN PENSTOCK
FP= 0.009

C SINGULAR LOSS BETWEEN UPPER AND LOWER GATE CHAMBER
EK= 0.3

C AIR FLOW QUANTITIES
-------------------

C TEMPERATURE
TEMP= 4.5

C ATMOSPHERIC PRESSURE (PSI OR KPA)
PATM= 77.6

C INCOMPRESSIBLE FRICTION FACTOR FOR THE AIR FLOW IN
C THE DUCT (FL/D)
FRIC= 0.93

C COMPRESSIBLE DISCHARGE COEFFICIENT OF AIR
C AT DUCT INLET
CINC= 0.5

C GEOMETRY
--------

C RESERVOIR WATER LEVEL
RES= 2181.69

C TAIL WATER LEVEL
TW= 2059.23

C ELEVATION UPPER GATE CHAMBER TO LOWER GATE CHAMBER
UGCLOC= 2168.04

C ELEVATION INVERT AT GATE
ZP= 2155.93

C ELEVATION OF TOP OF GATE CHAMBER
TGC= 2183.89

C AREA UPPER GATE CHAMBER
AU= 13.80

C AREA LOWER GATE CHAMBER
AD= 13.33

C AREA GATE
AG= 20.64

C CROSS-SECTIONAL AREA OF EACH AIR VENT
AVENT= 0.766

C NUMBER OF VENTS
NVENTS= 1

C CONSTANTS WHICH DESCRIBE THE VOLUME OF THE
C PENSTOCK AS A FUNCTION OF THE WATER SURFACE
C SEE THE DOCUMENTATION FOR THE DEFINITION OF
C THESE CONSTANTS
VOLC1= 46.16
VOLC2= 13.3
APPENDIX III

PROGRAM HFVENT

C PENSTOCK LENGTH
PL = 143.56
C PENSTOCK LENGTH AS A FUNCTION OF WATER SURFACE PARAMETER
PLEN = 2016.48
C PENSTOCK DIAMETER
DP = 4.12
C HEIGHT OF GATE
SO = 5.01
C HYDRAULIC DIAMETER OF LOWER GATE CHAMBER
HYDDIA = 1.41
C LUNISIANIS WHICH DESCRIBE THE FREE WATER SURFACE AREA
C IN THE PENSTOCK. SEE THE DOCUMENTATION FOR THE
C DEFINITION OF THESE CONSTANTS
CF = 48.83
PER = 8.23
CAREA = 13.5
C END OF INPUT DATA
C
C COMPUTED CONSTANTS
C
P1 = 3.14159
K = INT((TMAX-T(111))/(40.*DELT)+1)
grav = 32.2
GASCTE = 53.3
PTOHD = 2.30769
VAPOR = PTOHD*0.0256*10.**(0.0162*TEMP)
AGCCTE = 144.
ACURAC = 0.01
PGCINC = 0.01
ABSTEM = TEMP+59.67
RHOA = AGCCTE*PMT/(GASCTE*ABSTEM)
CD = 0.9303
IF(METRIC.EQ.0 GO TO 4
GASCTE = 287.
PTOHD = 0.000102
VAPOR = PTOHD*0.582*10.**(0.0292*TEMP+3.1
AGCCTE = 1000.
PMT = PATM*AGCCTE
ACURAC = 0.003
PGCINC = 50.
grav = 9.807
ABSTEM = TEMP+273.15
RHOA = PATM/(GASCTE*ABSTEM)
4 ADBPGC = PATM
AREAP = PI*DP*DP/4.
HEAD = RES-TW
POD = 1.
RH = SQRT((AREAP/PI)*DP/DP/2.
CC = (CD+CD*RH+SQRT(CD**4*RH**2+4.*CD**2))/2.
CONST = TLOSS-1.*FP*PL/DP*(AREAP/(AG*CD))**2-4.*AREAP**2/(AG+AREAP)+(AG+AREAP)*CC*POD*AG)/4.*AREAP/(AG+AREAP)
QR = AREAP*SQRT(2.*GRAV*HEAD/CONST)
VHP = (QR/AREAP)**2./2.*GRAV
VHR = VHP
PR = RES-2P-VHR*(AREAP/AG/CD)**2
AIR-WATER FLOW IN HYDRAULIC STRUCTURES

PROGRAM HFVENT

\[
WS = ZP + PR \\
PP = PR + 2 \cdot AG/(AG + AREAP) \cdot (2 \cdot CC \cdot VHR \cdot (AREAP/AG) \cdot 2 - 2 \cdot VHP \cdot AREAP/AG)
\]

WSREF = WS
HCOL = WSREF - UGCLGC
GCLP = ZP + DP
GCLL = UGCLGC - GCLP
QF = QR
QGC = 0.

AVOLRE = (TGC - WSI) * AU
WTAIR = RHOA * AVOLRE
AVOL = AVOLRE

PIN = PATM
PNC = PTNHD + PATM

CONST = PATM / RHOA**1.4
CKA = 0.

JFIRST = 1
DELTIM = 0.
JCK = 1
NT = 1

IF(NSPEED.EQ.0) HMOTOR = 0.

SLOSS = 1 - HMOTOR / (TLOSS * VHP)

PVAPOR = VAPOR - POC

**

SIGNIFICANCE OF JFIRST

1 - FIRST TIME THRU MAIN LOOP
2 - SECOND TIME THRU MAIN LOOP,
3 - WATER SURFACE IN UPPER GATE CHAMBER
4 - LAST TIME INCREMENT IN UPPER GATE CHAMBER
5 - FIRST TIME INCREMENT IN LOWER GATE CHAMBER
6 - LAST TIME INCREMENT IN LOWER GATE CHAMBER
7 - FIRST TIME INCREMENT IN PENSTOCK
8 - WATER SURFACE IN PENSTOCK
9 - WATER ELEVATION LESS THAN TAIL WATER SURFACE
10 - VAPOR PRESSURE FORMED AT GATE
11 - SONIC VELOCITY IN AIR VENT

*** COMPUTATIONS IN MAIN LOOP ***

DO 46 NTIM = 1, K

IF(NTIM.EQ.0) NTIM = NT - 2

IF(NTIM.LE.1) GO TO 5

DATA INITIALIZATION FOR SUBSEQUENT PASSES THRU MAIN LOOP

T(I) = T(I)
QGA(I) = QGA(I)
AP(I) = AP(I)
VP(I) = VP(I)
VGC(I) = VGC(I)
X(I) = X(I)
Y(I) = Y(I)
AS(I) = AS(I)
AGC(I) = AGC(I)
WIN(I) = WIN(I)
VOUT(I) = VOUT(I)
PGA(I) = PGA(I)
CA(I) = CA(I)
ENRTAP(I) = ENRTAP(I)
ERTAGC(I) = ERTAGC(I)
NP1(I) = NP1(I)
DVC(I) = DVC(I)
APPENDIX III

PROGRAM HFVENT

PA(I) = PA(I-1)
MACHA(I) = MACHA(I-1)
MACHGA(I) = MACHGA(I-1)
RHOAG(I) = RHOAG(I-1)
SPVOL(I) = SPVOL(I-1)
WTFLA(I) = WTFLA(I-1)
CKB(I) = CKB(I-1)
P3(I) = P3(I-1)
JCT(I) = JCT(I-1)

COMPUTATIONAL LOOP FOR 40 TIME INCREMENTS

DO 25 M=NT,L
J= M
JN= J-1

IF(NTIM.EQ.2.AND.J.LE.3) GO TO 6
GO TO 7

JM2= J+38
IF(J.GE.3)JM2= J-2
JM1= J+39
IF(J.GE.2)JM1= J-1

TRIAL DETERMINATION OF WATER SURFACE

IF(JFIRST.EQ.1) GO TO 19
IF(JFIRST.EQ.6) GO TO 10
IF(JFIRST.EQ.3) GO TO 0
IF(J.LE.4) GO TO 17

WSTEST = AS(J-2)*(VGC(JN)+VGC(J-2)+DELT*FUNCT2(Y(JN),VP(JN),VGC(JN)))*DELT/3.

GO TO 9

IF(JFIRST.EQ.3) WSTEST = AS(JN)-(DELT-DELTIM)*VGC(JN)*AU/AD

IF(JFIRST.EQ.3)GO TO 9

WSTEST = AS(JN)-DELT*(VGC(JN)+DELT*FUNCT2(Y(JN),VP(JN),VGC(JN)))/2.

IF(JFIRST.EQ.4) WSTEST = AS(JN)-(DELT+DELTIM)*VGC(JN)

IF(WSTEST.LE.GCLP) GO TO 10
IF(WSTEST.LE.UGCLGC) GO TO 11
GO TO 17

WATER SURFACE IN PENSTOCK

IF(JFIRST.EQ.9) GO TO 11
IF(JFIRST.EQ.7) GO TO 12
IF(JFIRST.EQ.8) GO TO 13

LAST INCREMENT BEFORE PENSTOCK FLOW

VOLGC = (AS(JN)-GCLP)

IF(J.GE.3) JM2 = J-2
TOUT = T(JN)+VOLGC*2./(VGC(JN)+VGC(JM2))

IF(JFIRST.EQ.4) TOUT= T(JN)+VOLGC/2.*AU/(AD*(VGC(JN)+VGC(JM2)))

VEL2 = VGC(JN)+FUNCT2(Y(JN),VP(JN),VGC(JN))

IF(JFIRST.EQ.4)DELT= DELT+DELTIM
DELTIM= VOLGC*2./(VEL2*VGC(JN))

IF(DELTIM.GT.DELT) GO TO 17
IF(DELTIM.LE.0.)DELTIM= TOUT-T(JN)
JFIRST= 0
CALL DE2(X,Y,VP,VGC,T,DELTIM,JN)

AS(J)= UGCLGC-Y(J)
IF(AS(J).LT.GCLP)AS(J)=GCLP
WS= AS(J)
X(J) = 0.
CALL AMACHIC(INC,KMUL,AVENT,DELTIM,PGCINC,WTAIR,CONST,

APPENDIX III
PROGRAM HFVENT

C TIME INCREMENT AS WATER ENTERS PENSTOCK
GO TO 20
11 DELT = DELT - DELTIM
C FIRST = 7
GO TO 13
12 DELT = DELT + DELTIM
C FIRST = 8
13 CALL DEL(X, VP, DELT)
AS(J) = WS
GO TO 10
C WATER SURFACE IN LOWER GATE CHAMBER
C
14 IF(JFIRST .EQ. 3) GO TO 15
15 IF(JFIRST .EQ. 4) GO TO 16
16 IF(JFIRST .EQ. 5) GO TO 17
C LAST INCREMENT IN UPPER GATE CHAMBER
VOLGC = (AS(JN) - UGCLGC)
TOUT = T(JN) + VOLGC/VGC(JN)
VEL2 = VGC(JN) + FUNCTION(Y(JN), VP(JNI, VGC(JN))
TOUT - T(JN))
DELTIM = VOLGC * 2. / (VEL2 + VGC(JN))
C FIRST = 3
CALL DE2(X, Y, VP, VGC, T, DELTIM, JN)
AS(J) = WSREF - Y(J)
WS = AS(J)
CALL AMACH(CINC, FRIC, AVENT, DELTIM, PGCINC, WTAIR, CONST, +
CKA)
GO TO 20
C TIME INCREMENT AS WATER ENTERS LOWER GATE CHAMBER
C
15 DELT = DELT - DELTIM
C FIRST = 4
WSREF = UGCLGC
JCK = 2
GO TO 17
16 DELT = DELT + DELTIM
C FIRST = 5
C WATER SURFACE IN UPPER OR LOWER GATE CHAMBER
C
17 CALL DE2(X, Y, VP, VGC, T, DELT, JN)
AS(J) = WSREF - Y(J)
WS = AS(J)
C DETERMINATION OF AIR FLOW RATE IN VENT
C
18 CALL AMACH(CINC, FRIC, AVENT, DELT, PGCINC, WTAIR, CONST, +
CKA)
IF(JFIRST .GE. 7) P3(J) = PTOHD * (ABSPGC - PATM)
GO TO 20
C ASSIMILATION OF RESULTS FOR FIRST TIME THRU
C
19 QGA(1) = QGC
AR(1) = QR
AP(1) = DP
VP(1) = AP(1) / AREAP
VGC(1) = 0.
X(1) = 0.
Y(1) = 0.
DX(1) = 0.
DY(1) = 0.
AS(1) = WS
AS(2) = WS
AOC(1) = ABSPGC
IF(METRIC.EQ.1) AOC(1) = AOC(1) / AGCCTE
APPENDIX III

PROGRAM HFVENT

VIN(I) = 0.
VOUT(I) = 0.
ENRTAP(I) = FUNCTION(VP(I), VGC(I), T(I))
ERTAGC(I) = FUNCTION2(Y(I), VP(I), VGC(I))
DVC(I) = P3(I) + DP
PGA(I) = PGO
CA(I) = CD
PA(I) = PP
MACHA(I) = 0.
MACHGA(I) = 0.
RHOAG(I) = RHOA
SPVOL(I) = 1. / RHOA
WTFLA(I) = 0.
CKBI(I) = 0.
JCT(I) = JCK
VOLR = 0.
JFIRST = 2
DPL(I) = TLOSS * SLOSS * (AP(I) / (.705 * DP * DPL)) * 2. / (2. * GRAV) + HMOTOR
GO TO 25

C
C ASSIMILATION OF RESULTS FOR REMAINING TIME INCREMENTS
C

20 CALL Q(QGC, VP, VGC)
QBA(I) = QGC
PCHECK = ABS(P3(I) - PVAPOR)
IF (PCHECK.LE.0.0001) JFIRST = 10
IF (JFIRST.EQ.10) GO TO 26
IF (JFIRST.EQ.11) GO TO 26

425 IF (WS.LE.TW.0. OR. QP.LT.0.) JFIRST = 9
IF (JFIRST.EQ.9) GO TO 26
AR(J) = QR
AP(J) = QP
AGC(J) = ABS_PGQ
IF (METRIC.EQ.1) AGC(J) = AGC(J) / AGCCTE
IF (JFIRST.GE.7) GO TO 21
ENRTAP(J) = FUNCTION(X(J), VP(J), T(J))
ERTAGC(J) = FUNCTION2(Y(J), VP(J), VGC(J))
IF (JFIRST.GE.4) ERTAGC(J) = ERTAGC(J) * (GCLP-WS) / GRAV
GO TO 22

21 ENRTAP(J) = FUNCTION3(X(J), VP(J), T(J))
ERTAGC(J) = 0.

22 WTAIR = WTAIR + WTFLA(J) + WTFLA(J) * DELT/2.
IF (JFIRST.EQ.3) OR. JFIRST.EQ.6) WTAIR = WTAIR + WTFLA(J)
RHOAG(J) = WTAIR / AVOL
SPVOL(J) = 1. / RHOAG(J)
VOLR = VOLR + FX + AR + JNJ * DELT/2.
IF (JCK.LT.4) VOLR = 0.

445 DPL(J) = TLOSS * SLOSS * (AP(J) / (.705 * DP * DPL)) * 2. / (2. * GRAV) + HMOTOR
DVC(J) = PR
PGA(J) = PGO
PA(J) = PP
CA(J) = CD
JCT(J) = JCK
DX(J) = DELX
DY(J) = DELY
VOUT(J) = MACHA(J) * SQRT(GRAV * SPVOL(J) * 1.4 * AGC(J) * AGCCTE)
IF (METRIC.FG.0.1) VTOL(J) = VTOL(J) / SQRT(GRAV)

455 IF (PIN.GE.PATM) GO TO 23
VIN(J) = CVEL * SQRT(7. * GASCTE * GRAV * ABSTEM
(1. - (PIN/PATM)**(2. / 7.))
IF (METRIC.EQ.1) VIN(J) = VIN(J) / SQRT(GRAV)
IF (MACHA(J).LT.0.) VIN(J) = -1.1 * VIN(J)
GO TO 24

23 VIN(J) = VOUT(J)
24 CKBI(J) = CKA
**AIR-WATER FLOW IN HYDRAULIC STRUCTURES**

**PROGRAM HFVENT**

```
*** WRITE STATEMENTS FOR OUTPUT OF RESULTS ***
CONTINUE

465 26 J = JN+1
     IF(JFIRST.EQ.10) J=J-1
     IF(JFIRST.EQ.11) J=J-1
     IF(JFIRST.EQ.9) J=J-1
     IF(METRIC.EQ.1) PATH= PATM/AGCTE
     FLOW QUANTITIES
     IF(METRIC.EQ.1) GO TO 50
     WRITE(3,27)
     WRITE(3,2)
     WRITE(3,26) PATM, RHOA
     WRITE(3,3)

480 27 FORMAT (1HI,8X,  
     48H COMPUTATION OF AIR FLOW INTO THE GATE CHAMBER D  
     8HURING AN / 16X,  
     40H EMERGENCY GATE CLOSURE IN THE PENSTOCK- / 27X,  
     18H INTAKE STRUCTURE )

485 26 FORMAT (// 8X, 21H ATMOSPHERIC PRESSURE /,14X,  
     23H SPECIFIC MASS OF AIR / 13X,  
     66.2, 5H PSI ,  
     23X, 6.4, 11H LB/FT. / //)
     WRITE(3,29) (T(N), PGA(N), CA(N), AR(N), QGA(N), AP(N), N=1,J)

490 29 FORMAT (28X, 15H FLOW QUANTITIES // 
     8X,  
     34H TIME GATE COEFF Q  
     20H Q / 16X,  
     49H OPENING DISCH. RESERVOIR GATE PENSTOCK /  
     49X, 6.4H/SEC (0/0) (CFS) (CFS)  
     9H (CFS) /  
     (5X, F0.1, 0X, F0.1, 1X, F0.2, 1X, F0.1, 1X, F0.1, 3X, F0.1)  
     IF(JFIRST.EQ.10) WRITE(3,30) T(J+1)

500 30 FORMAT (4X, F0.1, 1X, 38H VAPOR PRESSURE FORMED IN GATE CHAMBER ) 
     IF(JFIRST.EQ.11) WRITE(3,31) T(J+1)

510 31 FORMAT (4X, F0.1, 9X, 23H SONIC VELOCITY IN VENT)

WATER PRESSURES

505 32 FORMAT (28X, 15H WATER PRESSURES // 
     8X,  
     45H HEAD END GATE VENA  
     12H PENSTOCK /  
     16X, 4.4H, 5P ACROSS CHAMFER CONTRACTA /  
     29X, 4.4HUNIT / 
     8X, 4.4H/SEC (FT) (FT) (FT) (FT) / 
     (5X, F0.1, 3X, F0.1, 2X, F0.2, 2X, F0.2, 2X, F0.2, 2X, F0.2, 2X)

520 33 FORMAT (7X, F0.1, 6X, 30H VAPOR PRESSURE FORMED IN GATE CHAMBER ) 
     IF(JFIRST.EQ.10) WRITE(3,33) T(J+1)

525 34 FORMAT (7X, F0.1, 9X, 23H SONIC VELOCITY IN VENT)

AIR FLOW PROPERTIES

WRITE(3,27)
WRITE(3,2)
```
APPENDIX III

PROGRAM HFVENT

530 WRITE(3,3)
WRITE(3,31)IT(N),VIN(N),VOUT(N),NFLA(N),RHOA(N),AGC(N),
+ N=1,J)
IF(JFIRST.EQ.IO)WRITE(3,33)T(J+1)
IF(JFIRST.EQ.11)WRITE(3,34)T(J+1)

535 FORMAT(4X,2IH) AIR FLOW PROPERTIES //
+ 8X,
+ 47H TIME INLET OUTLET AIR FLOW SPECIFIC
+ 8H GATE /
+ 8H,
+ 4WH AIR VFI AIR VFI RATE MASS CHAMBER /
+ 8X,17HOF AIR PRESSURE /
+ 8X,50H(SEC) (FT/SEC) (FT/SEC) (LB/MSEC) (LB/CU FT)
+ 7H (PSIA) /
+ (15,X,F8.1,3X,F8.1,2X,F8.2,2X,F8.4,2X,F8.2)

540 C COMPUTATIONAL PROPERTIES
C

545 WRITE(3,27)
WRITE(3,2) WRITE(3,29) PATM, RHOA
WRITE(3,3) WRITE(3,31) IT(N), CKB(N), DT(N), DX(N), JCT(N), N=1,J)

550 FORMAT(4X,2WH) TIME ACCURACY INTEGRATION ERROR FLOW /
+ 20X.4WH GATE GATE PENSTOCK CONDITION /
+ 20X.39H CHAMBER CHAMBER (* SEE /
+ 20X.40H PRESS LEGEND) /
+ 12X.36H (SEC) (PSI) (FT) (FT) /
+ (10,X,F8.1,2X,F8.3,3X,F8.4,2X,F8.4,4X,1[4]))

555 IF(JFIRST.EQ.IO)WRITE(3,33)T(J+1)
IF(JFIRST.EQ.I1)WRITE(3,34)T(J+1)
IF(NTIM.EQ.K.OR.JFIRST.EQ.9)WRITE(3,37)

560 FORMAT(4X,28H) LEGEND //
+ 8X,49H 1 WATER SURFACE IN UPPER GATE CHAMBER /
+ 8X,49H 2 WATER SURFACE BELOW TOP OF GATE /
+ 8X,49H 3 WATER SURFACE JUST ENTERING PENSTOCK /
+ 8X,49H 4 HYDRAULIC JUMP FILLS PENSTOCK, GATE SUBMERGED/
+ 8X,49H 5 HYDRAULIC JUMP FILLS PENSTOCK, GATE FREE FLOW/
+ 8X,49H 6 WATER SURFACE IN PENSTOCK /

570 IF(NTIM.EQ.K.OR.JFIRST.EQ.9)GO TO 49
GO TO 48
C METRIC WRITE STATEMENTS
C

575 WRITE(3,3) WRITE(3,29) PATM, RHOA
WRITE(3,31)IT(N), CKB(N), DT(N), DX(N), JCT(N), N=1,J)

580 FORMAT(// 8X.21H ATMOSPHERIC PRESSURE, 14X,)
+ 23H SPECIFIC MASS OF AIR / 13X
+ F.6.2, 5H KPA /
+ 2X,F8.4,11H KG/CLM. //)

585 FORMAT(28X,15H) FLOW QUANTITIES //
+ 8X,
+ 34H TIME GATE COEFF Q /
+ 20H Q Q .16X,
+ 49H OPENING DISCH. RESERVOIR GATE PENSTOCK /
+ 6X,8H CHAMBER /

590 FORMAT(8X.49H) SEC) (0/0) (CMS) (CMS)
+ 9H (CMS) /
+ (5,X,F8.1,2X,F8.1,1X,F0.2,4X,F0.3,1X,F0.3,3X,F8.3))

595 IF(JFIRST.EQ.IO)WRITE(3,34)T(J+1)

600 FORMAT(14X,F8.1,5X,30H VAPOR PRESSURE FORMED IN GATE CHAMBER)
PROGRAM HFVENT

595 IF(JFIRST.EQ.11) WRITE(3,42) (J+1)
  42 FORMAT(4X,F8.1,9X,23H SONIC VELOCITY IN VENT)

C WATER PRESSURES
C
600 WRITE(3,27)
WRITE(3,31) PATM,RHOA
WRITE(3,33)
WRITE(3,43) (T(N),AG(N),DPL(N),P3(N),DVC(N),PA(N),N=1,J)

605 43 FORMAT(28X,15H WATER PRESSURES /)
+ 6X,
+ 53H TIME <GATE END GATE VENA PENS
+ 4HTOCK /
+ 16X,40H ELEV ACRROSS CHAMBER CONTRACTA /
+ 29X,4HUNIT /
+ 8X,45H(SEC) (M) (M) (M)
+ 5X,5H (M) /
IF(JFIRST.EQ.11) WRITE(3,44) (J+1)

610 IF(JFIRST.EQ.11)WRITE(3,45) (J+1)
FORMAT(7X,F8.6,15X,38H VAPOR PRESSURE FORMED IN GATE CHAMBER)
 IF(JFIRST.EQ.11)WRITE(3,46) (J+1)
  45 FORMAT(7X,F8.1,9X,23H SONIC VELOCITY IN VENT)

C AIR FLOW PROPERTIES
C
620 WRITE(3,27)
WRITE(3,32)
WRITE(3,39) PATM,RHOA
WRITE(3,31)
WRITE(3,46) (T(N),VIN(N),VOUT(N),WTFLA(N),RHOA(N),AGC(N),
+ N=1,J)
IF(JFIRST.EQ.11) WRITE(3,44) (J+1)
IF(JFIRST.EQ.11) WRITE(3,45) (J+1)

625 46 FORMAT(24X,21H AIR FLOW PROPERTIES /)
+ 6X,
+ 47H TIME INLET OUTLET AIR FLOW SPECIFIC
+ 9X GATE /, 16X,
+ 48H AIR VEL AIR VEL RATE MASS CHAMBER /
+ 48X,17HOF AIR PRESSURE /
+ 8X,48H(SEC) (M/SEC) (M/SEC) (KG/SEC) (KG/CU M)
+ 7H (KPA) /
+ (5X,F8.1,3X,F8.2,1X,F8.2,2X,F8.2,2X,F8.2))

C COMPUTATIONAL PROPERTIES
C
640 WRITE(3,27)
WRITE(3,31)
WRITE(3,39) PATM,RHOA
WRITE(3,31)
WRITE(3,47) (T(N),CKB(N),DY(N),DX(N),JCT(N),N=1,J)

645 47 FORMAT(24X,24H COMPUTATIONAL PROPERTIES /)
+ 16X,44H TIME ACCURACY INTEGRATION ERROR FLOW /
+ 20X,41H GATE GATE PENSTOCK CONDITION /
+ 20X,39H CHAMBER CHAMBER ('LAST /
+ 29X,4H PRESS PAGE /
+ 24X,36H (SEC) (PA) (M) /
+ (10X,F8.1,1X,F8.0,4X,F8.4,2X,F8.4,4X,14))
IF(JFIRST.EQ.10) WRITE(3,44) (J+1)
IF(JFIRST.EQ.11) WRITE(3,45) (J+1)

650 IF(NTIM.EQ.K.OR.JFIRST.EQ.9) WRITE(3,37)
IF(NTIM.EQ.K.OR.JFIRST.EQ.9) GO TO 49
PATM= AGCCP*PATM

48 CONTINUE
49 CALL EXIT

END
APPENDIX III

129

SUBROUTINE Q

SUBROUTINE Q(QGC, VP, VGC)

C THIS PROGRAM COMPUTES THE DISCHARGE THROUGH THE EMERGENCY GATE AS A FUNCTION OF GATE OPENING, RESERVOIR ELEVATION, DOWNSTREAM PRESSURE (FREE OR SUBMERGED), AND THE FREE FLOW DISCHARGE COEFFICIENT.

REAL MACHA, MACHGA, NVENTS
DdimenSion VP(50), VGC(501
COMMON CD, CKV, ABSPGC, PATM, PGC, WSREF, HCOL, WS, QR, QP, GRAV, PTOHD, *PP, P3(50), N, PL, FP, TLOSS, AU, AD, AREAP, AG, EK, OCR, POG, JFIRST, WTLFD(50) *, MACHA(50), MACHGa(50), AVOL, PIN, CVEL, T(50), JCK, GCLP, PGC, ZP, DP, TW, +UGCLGC, GCLL, HYDIA, VOLC1, ELC1, VOLC2, ELC2, CF, PER, CAREA, HMOTOR, *NVENTS, TGC, ENRTAP(50), SO, VOLR, CKX, PR, FSAV, FSAVI, DELX, DELY, +*, ACURAC, CDTEM, GASCTE, VAPOR, AGCCTE, SLOSS, METRIC, PENLEN
QP = VP(N) * AREAP
IF (JFIRST.GT.6) GO TO 1
QGC = VGC(N) * AD
GO TO 2
1 QGC = 0.
2 QR = QP - QGC
IF (JFIRST.GE.7) QR = AG * CD * SQRT(2. * GRAV * ((RES - ZP) + PGC - PTOHD * PATM))
IF (JCK.EQ.0) QR = AG * CD * SQRT(2. * GRAV * ((RES - ZP) - PR + PGC - PTOHD * PATM))
IF (JFIRST.GE.7 AND QR.GE.QP) QR = QP
IF (QR.LE.0) QR = 0.
RETURN
END

SUBROUTINE DE2

SUBROUTINE DE2 (X, Y, VX, VY, U, DELT, N)

C THIS PROGRAM SOLVES TWO DIFFERENTIAL EQUATIONS OF MOTION SIMULTANEOUSLY. THE TWO EQUATIONS ARE DEFINED BY FUNCT1 AND FUNCT2. THE OUTPUTS ARE DISTANCE (X AND Y) AND VELOCITY (VX AND VY).


REAL MACHA, MACHGa, NVENTS
DIImenSion X(50), Y(50), VX(50), VY(50), RK(5), RKX(5), RKY(5), U(50)
COMMON CD, CKV, ABSPGC, PATM, PGC, WSREF, HCOL, WS, QR, QP, GRAV, PTOHD, *PP, P3(50), N, PL, FP, TLOSS, AU, AD, AREAP, AG, EK, OCR, POG, JFIRST, WTLFD(50) *, MACHA(50), MACHGa(50), AVOL, PIN, CVEL, T(50), JCK, GCLP, RES, ZP, DP, TW, +UGCLGC, GCLL, HYDIA, VOLC1, ELC1, VOLC2, ELC2, CF, PER, CAREA, HMOTOR, *NVENTS, U, V, ENRTAP(50), SU, VOLR, CKX, PR, FSAV, FSAVI, DELX, DELY
*, ACURAC, ABSTEM, GASCTE, VAPOR, AGCCTE, SLOSS, METRIC, PENLEN

RHX(1) = X(N)
AIR-WATER FLOW IN HYDRAULIC STRUCTURES

SUBROUTINE DE2

30
RKVY(1) = Y(N)
RKVX(1) = VX(N)
RKVV(1) = VY(N)

IF(JFST.EQ.4) RKVY(1) = 0.

35
RKI(J) = T(N)
H = DELT/N.

F(I) = FUNCT(RKVX(I), RKVY(I), RKT(I))
G(I) = FUNCT2(RKY(I), RKVX(I), RKVY(I))

DO 1 K = 1, L

40
AK1 = RKVX(K) * H
AL1 = RKVY(K) * H
AM1 = H * FUNCT1(RKVX(K), RKVY(K), RKT(K))
AP1 = H * FUNCT2(RKY(K), RKVX(K), RKVY(K))

45
AK2 = (RKVX(K) * AM1/2) * H
AL2 = (RKVY(K) * AP1/2) * H
B1 = RKY(K) + AL1/2.
C1 = RKKVX(K) + AM1/2.
D1 = RKVY(K) + AP1/2.
E1 = RKT(K) + H/2.

50
AM2 = FUNCT1(C1, D1, E1) * H
AP2 = FUNCT2(B1, C1, D1) * H
AK3 = (RKVX(K) + AM2/2.) * H
AL3 = (RKVY(K) + AP2/2.) * H
B2 = RKY(K) + AL2/2.
C2 = RKKVX(K) + AM2/2.
D2 = RKVY(K) + AP2/2.
E2 = RKT(K) + H/2.

55
AM3 = FUNCT1(C2, D2, E2) * H
AP3 = FUNCT2(B2, C2, D2) * H
AK4 = (RKVX(K) + AM3/2) * H
AL4 = (RKVY(K) + AP3/2) * H
B3 = RKY(K) + AL3.
C3 = RKKVX(K) + AM3.
D3 = RKVY(K) + AP3.
E3 = RKT(K) + H.

60
AK5 = FUNCT1(C3, D3, E3) * H
AP5 = FUNCT2(B3, C3, D3) * H
AM5 = (AK1 + 2.*AK2 + 2.*AK3 + AK4) / 6.
ADELT = (AK2 + 2.*AK3 + 2.*AK4 + AK5) / 6.

65
DELTX = (AK1 + 2.*AK2 + 2.*AK3 + AK4) / 6.
DELTVX = (AL1 + 2.*AL2 + 2.*AL3 + AL4) / 6.
DELTVY = (AM1 + 2.*AM2 + 2.*AM3 + AM4) / 6.
DELTV = (AP1 + 2.*AP2 + 2.*AP3 + AP4) / 6.

70
RKY(I+1) = RKT(I) + H
RKVX(I+1) = RKVX(I) + DELTX
RKVY(I+1) = RKVY(I) + DELTY
RKVX(I+1) = RKVX(I) + DELTVX
RKVY(I+1) = RKVY(I) + DELTVY

75
IF(JFST.EQ.4 AND K.EQ.4) JCK = 3
F(K+1) = FUNCT1(RKVX(K+1), RKVY(K+1), RKT(K+1))
G(K+1) = FUNCT2(RKY(K+1), RKVX(K+1), RKVY(K+1))

80
IF(K.EQ.2) FSAV = G(K+1)
IF(K.EQ.4) FSAV = G(K+1)

1 CONTINUE

C CORRECTION OF INITIAL VALUES

85
DELXS = 0.
DELYS = 0.
JCK = 2

90 CALL DELTD(F, DELF, DEL2F, DEL3F, DEL4F)
CALL DELTD(G, DELG, DEL2G, DEL3G, DEL4G)
RKVX(2) = RKVX(1) + H * F(1) + DELF/12. + DEL3F/24. + DEL4F/40.
RKVY(2) = RKVY(1) + H * G(1) + DELG/12. + DEL2G/24. + DEL4G/40.
RKX(2) = RKX(1) + H * (RKVX(1) + RKVX(2)) / 2.
RKY(2) = RKY(1) + H * (RKVY(1) + RKVY(2)) / 2.
APPENDIX III

SUBROUTINE DE2

DO 2 J=1,3
F(J+1) = FUNCT1(RKX(J+1), RKVX(J+1), RKT(J+1))
G(J+1) = FUNCT2(RKY(J+1), RKVX(J+1), RKVY(J+1))
RKVX(J+2) = RKVX(J) + (F(J+1) + F(J)) * H/3.
RKVY(J+2) = RKVY(J) + (G(J+1) + G(J)) * H/3.

RKX(J+2) = RKX(J) + (RKVX(J+2) + 4. * RKVX(J+1) + RKVX(J)) * H/3.
RKY(J+2) = RKY(J) + (RKVY(J+2) + 4. * RKVY(J+1) + RKVY(J)) * H/3.

IF(J.EQ.1) JCK = 3
FC = FUNCT1(RKX(J+2), RKVX(J+2), RKT(J+2))
GC = FUNCT2(RKY(J+2), RKVX(J+2), RKVY(J+2))

IF(J.EQ.1) OLDX = RKX(J+2)
OLDY = RKY(J+2)
IF(J.EQ.2) OLDX = RKX(J+2)
OLDY = RKY(J+2)

IF(J.EQ.1) FSAV = FC
IF(J.EQ.2) FSAV = GC

RKX(J+2) = RKX(J+1) + H * (RKVX(J+2) + RKVX(J+1)) / 2.
RKY(J+2) = RKY(J+1) + H * (RKVY(J+2) + RKVY(J+1)) / 2.

IF(DELX.GE.DELX) DELX = DELX
IF(DELY.GE.DELY) DELY = DELY

DO 2 CONTINUE

VALUES AT END OF INTEGRATION PERIOD

VX(N+1) = RKVX(5)
VY(N+1) = RKVY(5)
X(N+1) = RKX(5)
Y(N+1) = RKY(5)
T(N+1) = T(N) + DELT

IF THE INTEGRATION ACCURACY IS EXCEEDED, THE
PREVIOUS VALUE OF VX AND OR VY IS USED TO
EXTRAPOLATE THE NEW VALUE OF X AND OR Y.

IF(DELX.LE.0.01) GO TO 3
VX(N+1) = VX(N) + DELT / (T(N) - T(N-1)) * (VX(N) - VX(N-1))
P3(N+1) = P3(N) + DELT / (T(N) - T(N-1)) * (P3(N) - P3(N-1))
X(N+1) = VX(N) + VX(N+1) * DELT / 2. * X(N)

3 CONTINUE

IF(DELY.LE.0.01) GO TO 4
VY(N+1) = VY(N) + DELT / (T(N) - T(N-1)) * (VY(N) - VY(N-1))
P3(N+1) = P3(N) + DELT / (T(N) - T(N-1)) * (P3(N) - P3(N-1))
Y(N+1) = VY(N) + VY(N+1) * DELT / 2. * Y(N)

END OF EXTRAPOLATION

4 CKX = VX(N+1)
CKY = VY(N+1)
RETURN

END
SUBROUTINE DELTD

C THIS PROGRAM COMPUTES THE 4TH DIFFERENCE OF A SERIES
C OF FIVE EQUALLY SPACED QUANTITIES

C

REAL MACHA,MACHGA,NVENTS
COMMON CD,CKV,ABSPGC,PATM,PGC,WSMLT,HCOL,WS,UK,UP,GRAV,PTOHD,
+PP,P3(50),J,PL,FP,TLOSS,AU,AD,AREAP,AG,ER,OCR,PGO,JFIRST,WFLA(50)
+MACHA(50),MACHGA(50),AVOL,PIN,CVEL,T(50),JCK,GCLP,RES,ZP,DP,TW,
+VUML,CML,HYDRA,VELC1,ELC1,VELC2,ELC2,CF,PER,CARA,HMOTOR,
+NVENTS,TGC,ENRTAP(50),SO,VR,CKX,PR,FSAV,FSAVI,DELX,DELY
+AURAC,ABSTEM,GASCTE,VAPOR,AGCCTE,SLOSS,METRIC,PENLEN
DIMENSION A(5),DEL('+),DEL2(3),DEL3(2)

DO 1 I=1,4
1 DEL(I)= A(I+1)-A(I)
DO 2 K=1,3
2 DEL2(K)= DEL(K+1)-DEL(K)
DO 3 JK=1,2
3 DEL3(JK)= DEL2(JK+1)-DEL2(JK)
B= DEL(1)
E= DEL2(1)
D= DEL3(1)
DEL4= DEL3(2)-DEL3(1)
RETURN
END

FUNCTION FUNCTI

C

REAL MACHA,MACHGA,NVENTS
COMMON CD,CKV,ABSPGC,PATM,PGC,WSMLT,HCOL,WS,UK,UP,GRAV,PTOHD,
+PP,P3(50),J,PL,FP,TLOSS,AU,AD,AREAP,AG,ER,OCR,PGO,JFIRST,WFLA(50)
+MACHA(50),MACHGA(50),AVOL,PIN,CVEL,T(50),JCK,GCLP,RES,ZP,DP,TW,
+VUML,CML,HYDRA,VELC1,ELC1,VELC2,ELC2,CF,PER,CARA,HMOTOR,
+NVENTS,TGC,ENRTAP(50),SO,VR,CKX,PR,FSAV,FSAVI,DELX,DELY
+AURAC,ABSTEM,GASCTE,VAPOR,AGCCTE,SLOSS,METRIC,PENLEN
PGO= 100.-OCR*ER
CD= (1.049E-04)+(7.062E-03)*PGO-(4.385E-05)*PGO**2-(2.39BE-06)*PGO**3+
+(3.578E-01)*PGO**4-(1.937E-01)*PGO**5
VHP= CA**2/(2.*GRAV)
VHR= (CA-DA*AU/AREAP)**2/2.*GRAV)
VHR= (CA-DA*AU/AREAP)**2/2.*GRAV)

FUNCTION FUNCTI(CA,DA,EA)

C

REAL MACHA,MACHGA,NVENTS
COMMON CD,CKV,ABSPGC,PATM,PGC,WSMLT,HCOL,WS,UK,UP,GRAV,PTOHD,
+PP,P3(50),J,PL,FP,TLOSS,AU,AD,AREAP,AG,ER,OCR,PGO,JFIRST,WFLA(50)
+MACHA(50),MACHGA(50),AVOL,PIN,CVEL,T(50),JCK,GCLP,RES,ZP,DP,TW,
+VUML,CML,HYDRA,VELC1,ELC1,VELC2,ELC2,CF,PER,CARA,HMOTOR,
+NVENTS,TGC,ENRTAP(50),SO,VR,CKX,PR,FSAV,FSAVI,DELX,DELY
+AURAC,ABSTEM,GASCTE,VAPOR,AGCCTE,SLOSS,METRIC,PENLEN
PGO= 100.-OCR*ER
CD= (1.049E-04)+(7.062E-03)*PGO-(4.385E-05)*PGO**2-(2.39BE-06)*PGO**3+
+(3.578E-01)*PGO**4-(1.937E-01)*PGO**5
VHP= CA**2/(2.*GRAV)
VHR= (CA-DA*AU/AREAP)**2/2.*GRAV)

FUNCTION FUNCTI(CA,DA,EA)

C
APPENDIX III

FUNCTION FUNCT2

FUNCTION FUNCT2(BB, CB, DB)

THIS PROGRAM GIVES THE VALUE OF THE SECOND ORDER
DIFFERENTIAL EQUATION OF DEPTH WITH RESPECT TO TIME
FOR THE WATER IN THE GATE CHAMBER. THE VALUE OF THE
DIFFERENTIAL IS FUNCT2.

REAL MACHA, MACHGA, NVENTS
COMMON CD, CKV, ABSPGC, PATM, PACR, HCOL, WC, QR, OP, OR, PR, VREF, PTOHD,
+ PP, PS(50), J, PL, FP, TLOSS, AU, AD, AREAP, AG, EK, OCR, PGO, JFIRST, WTLA(50)
+ MACHA(50), MACHGA(50), AVOL, PIN, CVEL, T(50), JCK, GCLP, RES, JP, DP, TW,
+ GCCLC, GCLL, HYDIA, VOLC, CLC, VOLE, ELC, CM, PER, CAREA, HMOTOR,
+ NVENTS, TGC, ENRTAP(50), 50, VOLR, CKX, PR, FS AV, FSAVI, DELX, DELY
+ , ACURAC, ABSTEM, GASCTE, VAPOR, AGCCTE, SLOSS, METRIC, PENLEN

VHGC = DB**2/(2.*GRAV)
VHR = (CB-DB*AU/AREAP)**2/(2.*GRAV)
IF(JFIRST.GE.4) VHR = (CB-DB*AD/AREAP)**2/(2.*GRAV)

M = (VHGC*FP/HYDIA)**2
PT = PR-DP
PVAPOR = VAPOR-PTOHD*PATM
IF(PT.GT.PVAPOR) GO TO 1

PT = PVAPOR

1 P3(J) = PT
IF(JFIRST.GE.4) GO TO 2

2 IF(JCK.EQ.3) GO TO 3

FUNCT2 = GRAV/(HCOL-BB+GCLL*AU/AD)*(PGC-PTOHD*PATM-PT+HCOL-BB
+ GCLL-VHGC*((1.+FP*GCLL/HYDIA+EK)*XO**2-1./2.).)
RETURN

2 IF(JCK.EQ.4) GO TO 2

FUNCT2 = GRAV**(PGC-PTOHD*PATM-PT+HCL-BB)**2
RETURN

EXTRAPOLATION OF FUNCT2 TO ENTRANCE OF PENSTOCK
(.ZERO LENGTH OF WATER COLUMN) JCK = 3 SIGNIFIES NO
MORE WATER IN GATE CHAMBER.

3 FUNCT2 = 2.*FSAV-FSAVI
RETURN

END

SUBROUTINE DE1

SUBROUTINE DE1(X, VX, DELT)

THIS PROGRAM SOLVES THE DIFFERENTIAL EQUATION OF MOTION
FOR FLOW IN THE PENSTOCK WHEN THE WATER SURFACE IS IN THE
PENSTOCK. THE OUTPUTS ARE DISTANCE (X) AND VELOCITY (VX).

REAL MACHA, MACHGA, NVENTS
COMMON CD, CKV, ABSPGC, PATM, PACR, HCOL, WC, QR, OP, OR, PR, VREF, PTOHD,
+ PP, PS(50), M, FL, FP, TLOSS, AU, AD, AREAP, AG, EK, OCR, PGO, JFIRST, WTLA(50)
SUBROUTINE DE1

C DETERMINATION OF INITIAL VALUES

N = M-1
RX(1) = X(N)
RVX(1) = VX(N)
RT(1) = T(N)
H = DELT/4.
RF(1) = FUNCT3(RX(1),RVX(1),RT(1))
DO 1 K = 1,N
   AK1 = FUNCT3(RX(K),RVX(K),RT(K))*H
   A1 = RX(K) + RVX(K)*H/2. + AK1*H/8.
   B1 = RVX(K) + AK1/2.
   C1 = RT(K) + H/2.
   AK2 = FUNCT3(A1,B1,C1)*H
   B2 = RVX(K) + AK2/2.
   AK3 = FUNCT3(A1,B2,C1)*H
   A3 = RX(K) + RVX(K)*H + AK3*H/2.
   B3 = RVX(K) + AK3
   C3 = RT(K) + H/4
   AK4 = FUNCT3(A3,B3,C3)*H
   DELTX = H*(RVX(K) + AK1 + AK2 + AK3 + AK4)/6.
   DELTVX = (AK1 + 2.*AK2 + 2.*AK3 + AK4)/6.
   RF(K+1) = FUNCT3(RX(K+1),RVX(K+1),RT(K+1))
1 CONTINUE

C CORRECTION OF INITIAL VALUES

DELY = 0.
DELXS = 0.
CALL DELTD(RVX,DELFX,DELFX,DELF,DELFX,DELF)
H(2) = RX(1) + H*(RVX(1) + DELTF/2. + DELF/24. + DELF/40.)
CALL DELTD(RF,DELFX,DELFX,DELF,DELF)
RVX(2) = RVX(1) + H*(RF(1) + DELF/2. + DELF/24. + DELF/40.)
DO 2 J = 1,3
   RF(J+1) = FUNCT3(RX(J+1),RVX(J+1),RT(J+1))
   RVX(J+2) = (RF(J+2) + 4.*RF(J+1) + RF(J)*H/3. + RX(J))/6.
   OLDX = RX(J+2)
   RX(J+2) = (RVX(J+2) + 4.*RVX(J+1) + RVX(J)) + 3.*RX(J)/3.
   DELX = ABS(OLDX - RX(J+2))
   IF(DELX .GE. DELTX) DELX = DELTX
2 CONTINUE

C VALUES AT END OF INTEGRATION PERIOD

VX(N+1) = RVX(5)
X(N+1) = RX(5)
CKX = X(N+1)
CKV = VX(N+1)
T(N+1) = T(N) + DELT
RETURN
END
APPENDIX III

FUNCTION FUNCT3

FUNCTION FUNCT3(AC,BC,DC)

This program gives the value of the second order differential equation of flow distance with respect to time for a free water surface in the penstock.

REAL MACHA,MACHGA,NVENTS
COMMON CD,CKV,ABSPGC,PATM,PGC,~SREF,HCOL,~S,QR,QP,GRAV,PTOHD,
+GR,GR15U),J,PL,FP,ILLUS,AG,AD,AREAP,AG,EK,GCR,PGD,JFIRST,WFLA(50)
+MACHA(50),MACHGA(50),AVOL,PIN,CVEL,T(50),JCK,CLC,R,RES,DP,DP,TW,
+UCLG,C1L,HYDIA,VP,E,VLC1,EVL2,EL2,F,PER,AREA,HMOTOR,
+NVENTS,TS,ENRTP(50),5O,VP0R,CX,PR,FSAY,FSAV,DELX,DELY
+ACURAC,ABSTEM,GASC,T,VAPO,AGGCT,ELOSS,METRIC,PENLEN
JN= J-1

VHP= BC**2/(2.*GRAV)
PG0= 100.-GCR*DC
IF(PGO.LE.0.)GO TO 1
CD= (1.049E-01+7.062E-03*PGO-5.83OE-05l*PGO**2-2.398E-06+PGO**3-(3.57BE-08)*PGO**4+1.987E-10l*PGO**5
PGD= PGO/100.
BH= PGD*SO/(RES-ZP)
GO TO 2
1 PGO= 0.
CD= 0.
PGD= 0.
CC= .611
2 HVC= SO*CC*PGD
IF(WS.LE.ZP)GO TO 6
AVOLCK= AC*AREAP-QR*(DC-T(JN))-VOLR
IF(AVOLCK.LT.O.)AVOLCK= .I.E-30
CKWS= GCLP-AVOLCK/VOLCl
IF(JCK.EQ.G)GO TO 6
IF(JCK.EQ.5lGO TO 3
JUMP IN CONDUIT, GATE SUBMERGED
DEPT= SQRT(AVOLCK/3.5/DP)
PR= DP-DEPT
QR= AD*CD*SQRT2.*GRAV*(((RES-ZP)-PR+PGC-PTOHD*PATM))
IF(PL.TT.HVC)GO TO 3
JCK= 4
IF(POGD.LE.0.)GO TO 4
PP= QR**2/(CC*PGD*AG*AREAP*GRAV)+PR*(DP-DEPT)/(2.*DP/2.+VHP
++VHP**2+PGC-PTOHD*PATM)
GO TO 5
C
C JUMP IN CONDUIT, GATE NOT SUBMERGED
3 PR= HVC
JCK=5
IF(POGD.LE.0.)GO TO 4
PP= (2.*CD**2*AG*RES-ZP*PGC-PTOHD*PATM)/(PGD+CC**2*AREAP)
+PR*CC*AG*PGD/(2.*AREAP)-2.*VHP+DP/2.+PGC-PTOHD*PATM
DEPT= QR/QR*(16.1*DP**2+CC*SO*PGD)+HVC*HVC-OP/QP/16.1*DP*AREAP
IF(PE.LE.DP.AND.DEPTE.GT.0.)PP= SQRT(DEPT2)+PGC-PTOHD*PATM
IF(DEPTE.LE.0.)GO TO 6
GO TO 5
4 PP= PR+PGC-PTOHD*PATM
5 WS= ZP*PR
IF(PP.LE.DP)WS= ZP+PP
PL= HS.PENLEN
IF(CKWS.LT.ZP*PR.AND.JFIRST.EQ.8lGO TO 6
FUNCT3= GRAV/PL*(PP-VHP*(TLOSS+SLOSS-1.+FP/PL/DP)+ZP-TW-HMOTOR)
RETURN

C
FUNCTION FUNCT3
C WATER SURFACE BELOW GATE SEAT
C
6 QRTST = AG*CD*SQRT(2.*GRAV*(RES-ZP+PTOH+1/PATM))
7 IF(QRTST.GT.QP)QRTST = QP
JCK = 6
8 DT = (DC-T(JN))
9 IF(DT.LE.O.1)GO TO 7
10 DQRDT = (QR-QRTST)/DT
11 WS = ZP+DP-(AC*AREAP-(2.*QRTST-DQRDT*DT/2.-VOLR)/VOLC1
12 IF(WS.LE.ELC1)WS = ELC1-(AC*AREAP-(2.*QRTST-DQRDT*DT/2.-
13 VOLR)/VOLC2
14 IF(WS.GE.GCLP)GO TO 8
15 PL = WS-PENLEN
16 SURAR = CF*SIN((GCLP-WS)/PER1
17 IF(SURAR.LE.CAREA)SURAR = CAREA
18 IF(ABS(GCLP-WS).LE.DP/4.)SURAR = AREAP
19 VWS = VWS+2/(2.*GRAV)
20 IF(PGD.GT.O.1 GO TO 9
21 VHVC = 0.
22 GO TO 10
23 VHVC = (QRTST/1AG*CC)**2/(2.*GRAV)
24 IF(ABS(GCLP-WSI.LE.DP/q/1SURAR= AREAP
25 VWS= (AREAP**2-1/VWS**2)
26 IF(PGD.GT.0.1)GO TO 9
27 IF(ABS(GCLP-WSI.LE.DP/q/1SURAR= AREAP
28 VWS= (AREAP**2-1/VWS**2)
29 PP= PTOHD*(ABSPGC-PATM)
30 IF(WS.GE.ZP)PP= W'SZP
31 PR=HVC
32 RETURN
33 END

SUBROUTINE AMACH
1 SUBROUTINE AMACH(CINC,FRICT,AVENT,DELT,PGCINC,WTAIR,CONST,
2 +CKA)
3 C THIS PROGRAM GIVES THE AIR FLOW RATE IN THE AIR VENT AS A
4 C FUNCTION OF THE WATER SURFACE ELEVATION.
5 C
6 REAL MIN,MIN,MA Cha,MACGa,NEVENTS
7 COMMON CD,CKV,ABSPGC,PATG,PGC,WSREF,HCOL,WS,QR,QR,GRAV,PTOHD,
8 PP,POS,GO,J,PL,PP,TL0SS,AU,AD,AREAP,A0,EX,OLM,POS,1FIRST,MTFLA(30)
9 +,MACHA(50),MACHGA(50),AVOL,PIN,CVEL,T(50),JCK,GCLP,RES,DP,TW,
10 +UGCLGC,GCLL,HYDDIA,VELO,CLC1,VELO2,CLC2,CF,PER,CAREA,HMOTOR,
11 +NEVENTS,T0C,ENRTAP(50),SO,VOLR,CKX,PR,FSAV,FSAV1,DELX,DELY
12 +,ACURAC,ABSTEM,GASCTE,VAPOR,AGCTE,SLOSS,METRIC,FENLEN
13 EQ(MACH)= (1.-MACH*MACH)/(1.4*MACH*MACH)+8571*
14 ALOG(1.2*MACH*MACH)/(1.4*MACH*MACH)+8571*
15 ALOG(1.2*MACH*MACH)/(1.4*MACH*MACH)+.8571*
16 JN= J-1
17 MACHN= MACHA(JN)
18 MACHGC= MACHGA(JN)
19 IF(ABS(MACHA(JN)).LE.0.0001)MACHN= .0001
20 IF(ABS(MACHA(JN)).LE.0.0001)MACHGC=.0001
21 C SETS AIR FLOW DIRECTION INTO VENT
22 C KSN= 1.
23 IF(CKV.LT.0.)CKSN= -1.
24 IF(CKSN.LT.0.)MACHN= CKSN*ABSMACHIN
25 IF(CKSN.LT.0.)MACHGC= CKSN*ABSMACHGC
SUBROUTINE AMACH

M1I = 0.50
CKASAV = 0.001
DM1I = -0.24999

30 C
C COMPUTATION OF AIR VOLUME IN GATE CHAMBER AND PENSTOCK

RC = 1./((1.2)**3.5)
OVER = 0
AVOL = AU*-(TGC-WS)
AVOL = AVOL + 0.5*(UGCLLC-WS)*AD
IF(WS.LE.UGCLLC)AVOL = AVOL + AD*GCLL + (GCLP-WS)*VOLC
IF(JFIRST.EQ.G)AVOL = AVOL + AD*GCLL + AREAP*CKX-VOLR-QDEL

40 IF(JCK.GE.4)AVOL = AVOL + AD*GCLL + AREAP*CKX-VOLR-QDEL
C C
C COMPUTATION OF THE PRESSURE RATIO, THE COMPRESSIBLE DISCHARGE
C COEFFICIENT, THE REAL MACH NUMBER AT I, AND THE AIR FLOW RATE

45 C GIVEN THE IDEAL MACH NUMBER AT THE BEGINNING OF THE DUCT REGION

50 DO 14 NDO=1,30
R = 1./((1.+2.*M1I**2)**3.5)
IF(R/RC.LT.1.)GO TO 1

55 C COMPRESSIBLE DISCHARGE COEFFICIENT
C = 1.-CINC*CINC*(1.-CINC)**7*(CINC-1.)**4*(1./R-1.)/(1./RC-1.)
GO TO 2

1 C = 1.-CINC*(1.-7.*(CINC-1.)*(1.-CINC)**7)+(1.-R/RC)**21
2 RADICL = 1.+8.*C*C*(M1I*M1I+2.*M1I*M1I*M1I)
ROOT = SQRT(RADICL)
M1R = SQRT((-1.+ROOT)/.4)
IF(ABS(M1R).LE.0.00001)M1R = -1E-10*CKSN
IF(CKSN.LT.0.)M1R = -1.*M1R
MACHIN = M1R

60 C ENTROPY INCREASE IN INLET REGION
ENT = (C*M1I/M1R)**7.
IF(ABS(M1I).LE.0.001)ENT = 1.

65 C STAGNATION PRESSURE AT END OF INLET
PIN = ENT*PATM
IF(CKSN.LT.0.)PIN = ENT*PGCTRL

C VELOCITY COEFFICIENT
CVEL = (M1R/M1I)**2/C

70 C MASS FLOW RATE
WTFLA(J) = AGCCTE/SQRT(AESTEM*GASCTE/(1.4*GRAV))*AVENT*PATM*ENT*
+ MIR*NVENTS/(1.+2.*MIR)**21**3
IF(CKSN.LT.0.)WTFLA(J) = AGCCTE/SQRT(AESTEM*GASCTE/(1.4*GRAV))*
+ AVENT*MUCLN*ENT*CKSN**7.2*M1R**7.2**2
IF(METRIC.EQ.1.)WTFLA(J) = WTFLA(J)/(AGCCTE*SQRT(GRAV))

75 C COMPUTATION OF MACH NO. AT OUTLET OF AIR DUCT
C CONSIDERING THE FRICTION LOSSES IN THE DUCT

80 C FMAX = EQ(MACHIN)
IF(FRICT.GT.FMAX)GO TO 5
MACHGC = MACHIN
CKM = -0.01
DM = MACHGC/2.
IF(CKM .GE. 0.5)DM = (1.-MACHGC)/?2

85 C EQRH = FMAX
EQHL = EQ(MACHGC)

C THIS LOOP FINDS THE VALUE OF MACHGC
DO 4 K=1,35
EQHL = EQ(MACHGC)

C K = CMII EQU IT FRICT
CALL NEWX(CK,CKM,MACHGC,DM)
IF(K.EQ.1)GO TO 3
AIR-WATER FLOW IN HYDRAULIC STRUCTURES

SUBROUTINE AMACH

T1 = ABS(CK/FRICT)

IF(T1.LE.0.002)GO TO 12

CKM = CK

CONTINUE

GO TO 12

COMPUTATION OF AIR FLOW RATE WITH MACH 1 AT DUCT OUTLET

MACHGC = 1.
MACHIN = 1.
DMIN = -0.25
C2 = -0.01

DO 6 N=1,30

EQRH = EQ(MACHIN)
Cl = EQRH - FRICT
CALL NEWX(Cl, C2, MACHIN, DMIN)

T1 = ABS(Cl/FRICT)

IF(T1.LE.0.005)GO TO 7

CONTINUE

MII = MACHIN
DMIN = -0.25
C2 = -0.01

MII = 1.0

DO 10 MOON = 1,30

R = 1./(1.+2.*MII**2)**3.5

IF((R/RC).LE.1.)GO TO 8

C = 1.-1.*CINC*(1.-.7*(CINC-1.)*(1./R-1.))-(2.*CINC*(1.-1.*(R-1.))**2)

GO TO 9

RADICL = 1.+B*C**2*(MII**2+2.*MII**4)

ROOT = SQRT(RADICL)

MIR = SQRT((-1.+ROOT)/.4)
T1 = ABS(C/MIR)

IF(T1.LE.0.005)GO TO 11

CONTINUE

ENT = (C*MII/MIR)**7.
PIN = ENT*PATM
CVEL = (MIR/MII)**2/C

WTFLA(J) = AGCCTE/SQRT(ABSTEM*GASCTE/(1.4*GRAV)*AVOL)

IF(METRIC.EQ.1)WTFLA(J) = WTFLA(J)/(AGCCTE*SQRT(GRAV))

JFIRST = 11

GO TO 15

ADIABATIC EXPANSION OF AIR IN THE GATE CHAMBER GIVES THE GATE CHAMBER PRESSURE PGCTST

RHOA = (2.*WTAIR+(WTFLA(JN)+WTFLA(J))*DELT)/2.*AVOL
PGCTST = CONST*RHOA**1.4

IF(JFIRST.EQ.4.OR.JFIRST.EQ.7)PGCTST = CONST*(WTAIR+WTFLA(J))**1.4

PGCTRL = PIN*MACHIN/MACHGC*SQRT((1.+.2*MACHGC**2)/(1.+2.*MACHIN**2)**6)

IF(CKSN.LT.0.)PGCTRL = PATM/ENT*MACHIN/MACHGC*SQRT((1.+.2*MACHGC**2)/(1.+2.*MACHIN**2)**6)

CKA = PGCTRL - PGCTST

ABSPGC = (PGCTRL + PGCTST) / 2.

PGC = PTOHD*ABSPGC
SUBROUTINE AMACH

160 IF (ABS(CKA).LE.PGCINC) GO TO 15
CALL NEWX(CKA,CKASAV,MII,DMII)
CKASAV = CKA
165 IF(MII.GE.0.0.AND.CKV.GE.0.0)GO TO 13
IF(MII.LT.0.0.AND.CKV.LT.0.0)GO TO 13
MII = ABS(MII)CKSN/2.
DMII = DMII/2.
13 IF (ABS(MII)).GE.1.238) MII = 1.238*CKSN
CONTINUE
OVER = 1
15 MACHA(J) = MIR
MACHOA(J) = MACHGC
170 IF (CKSN.LT.0.) MACHGA(J) = MIR
IF (CKSN.LT.0.) MACHA(J) = MACHGC
175 IF (OVER.EQ.1) WRITE(3,161)
OVER = 1
16 FORMAT(1HI,19X,35HONE OR MORE LOOPS IN AMACH EXCEEDED /
+ 30X,LHS = ,F9.2)
RETURN
END

SUBROUTINE NEWX

1 SUBROUTINE NEWX(C1,C2,X,D)
C
C THIS SUBROUTINE DETERMINES THE NEXT TRIAL VALUE OF X WHICH
C SATISFIES A FUNCTION C = F(X). IF THE CORRECT ROOT LIES
C BETWEEN THE OLD VALUE OF THE FUNCTION (C1) AND THAT VALUE
C JUST COMPUTED (C2), THEN THE INCREMENT IS HALVED. OTHERWISE,
C X IS INCREASED BY D. THE LOGIC OF THIS PROGRAM RESULTS
C IN INCREASES OF C AS X INCREASES. IF THE OPPOSITE IS TRUE
C THEN D MUST BE MADE NEGATIVE IN THE CALLING ROUTINE. IN
C THE CALLING ROUTINE THE INITIAL VALUE OF C1 SHOULD BE EQUAL
C TO ZERO.
C
C C1 = PREVIOUS VALUE OF FUNCTION
C C2 = VALUE OF FUNCTION JUST COMPUTED
C X = INDEPENDENT VARIABLE
C D = INCREMENT OF X
C
IF(C1.GT.0.)GO TO 2
IF(C2.LT.0.)GO TO 1
D = D/2.
GO TO 3
1 X = X + D
RETURN
2 IF(C2.GT.0.)GO TO 3
D = D/2.
GO TO 1
3 X = X - D
RETURN
END
Computation of Air Flow into the Gate Chamber during an Emergency Gate Closure in the Penstock-Intake Structure
Morrow Point Dam

Atmospheric Pressure
77.60 kPa
Specific Mass of Air
.9738 kg/cu.m.

One 840mm by 915mm Air Vent
Flow Quantities

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APPENDIX III

COMPUTATION OF AIR FLOW INTO THE GATE CHAMBER DURING AN EMERGENCY GATE CLOSURE IN THE PENSTOCK-INTAKE STRUCTURE MORROW POINT DAM

ATMOSPHERIC PRESSURE
77.60 KPA

SPECIFIC MASS OF AIR
0.9738 KG/ CU.M.

ONE 840MM BY 915MM AIR VENT WATER PRESSURES

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## Air-Water Flow in Hydraulic Structures

### Computation of Air Flow Into the Gate Chamber During an Emergency Gate Closure in the Penstock-Intake Structure: Morrow Point Dam

**Atmospheric Pressure**: 77.60 kPa  
**Specific Mass of Air**: 0.9738 kg/ cu. m.

### One 80mm by 915mm Air Vent

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APPENDIX III

COMPUTATION OF AIR FLOW INTO THE GATE CHAMBER DURING AN EMERGENCY GATE CLOSURE IN THE PENSTOCK-INTAKE STRUCTURE MORMOW POINT DAM

ATMOSPHERIC PRESSURE
77.60 KPA

SPECIFIC MASS OF AIR
0.9738 KG/CU.M.

ONE 840MM BY 915MM AIR VENT

COMPUTATIONAL PROPERTIES

TIME

ACCURACY

INTEGRATION ERROR

FLOW CONDITION

GATE PENSTOCK

CHAMBER CHAMBER (*LAST PAGE)

(SEC) (PA) (M) (M)

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3.0 -43.0000 0.0000 2
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LEGEND

* 1 WATER SURFACE IN UPPER GATE CHAMBER

2 WATER SURFACE BELOW TOP OF GATE

3 WATER SURFACE JUST ENTERING PENSTOCK

4 HYDRAULIC JUMP FILLS PENSTOCK, GATE SUBMERGED

5 HYDRAULIC JUMP FILLS PENSTOCK, GATE FREE FLOW

6 WATER SURFACE IN PENSTOCK

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On November 6, 1979, the Bureau of Reclamation was renamed the Water and Power Resources Service in the U.S. Department of the Interior.
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