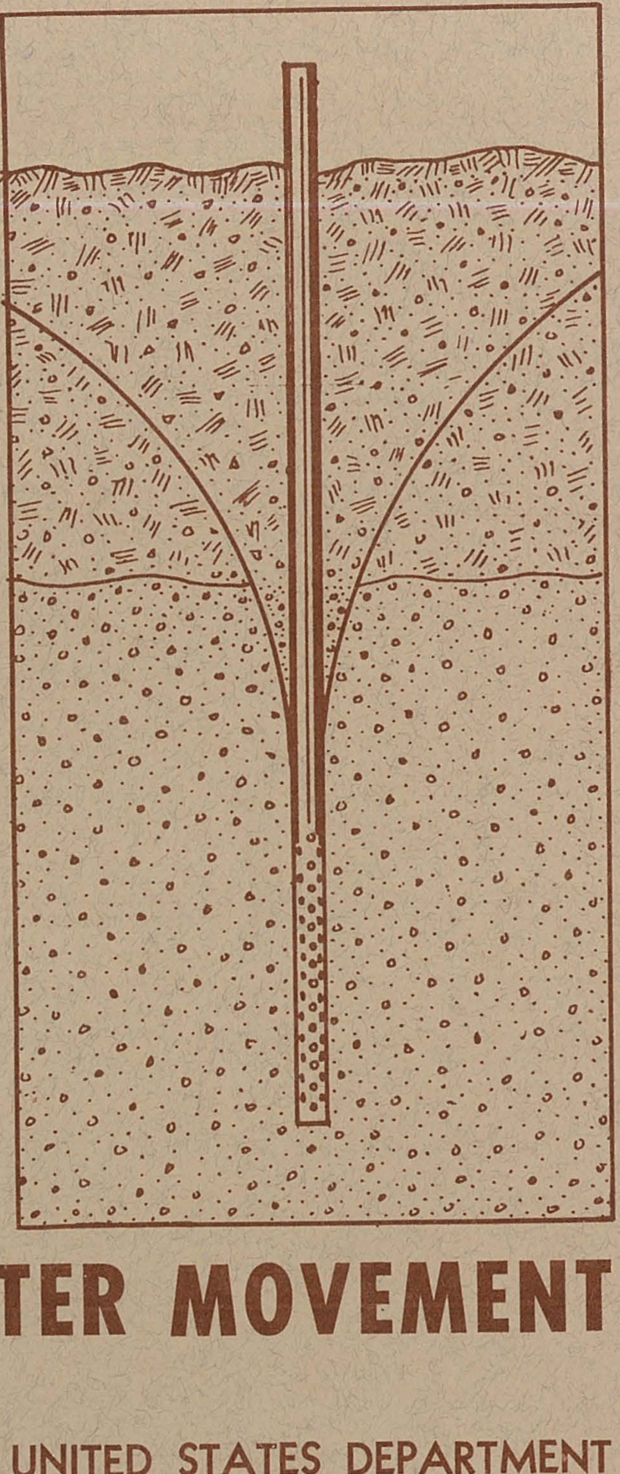
A WATER RESOURCES TECHNICAL PUBLICATION

ENGINEERING MONOGRAPH No. 31



GROUND-WATER MOVEMENT

UNITED STATES DEPARTMENT
OF THE INTERIOR
BUREAU OF RECLAMATION

A WATER RESOURCES TECHNICAL PUBLICATION

Engineering Monograph No. 31

Ground-Water Movement

By R. E. GLOVER

Denver, Colorado

United States Department of the Interior

BUREAU OF RECLAMATION

Contents

LIST OF FIGURES

LIST OF TABLES

Units Used

THE FORMULAS to be described and elaborated upon in this monograph apply to certain important cases of ground-water movement. The formulas are expressed in a notation which has been selected, on the basis of experience, to serve the needs of this subject. Units are specified in the notation as a means of identifying physical dimensions, but the formulas are written in consistent form and will, therefore, apply in any consistent system of units. A system of units is consistent when no more than one unit of a kind is permitted. In general, these formulas involve only the units of length and time. The use of consistent units secures the advantages of simplicity and flexibility.

An example of a consistent unit system based upon the units of length in feet and time in seconds is:

Length: feet
Time: seconds
Flow: cubic feet per second
Permeability: feet per second
Drawdown: feet
Thickness of aquifer: feet
Radius: feet

This system, for example, would become inconsistent if flow were expressed in gallons per minute, because the gallon unit of volume does not agree with the chosen unit of length and the minute unit does not agree with the chosen unit of time.

Graphs appearing in the text have been prepared by using dimensionless parameters. Such parameters are often composed of a group of quantities which have units but they are so arranged that the parameter, as a whole, has none. Using such parameters is advantageous in that they permit the construction of generalized charts which can be used with any system of consistent units.

Notation and Definition of Terms

The following notation is used throughout the text:

a a well or drain radius (feet)

b an outer radius (feet)

$$c=\frac{\Gamma\left(\frac{7}{6}\right)}{\Gamma\left(\frac{5}{3}\right)\sqrt{\pi}}=0.5798 \text{ (dimensionless)}$$

D the initial saturated thickness of an aquifer (feet)

D_a an average saturated thickness (feet)

d the vertical distance between the centerline of a drain and an impermeable barrier or a saturated thickness below some maintained minimum water level (feet)

$e=2.71828+$. The base of the natural system of logarithms

$E=\frac{\pi K}{2d \log_e \left(\frac{d}{a}\right)}$ $\frac{1}{\text{sec}}$

f a pumping rate distributed over an area (ft per sec)

F a flow of ground water through a unit width of aquifer (ft^2 per sec)

F_L a flow to a drain, per unit length of drain, as limited by a local resistance (ft^2 per sec)

F_o a value of F at $x=0$ (ft^2 per sec)

$G\left(\frac{\sqrt{4\alpha t}}{a}\right)$ a function of the parameter $\left(\frac{\sqrt{4\alpha t}}{a}\right)$. The discharge of a flowing artesian well is given in terms of this function in the form $Q=2\pi KDs_oG\left(\frac{\sqrt{4\alpha t}}{a}\right)$. The function $G\left(\frac{\sqrt{4\alpha t}}{a}\right)$ is dimensionless

h_r and H_r transient and maximum amplitudes of reservoir fluctuation (feet)

h in the Dupuit-Forchheimer idealization, a drainable depth of water in an aquifer (feet). In the Laplace idealization, a pressure in excess of hydrostatic, expressed in terms of the pressure due to a unit depth of water

h_a and h_b drainable depths as used in the method of Brooks

H an initial drainable depth (feet)

i an infiltration rate (ft per sec)

$I_o(x)$ a modified Bessel function of the parameter x of zero order and the first kind

$J_o(x)$, $J_1(x)$ Bessel's functions, of order zero and one, of the parameter x (dimensionless) (Notation of Reference 4)

K permeability of an aquifer (ft per sec)

KD the transmissibility of an aquifer (ft^2 per sec)

$K_o(x)$, $K_1(x)$ modified Bessel's functions, of orders zero and one, of the parameter x of the second kind (dimensionless)

L the distance between parallel drains (feet)

L_c the length of a leaky canal (feet)

m and n consecutive whole numbers used in specifying the terms of a series

m the thickness of a horizontal bed or member which offers a high resistance to the flow of ground water (feet)

p the permeability of a bed which offers a high resistance to the flow of ground water (ft per sec)

p_1 the part of the drainable water which remains in the aquifer at the time t (dimensionless)

q_d a portion of the flow of a well which is taken from an identified source (ft^3 per sec)

q_1 a flow per unit length of a line source or a flow to a unit length of a drain (ft^2 per sec)

r a radius (feet)

R a total return flow up to a time t per unit width of aquifer ($feet^2$)

S an increment of storage capacity contributed by bank storage per unit length of bank ($feet^2$)

s drawdown (feet)

s_o for a flowing artesian well, the initial pressure reduction at the well when flow began, expressed in feet of water

t time (seconds)

t_e an equivalent time. See Figure 10 (seconds)

t_1 a time between irrigations (seconds)

t_L a time during which a flow to a drain is limited by a local resistance (seconds)

T a period (seconds)

u a dimensionless variable

$U=\frac{h}{H}$ (dimensionless)

V voids ratio. The ratio of drainable or fillable voids to the total volume (dimensionless)

W a factor in the equation $U=WY$ (W is dimensionless)

x and y rectangular coordinates (ft)

x a symbol used to indicate a dimensionless parameter

$Y_0(x)$, $Y_1(x)$ Bessel's functions of the zero and first orders of the second kind (Notation of Reference 4)

Y a factor in the equation $U=WY$ (Y is dimensionless)

y_{c0}, y_{ct}, y_{x0} drainable depths as used in the drain spacing procedure of Dumm, Tapp, and Moody

y_{c0} an initial drainable depth midway between drains

y_{ct} a drainable depth at the point midway between drains at the time t

y_{x0} an initial drainable depth at the point x

$\alpha=\frac{KD}{V}$ the diffusivity (ft² per sec)

β_n a root of a Bessel's equation, defined where used. The dimensions of β are $\frac{1}{\text{feet}}$

$\omega=\frac{2\pi}{T}\frac{1}{\text{sec}}$

$\sigma=\frac{Q}{2\pi KD^2}$ (dimensionless)

$\eta=\left(\frac{p}{mV}\right)t$ (dimensionless)

$\eta_1=\left(\frac{KH}{VL^2}\right)t$ (dimensionless)

$\mu=\frac{s}{\left(\frac{Q}{2\pi KD}\right)}$ (dimensionless)

$\rho=r\sqrt{\frac{p}{mKD}}$ (dimensionless)

$\xi=\frac{x}{L}$ (dimensionless)

$U_0(\beta_n r)=J_0(\beta_n r)Y_0(\beta_n a)-J_0(\beta_n a)Y_0(\beta_n r)$

$\pi=3.14159+$ (dimensionless)

$\Gamma(x)=$ a gamma function of the parameter x (dimensionless)

Definitions

Aquifer — A water-bearing bed or stratum.

Diffusivity — A quantity $\alpha=\frac{KD}{V}$ used with the Dupuit-Forchheimer idealization to specify the transient behavior of an aquifer.

Transmissivity — A quantity expressed in the Dupuit-Forchheimer idealization as the product KD. It defines the ability of an aquifer to transmit ground water under the influence of a gradient.

Exponential integral — A tabulated function defined by the integral

$$\int_x^\infty \frac{e^{-u}}{u}\,du$$

Probability integral — A tabulated function defined by the integral

$$\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du.$$

Introduction

THIS MONOGRAPH presents analyses of a variety of ground-water problems encountered in the planning and development of Bureau of Reclamation water resources projects in the western United States. These problems include analysis of depletions caused by pumping, estimates of seepages, computation of return flows, analysis of drawdown, and estimates of permeabilities for selection of pump capacities. In analyzing these and other ground-water problems, theoretical assumptions and limitations are outlined and specific problems are computed through the use of charts, tables, and the solution of equations derived from the theoretical considerations.

Many of the problems presented were originally analyzed to serve an immediate need and were made available to interested Bureau of Reclamation engineers through informal memoranda. In 1960, these memoranda were compiled, edited, and issued as Technical Memorandum No. 657, "Studies of Ground-Water Movement," to assist the Bureau's engineering staff in its analyses of ground-water problems. To extend the coverage of situations encountered on Reclamation projects, to assure an orderly presentation and to provide solutions for a wide variety of ground-water conditions, this monograph includes the information contained in the technical memorandum, suitably revised and rearranged, and in addition, includes accounts of other data and analyses prepared both by Bureau of Reclamation engineers and by others.

The solutions to the problems described are generally based upon the Dupuit-Forchheimer idealization, which involves the assumption that the gradient at the water table is effective through the entire saturated thickness of an unconfined aquifer. This assumption is justified on theoretical grounds only if the gradients are small as compared to unity. It is equivalent to an assumption that the vertical gradients can be neglected.

An alternative approach can be based upon the concept that, for each element of volume below the water table, the flows of water into and out of it must be equal. This formulation yields a differential equation of the Laplace type.

These two formulations will be dealt with in detail subsequently, but it will be of interest here to consider the relative merits of these two approaches. The Laplace formulation, while admittedly the more general of the two, suffers from the limitations imposed by serious mathematical difficulties. On the other hand, if the Dupuit-Forchheimer idealization is used and an additional simplification is introduced, by neglecting the effect of draw-down on the areas available for flow of ground water, the differential equations obtained become identical in form with those which have long been studied in the theory of heat conduction solids. There is, therefore, a great tactical advantage in making this latter choice, since the wealth of resources available to this older discipline becomes immediately available for exploitation in the new field. These advantages are bought at a price, however, since the simplifications introduced appear, from a theoretical standpoint, to limit the validity of the solutions that are obtained to those cases where the gradients are small compared to unity and the changes of ground-water level are small compared to the original saturated depth in the aquifer. Under these conditions it is fair to raise the question as to how well the solutions obtained will hold up under conditions met in the field. Some comparisons of observed and computed values are presented later to permit the reader to judge for himself the effect of these simplifications.

The cases presented herein represent transient conditions. In general, steady states are considered only where they represent a terminal condition.

Analytical Formulations

DIFFERENTIAL EQUATIONS provide a basis for the development of formulas to account for the flow of ground water. These equations express the requirement that if the flows into and out of an element of aquifer volume are different, then there must be a corresponding loss or accumulation of water in the element. The differential equations, therefore, express the important fundamental fact that water volumes are conserved. As a consequence, it will be found that even though the differential equations may fail to account for some factors known to be present, they do impose upon the solutions a rigid accounting for water volumes. This is true without exception so long as the solutions are exact in the mathematical sense. All of the solutions presented herein are of this type unless otherwise explicitly noted. The budgetary requirement expressed by the differential equation will be referred to hereafter as the continuity condition. The following conditions are assumed to prevail:

(1) The aquifer is homogeneous and isotropic and has a permeability K.

(2) When the water table descends, it leaves the aquifer drained above it and when it rises, it fills the aquifer to its own level but contributes nothing to the water content above its own level. The ratio of the drainable or fillable volume to the gross volume is V.

(3) All flow takes place below the water table.

(4) The aquifer rests upon an impermeable horizontal bed.

The Dupuit-Forchheimer idealization applies the water-table gradient to the entire saturated depth of the aquifer. The continuity condition is developed as follows. The notation is shown in Figure 1.

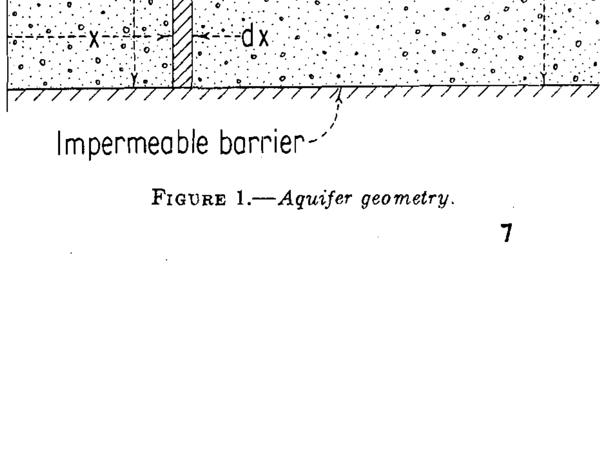


FIGURE 1.—*Aquifer geometry.*

The flow F through a unit width and the height h at the distance x from the origin is

$$F=Kh\frac{\partial h}{\partial x}. \tag{1}$$

The continuity condition is

$$\frac{\partial F}{\partial x}dx\,dt=V\frac{\partial h}{\partial t}dt\,dx.$$

By substitution and rearrangement

$$K\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right)=V\frac{\partial h}{\partial t}.$$

If, as an approximation, the quantity $h\frac{\partial h}{\partial x}$ is replaced by $D\frac{\partial h}{\partial x}$ and

$$\alpha=\frac{KD}{V}, \tag{2}$$

the above relation reduces to

$$\alpha\frac{\partial^2 h}{\partial x^2}=\frac{\partial h}{\partial t}. \tag{3}$$

If y represents a coordinate whose direction is horizontal and normal to that of x and if there are gradients $\frac{\partial h}{\partial y}$, the above relation takes the form

$$\alpha\left(\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}\right)=\frac{\partial h}{\partial t}. \tag{4}$$

These are the linearized forms of the Dupuit-Forchheimer continuity equations. It may be noted that the simplification introduced above introduces the restriction that h must be small compared to D.

In radially symmetrical cases, the differential equation takes the form*

$$\alpha\left(\frac{\partial^2 s}{\partial r^2}+\frac{1}{r}\frac{\partial s}{\partial r}\right)=\frac{\partial s}{\partial t}. \tag{5}$$

*In some cases it is expedient to work in terms of the quantity h and in other cases the quantity s. Reasons for each choice will become clear when a specific case is being considered.

Linearized in this way, Formulas (3), (4), and (5) are identical in form with the differential equations of heat conduction in solids. The great advantage of the Dupuit-Forchheimer formulation is a result of this correspondence, which makes the resources of the older development available for computation of ground-water movements.

The Laplace formulation deals with the condition of continuity of an infinitesimal unit of volume $dx\,dy\,dz$ where x, y, and z are rectangular coordinates. In this treatment it will be assumed that the coordinates x and y lie in a horizontal plane and the z coordinate is vertical and positive in the upward direction. The condition that the flow into the element of volume must equal the flow out of it is

$$K\left(\frac{\partial^2 h}{\partial x^2}dx\,dy\,dz+\frac{\partial^2 h}{\partial y^2}dy\,dz\,dx+\frac{\partial^2 h}{\partial z^2}dz\,dx\,dy\right)=0,$$

or

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=0 \tag{6}$$

where h represents the departure of the pressures from some stable hydrostatic configuration. It is expressed in terms of the pressure due to a unit depth of water. Equation (6) is of the Laplace form. Many solutions are available for steady state cases, but in the free water table transient cases, where the moving upper boundary must be accounted for, serious mathematical difficulties are encountered. It is the difficulty with the moving boundary which has kept this formulation from coming into general use. If solutions of Equation (6) meeting the appropriate initial and boundary conditions could be found, the vertical gradient would be accounted for and the limitations imposed by the Dupuit-Forchheimer idealization would not appear.

If the flow is radially symmetrical this continuity equation takes the form

$$\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}+\frac{\partial^2 h}{\partial z^2}=0. \tag{7}$$

The treatment of specific cases can now be attempted.

Radially Symmetrical Cases

Pumped well—confined aquifer. The case of a well in a confined aquifer may be met in an artesian area where the pressures have declined to the point where pumps must be used. The aquifer of permeability K and thickness D is confined above and below between impermeable formations. The pump maintains the flow Q. The condition of continuity is

$$\alpha\left(\frac{\partial^2 s}{\partial r^2}+\frac{1}{r}\frac{\partial s}{\partial r}\right)=\frac{\partial s}{\partial t}.$$

A solution which satisfies the continuity requirement and the conditions

$$s=0 \text{ when } t=0 \text{ for } r>0,$$

$$s\rightarrow 0 \text{ when } r\rightarrow\infty, \qquad (8)$$

is

$$s=\frac{Q}{2\pi KD}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u}\,du. \qquad (9)$$

The integral which appears in Equation (9) is a form of the exponential integral. Values of this function have been tabulated. In terms of the exponential integral function its value is

$$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u}\,du=-\frac{1}{2}\,Ei\left(-\frac{r^2}{4\alpha t}\right). \qquad (10)$$

Values of

$$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u}\,du$$

can be obtained from Table 1 or they can be computed from the series

$$\int_{x}^{\infty}\frac{e^{-u^2}}{u}\,du=-0.288608-\log_e x$$
$$+\frac{x^2}{2}-\frac{x^4}{2!4}+\frac{x^6}{3!6}-\cdots \qquad (11)$$

In this series x represents an argument for which the function is to be evaluated. In the series (11) the error committed by stopping at any term is less than the first term omitted. When used for finding values to use in Equation (9), $x=\frac{r}{\sqrt{4\alpha t}}$. This integral can also be evaluated by use of the tabulated exponential integral as $-\frac{1}{2}\,Ei\left(-\frac{r^2}{4\alpha t}\right)$ as noted previously.

Pumped well—unconfined aquifer. A well that is to be pumped from an unconfined aquifer occurs commonly. The aquifer rests on an impermeable bed and the saturated portion of the aquifer terminates at the top in a water table. Such conditions are often found, for example, in the alluvial sediments of river valleys.

The aquifer in these instances is composed of sands and gravels deposited by the stream. The stream runs over the surface of the aquifer and is in contact with the ground water stored in it. A moment's consideration will show that Formula (9) can be used to provide an approximate treatment for this case if the drawdown s is everywhere small compared to D. This is the customary treatment for the water-table case.

Example 1

A well of 2 feet effective diameter is sunk into an aquifer of permeability $K=0.0020$ foot per second and is pumped at the rate of 500 gallons per minute. The original saturated depth is 70 feet and the ratio of drainable voids to the gross volume of the sediments is $V=0.20$. Compute the drawdown at radii 50 and 100 feet from the well after it has been pumped for a period of 72 hours.

TABLE 1.—*Values of the integral* $\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$

$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}du$
0.00010	8.92173	0.00054	7.23533	0.00098	6.63935	0.00520	4.97050
0.00011	8.82642	0.00055	7.21698	0.00099	6.62920	0.00530	4.95145
0.00012	8.73941	0.00056	7.19897	0.00100	6.61915	0.00540	4.93276
0.00013	8.65937	0.00057	7.18127	0.00110	6.52384	0.00550	4.91441
0.00014	8.58526	0.00058	7.16387	0.00120	6.43683	0.00560	4.89640
0.00015	8.51627	0.00059	7.14678	0.00130	6.35678	0.00570	4.87870
0.00016	8.45173	0.00060	7.12997	0.00140	6.28268	0.00580	4.86131
0.00017	8.39110	0.00061	7.11344	0.00150	6.21368	0.00590	4.84421
0.00018	8.33395	0.00062	7.09718	0.00160	6.14914	0.00600	4.82741
0.00019	8.27988	0.00063	7.08118	0.00170	6.08852	0.00610	4.81088
0.00020	8.22859	0.00064	7.06543	0.00180	6.03136	0.00620	4.79462
0.00021	8.17980	0.00065	7.04993	0.00190	5.97730	0.00630	4.77862
0.00022	8.13327	0.00066	7.03466	0.00200	5.92600	0.00640	4.76287
0.00023	8.08882	0.00067	7.01962	0.00210	5.87721	0.00650	4.74737
0.00024	8.04626	0.00068	7.00481	0.00220	5.83069	0.00660	4.73210
0.00025	8.00544	0.00069	6.99021	0.00230	5.78624	0.00670	4.71706
0.00026	7.96622	0.00070	6.97582	0.00240	5.74368	0.00680	4.70225
0.00027	7.92848	0.00071	6.96164	0.00250	5.70286	0.00690	4.68765
0.00028	7.89211	0.00072	6.94765	0.00260	5.66364	0.00700	4.67326
0.00029	7.85702	0.00073	6.93386	0.00270	5.62590	0.00710	4.65908
0.00030	7.82312	0.00074	6.92025	0.00280	5.58953	0.00720	4.64509
0.00031	7.79033	0.00075	6.90683	0.00290	5.55444	0.00730	4.63130
0.00032	7.75858	0.00076	6.89358	0.00300	5.52054	0.00740	4.61769
0.00033	7.72781	0.00077	6.88051	0.00310	5.48775	0.00750	4.60427
0.00034	7.69796	0.00078	6.86761	0.00320	5.45600	0.00760	4.59103
0.00035	7.66897	0.00079	6.85487	0.00330	5.42523	0.00770	4.57796
0.00036	7.64080	0.00080	6.84229	0.00340	5.39538	0.00780	4.56505
0.00037	7.61340	0.00081	6.82987	0.00350	5.36639	0.00790	4.55232
0.00038	7.58673	0.00082	6.81760	0.00360	5.33822	0.00800	4.53974
0.00039	7.56076	0.00083	6.80548	0.00370	5.31082	0.00810	4.52732
0.00040	7.53544	0.00084	6.79350	0.00380	5.28415	0.00820	4.51505
0.00041	7.51075	0.00085	6.78167	0.00390	5.25818	0.00830	4.50293
0.00042	7.48665	0.00086	6.76997	0.00400	5.23286	0.00840	4.49095
0.00043	7.46312	0.00087	6.75841	0.00410	5.20817	0.00850	4.47912
0.00044	7.44013	0.00088	6.74698	0.00420	5.18407	0.00860	4.46742
0.00045	7.41765	0.00089	6.73568	0.00430	5.16054	0.00870	4.45586
0.00046	7.39568	0.00090	6.72451	0.00440	5.13755	0.00880	4.44443
0.00047	7.37417	0.00091	6.71346	0.00450	5.11508	0.00890	4.43314
0.00048	7.35312	0.00092	6.70253	0.00460	5.09310	0.00900	4.42196
0.00049	7.33250	0.00093	6.69172	0.00470	5.07160	0.00910	4.41091
0.00050	7.31229	0.00094	6.68102	0.00480	5.05054	0.00920	4.39999
0.00051	7.29249	0.00095	6.67044	0.00490	5.02992	0.00930	4.38918
0.00052	7.27307	0.00096	6.65997	0.00500	5.00972	0.00940	4.37848
0.00053	7.25403	0.00097	6.64961	0.00510	4.98992	0.00950	4.36790

TABLE 1.—*Values of the integral* $\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$ —Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$
0. 00960	4. 35743	0. 05900	2. 54335	0. 11200	1. 90690	0. 16500	1. 52672
0. 00970	4. 34707	0. 06000	2. 52660	0. 11300	1. 89812	0. 16600	1. 52084
0. 00980	4. 33681	0. 06100	2. 51013	0. 11400	1. 88943	0. 16700	1. 51500
0. 00990	4. 32666	0. 06200	2. 49393	0. 11500	1. 88081	0. 16800	1. 50920
0. 01000	4. 31661	0. 06300	2. 47800	0. 11600	1. 87226	0. 16900	1. 50343
0. 01100	4. 22131	0. 06400	2. 46231	0. 11700	1. 86379	0. 17000	1. 49770
0. 01200	4. 13431	0. 06500	2. 44687	0. 11800	1. 85540	0. 17100	1. 49200
0. 01300	4. 05428	0. 06600	2. 43167	0. 11900	1. 84708	0. 17200	1. 48634
0. 01400	3. 98019	0. 06700	2. 41670	0. 12000	1. 83883	0. 17300	1. 48071
0. 01500	3. 91121	0. 06800	2. 40195	0. 12100	1. 83065	0. 17400	1. 47512
0. 01600	3. 84669	0. 06900	2. 38742	0. 12200	1. 82254	0. 17500	1. 46956
0. 01700	3. 78608	0. 07000	2. 37310	0. 12300	1. 81450	0. 17600	1. 46403
0. 01800	3. 72894	0. 07100	2. 35898	0. 12400	1. 80652	0. 17700	1. 45854
0. 01900	3. 67489	0. 07200	2. 34507	0. 12500	1. 79862	0. 17800	1. 45308
0. 02000	3. 62361	0. 07300	2. 33135	0. 12600	1. 79077	0. 17900	1. 44765
0. 02100	3. 57485	0. 07400	2. 31782	0. 12700	1. 78299	0. 18000	1. 44226
0. 02200	3. 52835	0. 07500	2. 30447	0. 12800	1. 77528	0. 18100	1. 43690
0. 02300	3. 48392	0. 07600	2. 29130	0. 12900	1. 76762	0. 18200	1. 43157
0. 02400	3. 44138	0. 07700	2. 27830	0. 13000	1. 76003	0. 18300	1. 42627
0. 02500	3. 40058	0. 07800	2. 26548	0. 13100	1. 75249	0. 18400	1. 42100
0. 02600	3. 36139	0. 07900	2. 25281	0. 13200	1. 74502	0. 18500	1. 41576
0. 02700	3. 32367	0. 08000	2. 24032	0. 13300	1. 73760	0. 18600	1. 41055
0. 02800	3. 28733	0. 08100	2. 22797	0. 13400	1. 73025	0. 18700	1. 40537
0. 02900	3. 25227	0. 08200	2. 21578	0. 13500	1. 72294	0. 18800	1. 40022
0. 03000	3. 21840	0. 08300	2. 20375	0. 13600	1. 71570	0. 18900	1. 39510
0. 03100	3. 18564	0. 08400	2. 19185	0. 13700	1. 70851	0. 19000	1. 39001
0. 03200	3. 15392	0. 08500	2. 18010	0. 13800	1. 70137	0. 19100	1. 38495
0. 03300	3. 12318	0. 08600	2. 16849	0. 13900	1. 69429	0. 19200	1. 37992
0. 03400	3. 09336	0. 08700	2. 15702	0. 14000	1. 68726	0. 19300	1. 37491
0. 03500	3. 06441	0. 08800	2. 14567	0. 14100	1. 68028	0. 19400	1. 36993
0. 03600	3. 03628	0. 08900	2. 13446	0. 14200	1. 67335	0. 19500	1. 36498
0. 03700	3. 00891	0. 09000	2. 12338	0. 14300	1. 66648	0. 19600	1. 36006
0. 03800	2. 98228	0. 09100	2. 11242	0. 14400	1. 65965	0. 19700	1. 35516
0. 03900	2. 95635	0. 09200	2. 10158	0. 14500	1. 65287	0. 19800	1. 35029
0. 04000	2. 93107	0. 09300	2. 09086	0. 14600	1. 64614	0. 19900	1. 34545
0. 04100	2. 90642	0. 09400	2. 08026	0. 14700	1. 63946	0. 20000	1. 34063
0. 04200	2. 88236	0. 09500	2. 06977	0. 14800	1. 63283	0. 20100	1. 33584
0. 04300	2. 85887	0. 09600	2. 05940	0. 14900	1. 62624	0. 20200	1. 33108
0. 04400	2. 83593	0. 09700	2. 04913	0. 15000	1. 61970	0. 20300	1. 32634
0. 04500	2. 81350	0. 09800	2. 03897	0. 15100	1. 61320	0. 20400	1. 32162
0. 04600	2. 79156	0. 09900	2. 02892	0. 15200	1. 60675	0. 20500	1. 31693
0. 04700	2. 77010	0. 10000	2. 01896	0. 15300	1. 60035	0. 20600	1. 31227
0. 04800	2. 74910	0. 10100	2. 00911	0. 15400	1. 59398	0. 20700	1. 30763
0. 04900	2. 72853	0. 10200	1. 99936	0. 15500	1. 58766	0. 20800	1. 30301
0. 05000	2. 70837	0. 10300	1. 98971	0. 15600	1. 58139	0. 20900	1. 29842
0. 05100	2. 68862	0. 10400	1. 98015	0. 15700	1. 57515	0. 21000	1. 29385
0. 05200	2. 66925	0. 10500	1. 97068	0. 15800	1. 56896	0. 21100	1. 28930
0. 05300	2. 65026	0. 10600	1. 96131	0. 15900	1. 56280	0. 21200	1. 28478
0. 05400	2. 63162	0. 10700	1. 95203	0. 16000	1. 55669	0. 21300	1. 28028
0. 05500	2. 61333	0. 10800	1. 94283	0. 16100	1. 55062	0. 21400	1. 27581
0. 05600	2. 59536	0. 10900	1. 93372	0. 16200	1. 54459	0. 21500	1. 27136
0. 05700	2. 57772	0. 11000	1. 92470	0. 16300	1. 53859	0. 21600	1. 26693
0. 05800	2. 56038	0. 11100	1. 91576	0. 16400	1. 53264	0. 21700	1. 26252

TABLE 1.—*Values of the integral* $\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$
0.21800	1.25813	0.27100	1.05309	0.32400	0.88955	0.37700	0.75552
0.21900	1.25377	0.27200	1.04966	0.32500	0.88677	0.37800	0.75322
0.22000	1.24943	0.27300	1.04626	0.32600	0.88401	0.37900	0.75093
0.22100	1.24511	0.27400	1.04286	0.32700	0.88126	0.38000	0.74865
0.22200	1.24081	0.27500	1.03949	0.32800	0.87851	0.38100	0.74638
0.22300	1.23653	0.27600	1.03612	0.32900	0.87578	0.38200	0.74411
0.22400	1.23228	0.27700	1.03277	0.33000	0.87306	0.38300	0.74185
0.22500	1.22804	0.27800	1.02943	0.33100	0.87034	0.38400	0.73960
0.22600	1.22383	0.27900	1.02611	0.33200	0.86764	0.38500	0.73736
0.22700	1.21963	0.28000	1.02280	0.33300	0.86495	0.38600	0.73512
0.22800	1.21546	0.28100	1.01951	0.33400	0.86227	0.38700	0.73289
0.22900	1.21131	0.28200	1.01623	0.33500	0.85959	0.38800	0.73067
0.23000	1.20717	0.28300	1.01296	0.33600	0.85693	0.38900	0.72846
0.23100	1.20306	0.28400	1.00970	0.33700	0.85428	0.39000	0.72625
0.23200	1.19896	0.28500	1.00646	0.33800	0.85163	0.39100	0.72406
0.23300	1.19489	0.28600	1.00323	0.33900	0.84900	0.39200	0.72186
0.23400	1.19083	0.28700	1.00002	0.34000	0.84637	0.39300	0.71968
0.23500	1.18680	0.28800	0.99681	0.34100	0.84376	0.39400	0.71750
0.23600	1.18278	0.28900	0.99362	0.34200	0.84115	0.39500	0.71533
0.23700	1.17878	0.29000	0.99045	0.34300	0.83856	0.39600	0.71317
0.23800	1.17480	0.29100	0.98728	0.34400	0.83597	0.39700	0.71102
0.23900	1.17084	0.29200	0.98413	0.34500	0.83339	0.39800	0.70887
0.24000	1.16690	0.29300	0.98100	0.34600	0.83082	0.39900	0.70673
0.24100	1.16297	0.29400	0.97787	0.34700	0.82826	0.40000	0.70459
0.24200	1.15907	0.29500	0.97476	0.34800	0.82571	0.40100	0.70247
0.24300	1.15518	0.29600	0.97165	0.34900	0.82317	0.40200	0.70035
0.24400	1.15131	0.29700	0.96857	0.35000	0.82064	0.40300	0.69823
0.24500	1.14746	0.29800	0.96549	0.35100	0.81811	0.40400	0.69613
0.24600	1.14362	0.29900	0.96242	0.35200	0.81560	0.40500	0.69403
0.24700	1.13980	0.30000	0.95937	0.35300	0.81310	0.40600	0.69194
0.24800	1.13600	0.30100	0.95633	0.35400	0.81060	0.40700	0.68985
0.24900	1.13222	0.30200	0.95330	0.35500	0.80811	0.40800	0.68777
0.25000	1.12845	0.30300	0.95029	0.35600	0.80563	0.40900	0.68570
0.25100	1.12471	0.30400	0.94728	0.35700	0.80316	0.41000	0.68364
0.25200	1.12097	0.30500	0.94429	0.35800	0.80070	0.41100	0.68158
0.25300	1.11726	0.30600	0.94131	0.35900	0.79825	0.41200	0.67953
0.25400	1.11356	0.30700	0.93834	0.36000	0.79580	0.41300	0.67748
0.25500	1.10988	0.30800	0.93538	0.36100	0.79337	0.41400	0.67544
0.25600	1.10621	0.30900	0.93243	0.36200	0.79094	0.41500	0.67341
0.25700	1.10256	0.31000	0.92949	0.36300	0.78852	0.41600	0.67139
0.25800	1.09892	0.31100	0.92657	0.36400	0.78611	0.41700	0.66937
0.25900	1.09531	0.31200	0.92366	0.36500	0.78371	0.41800	0.66736
0.26000	1.09170	0.31300	0.92075	0.36600	0.78131	0.41900	0.66535
0.26100	1.08812	0.31400	0.91786	0.36700	0.77893	0.42000	0.66335
0.26200	1.08454	0.31500	0.91498	0.36800	0.77655	0.42100	0.66136
0.26300	1.08099	0.31600	0.91211	0.36900	0.77418	0.42200	0.65937
0.26400	1.07745	0.31700	0.90926	0.37000	0.77182	0.42300	0.65739
0.26500	1.07392	0.31800	0.90641	0.37100	0.76947	0.42400	0.65542
0.26600	1.07041	0.31900	0.90357	0.37200	0.76712	0.42500	0.65345
0.26700	1.06692	0.32000	0.90074	0.37300	0.76479	0.42600	0.65149
0.26800	1.06344	0.32100	0.89793	0.37400	0.76246	0.42700	0.64954
0.26900	1.05997	0.32200	0.89512	0.37500	0.76014	0.42800	0.64759
0.27000	1.05652	0.32300	0.89233	0.37600	0.75782	0.42900	0.64564

TABLE 1.—*Values of the integral* $\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$
0. 43000	0. 64371	0. 48300	0. 54931	0. 53600	0. 46897	0. 58900	0. 40023
0. 43100	0. 64178	0. 48400	0. 54767	0. 53700	0. 46757	0. 59000	0. 39903
0. 43200	0. 63985	0. 48500	0. 54604	0. 53800	0. 46618	0. 59100	0. 39783
0. 43300	0. 63794	0. 48600	0. 54441	0. 53900	0. 46479	0. 59200	0. 39664
0. 43400	0. 63603	0. 48700	0. 54279	0. 54000	0. 46340	0. 59300	0. 39545
0. 43500	0. 63412	0. 48800	0. 54117	0. 54100	0. 46202	0. 59400	0. 39427
0. 43600	0. 63222	0. 48900	0. 53956	0. 54200	0. 46064	0. 59500	0. 39309
0. 43700	0. 63033	0. 49000	0. 53795	0. 54300	0. 45927	0. 59600	0. 39191
0. 43800	0. 62844	0. 49100	0. 53635	0. 54400	0. 45790	0. 59700	0. 39073
0. 43900	0. 62656	0. 49200	0. 53475	0. 54500	0. 45654	0. 59800	0. 38956
0. 44000	0. 62468	0. 49300	0. 53316	0. 54600	0. 45517	0. 59900	0. 38840
0. 44100	0. 62281	0. 49400	0. 53157	0. 54700	0. 45382	0. 60000	0. 38723
0. 44200	0. 62095	0. 49500	0. 52999	0. 54800	0. 45246	0. 60100	0. 38607
0. 44300	0. 61909	0. 49600	0. 52841	0. 54900	0. 45111	0. 60200	0. 38491
0. 44400	0. 61724	0. 49700	0. 52684	0. 55000	0. 44977	0. 60300	0. 38376
0. 44500	0. 61539	0. 49800	0. 52527	0. 55100	0. 44843	0. 60400	0. 38261
0. 44600	0. 61355	0. 49900	0. 52370	0. 55200	0. 44709	0. 60500	0. 38146
0. 44700	0. 61172	0. 50000	0. 52214	0. 55300	0. 44575	0. 60600	0. 38031
0. 44800	0. 60989	0. 50100	0. 52059	0. 55400	0. 44442	0. 60700	0. 37917
0. 44900	0. 60806	0. 50200	0. 51904	0. 55500	0. 44310	0. 60800	0. 37803
0. 45000	0. 60625	0. 50300	0. 51749	0. 55600	0. 44178	0. 60900	0. 37690
0. 45100	0. 60443	0. 50400	0. 51595	0. 55700	0. 44046	0. 61000	0. 37577
0. 45200	0. 60263	0. 50500	0. 51441	0. 55800	0. 43914	0. 61100	0. 37464
0. 45300	0. 60083	0. 50600	0. 51288	0. 55900	0. 43783	0. 61200	0. 37351
0. 45400	0. 59903	0. 50700	0. 51135	0. 56000	0. 43653	0. 61300	0. 37239
0. 45500	0. 59724	0. 50800	0. 50983	0. 56100	0. 43522	0. 61400	0. 37127
0. 45600	0. 59546	0. 50900	0. 50831	0. 56200	0. 43392	0. 61500	0. 37016
0. 45700	0. 59368	0. 51000	0. 50680	0. 56300	0. 43263	0. 61600	0. 36905
0. 45800	0. 59191	0. 51100	0. 50529	0. 56400	0. 43134	0. 61700	0. 36794
0. 45900	0. 59014	0. 51200	0. 50378	0. 56500	0. 43005	0. 61800	0. 36683
0. 46000	0. 58838	0. 51300	0. 50228	0. 56600	0. 42876	0. 61900	0. 36573
0. 46100	0. 58662	0. 51400	0. 50078	0. 56700	0. 42748	0. 62000	0. 36463
0. 46200	0. 58487	0. 51500	0. 49929	0. 56800	0. 42621	0. 62100	0. 36353
0. 46300	0. 58312	0. 51600	0. 49781	0. 56900	0. 42493	0. 62200	0. 36244
0. 46400	0. 58138	0. 51700	0. 49632	0. 57000	0. 42366	0. 62300	0. 36135
0. 46500	0. 57965	0. 51800	0. 49485	0. 57100	0. 42240	0. 62400	0. 36026
0. 46600	0. 57792	0. 51900	0. 49337	0. 57200	0. 42114	0. 62500	0. 35918
0. 46700	0. 57619	0. 52000	0. 49190	0. 57300	0. 41988	0. 62600	0. 35810
0. 46800	0. 57447	0. 52100	0. 49044	0. 57400	0. 41862	0. 62700	0. 35702
0. 46900	0. 57276	0. 52200	0. 48898	0. 57500	0. 41737	0. 62800	0. 35594
0. 47000	0. 57105	0. 52300	0. 48752	0. 57600	0. 41612	0. 62900	0. 35487
0. 47100	0. 56935	0. 52400	0. 48607	0. 57700	0. 41488	0. 63000	0. 35380
0. 47200	0. 56765	0. 52500	0. 48462	0. 57800	0. 41364	0. 63100	0. 35274
0. 47300	0. 56596	0. 52600	0. 48317	0. 57900	0. 41240	0. 63200	0. 35167
0. 47400	0. 56427	0. 52700	0. 48173	0. 58000	0. 41117	0. 63300	0. 35061
0. 47500	0. 56259	0. 52800	0. 48030	0. 58100	0. 40994	0. 63400	0. 34956
0. 47600	0. 56091	0. 52900	0. 47887	0. 58200	0. 40871	0. 63500	0. 34850
0. 47700	0. 55924	0. 53000	0. 47744	0. 58300	0. 40749	0. 63600	0. 34745
0. 47800	0. 55757	0. 53100	0. 47602	0. 58400	0. 40627	0. 63700	0. 34640
0. 47900	0. 55591	0. 53200	0. 47460	0. 58500	0. 40505	0. 63800	0. 34536
0. 48000	0. 55425	0. 53300	0. 47319	0. 58600	0. 40384	0. 63900	0. 34432
0. 48100	0. 55260	0. 53400	0. 47178	0. 58700	0. 40263	0. 64000	0. 34328
0. 48200	0. 55095	0. 53500	0. 47037	0. 58800	0. 40143	0. 64100	0. 34224

TABLE 1.—*Values of the integral* $\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$
0. 64200	0. 34121	0. 69500	0. 29045	0. 74800	0. 24676	0. 80100	0. 20917
0. 64300	0. 34018	0. 69600	0. 28956	0. 74900	0. 24600	0. 80200	0. 20851
0. 64400	0. 33915	0. 69700	0. 28868	0. 75000	0. 24524	0. 80300	0. 20786
0. 64500	0. 33813	0. 69800	0. 28780	0. 75100	0. 24448	0. 80400	0. 20720
0. 64600	0. 33711	0. 69900	0. 28692	0. 75200	0. 24372	0. 80500	0. 20655
0. 64700	0. 33609	0. 70000	0. 28604	0. 75300	0. 24297	0. 80600	0. 20590
0. 64800	0. 33507	0. 70100	0. 28517	0. 75400	0. 24222	0. 80700	0. 20526
0. 64900	0. 33406	0. 70200	0. 28430	0. 75500	0. 24147	0. 80800	0. 20461
0. 65000	0. 33305	0. 70300	0. 28343	0. 75600	0. 24072	0. 80900	0. 20397
0. 65100	0. 33204	0. 70400	0. 28256	0. 75700	0. 23997	0. 81000	0. 20333
0. 65200	0. 33104	0. 70500	0. 28170	0. 75800	0. 23923	0. 81100	0. 20269
0. 65300	0. 33004	0. 70600	0. 28084	0. 75900	0. 23849	0. 81200	0. 20205
0. 65400	0. 32904	0. 70700	0. 27998	0. 76000	0. 23775	0. 81300	0. 20141
0. 65500	0. 32805	0. 70800	0. 27912	0. 76100	0. 23701	0. 81400	0. 20078
0. 65600	0. 32705	0. 70900	0. 27827	0. 76200	0. 23628	0. 81500	0. 20015
0. 65700	0. 32606	0. 71000	0. 27742	0. 76300	0. 23554	0. 81600	0. 19952
0. 65800	0. 32508	0. 71100	0. 27657	0. 76400	0. 23481	0. 81700	0. 19889
0. 65900	0. 32409	0. 71200	0. 27572	0. 76500	0. 23408	0. 81800	0. 19826
0. 66000	0. 32311	0. 71300	0. 27487	0. 76600	0. 23336	0. 81900	0. 19764
0. 66100	0. 32213	0. 71400	0. 27403	0. 76700	0. 23263	0. 82000	0. 19701
0. 66200	0. 32116	0. 71500	0. 27319	0. 76800	0. 23191	0. 82100	0. 19639
0. 66300	0. 32018	0. 71600	0. 27235	0. 76900	0. 23119	0. 82200	0. 19577
0. 66400	0. 31921	0. 71700	0. 27152	0. 77000	0. 23047	0. 82300	0. 19515
0. 66500	0. 31824	0. 71800	0. 27069	0. 77100	0. 22975	0. 82400	0. 19454
0. 66600	0. 31728	0. 71900	0. 26985	0. 77200	0. 22904	0. 82500	0. 19392
0. 66700	0. 31632	0. 72000	0. 26903	0. 77300	0. 22832	0. 82600	0. 19331
0. 66800	0. 31536	0. 72100	0. 26820	0. 77400	0. 22761	0. 82700	0. 19270
0. 66900	0. 31440	0. 72200	0. 26738	0. 77500	0. 22690	0. 82800	0. 19209
0. 67000	0. 31345	0. 72300	0. 26656	0. 77600	0. 22620	0. 82900	0. 19148
0. 67100	0. 31249	0. 72400	0. 26574	0. 77700	0. 22549	0. 83000	0. 19088
0. 67200	0. 31155	0. 72500	0. 26492	0. 77800	0. 22479	0. 83100	0. 19027
0. 67300	0. 31060	0. 72600	0. 26411	0. 77900	0. 22409	0. 83200	0. 18967
0. 67400	0. 30966	0. 72700	0. 26329	0. 78000	0. 22339	0. 83300	0. 18907
0. 67500	0. 30872	0. 72800	0. 26248	0. 78100	0. 22269	0. 83400	0. 18847
0. 67600	0. 30778	0. 72900	0. 26168	0. 78200	0. 22200	0. 83500	0. 18787
0. 67700	0. 30684	0. 73000	0. 26087	0. 78300	0. 22131	0. 83600	0. 18728
0. 67800	0. 30591	0. 73100	0. 26007	0. 78400	0. 22062	0. 83700	0. 18668
0. 67900	0. 30498	0. 73200	0. 25927	0. 78500	0. 21993	0. 83800	0. 18609
0. 68000	0. 30405	0. 73300	0. 25847	0. 78600	0. 21924	0. 83900	0. 18550
0. 68100	0. 30313	0. 73400	0. 25767	0. 78700	0. 21855	0. 84000	0. 18491
0. 68200	0. 30221	0. 73500	0. 25688	0. 78800	0. 21787	0. 84100	0. 18432
0. 68300	0. 30129	0. 73600	0. 25609	0. 78900	0. 21719	0. 84200	0. 18374
0. 68400	0. 30037	0. 73700	0. 25530	0. 79000	0. 21651	0. 84300	0. 18316
0. 68500	0. 29945	0. 73800	0. 25451	0. 79100	0. 21583	0. 84400	0. 18257
0. 68600	0. 29854	0. 73900	0. 25373	0. 79200	0. 21516	0. 84500	0. 18199
0. 68700	0. 29763	0. 74000	0. 25294	0. 79300	0. 21449	0. 84600	0. 18141
0. 68800	0. 29673	0. 74100	0. 25216	0. 79400	0. 21381	0. 84700	0. 18084
0. 68900	0. 29582	0. 74200	0. 25139	0. 79500	0. 21314	0. 84800	0. 18026
0. 69000	0. 29492	0. 74300	0. 25061	0. 79600	0. 21248	0. 84900	0. 17969
0. 69100	0. 29402	0. 74400	0. 24984	0. 79700	0. 21181	0. 85000	0. 17912
0. 69200	0. 29313	0. 74500	0. 24906	0. 79800	0. 21115	0. 85100	0. 17855
0. 69300	0. 29223	0. 74600	0. 24830	0. 79900	0. 21049	0. 85200	0. 17798
0. 69400	0. 29134	0. 74700	0. 24753	0. 80000	0. 20983	0. 85300	0. 17741

TABLE 1.—*Values of the integral* $\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$
0. 85400	0. 17684	0. 90700	0. 14910	0. 96000	0. 12532	1. 01300	0. 10500
0. 85500	0. 17628	0. 90800	0. 14861	0. 96100	0. 12491	1. 01400	0. 10465
0. 85600	0. 17572	0. 90900	0. 14813	0. 96200	0. 12450	1. 01500	0. 10430
0. 85700	0. 17516	0. 91000	0. 14765	0. 96300	0. 12408	1. 01600	0. 10395
0. 85800	0. 17460	0. 91100	0. 14717	0. 96400	0. 12367	1. 01700	0. 10359
0. 85900	0. 17404	0. 91200	0. 14669	0. 96500	0. 12327	1. 01800	0. 10325
0. 86000	0. 17349	0. 91300	0. 14622	0. 96600	0. 12286	1. 01900	0. 10290
0. 86100	0. 17293	0. 91400	0. 14574	0. 96700	0. 12245	1. 02000	0. 10255
0. 86200	0. 17238	0. 91500	0. 14527	0. 96800	0. 12205	1. 02100	0. 10221
0. 86300	0. 17183	0. 91600	0. 14479	0. 96900	0. 12164	1. 02200	0. 10186
0. 86400	0. 17128	0. 91700	0. 14432	0. 97000	0. 12124	1. 02300	0. 10152
0. 86500	0. 17073	0. 91800	0. 14385	0. 97100	0. 12084	1. 02400	0. 10117
0. 86600	0. 17018	0. 91900	0. 14339	0. 97200	0. 12044	1. 02500	0. 10083
0. 86700	0. 16964	0. 92000	0. 14292	0. 97300	0. 12004	1. 02600	0. 10049
0. 86800	0. 16910	0. 92100	0. 14245	0. 97400	0. 11964	1. 02700	0. 10015
0. 86900	0. 16855	0. 92200	0. 14199	0. 97500	0. 11924	1. 02800	0. 09981
0. 87000	0. 16801	0. 92300	0. 14153	0. 97600	0. 11885	1. 02900	0. 09948
0. 87100	0. 16748	0. 92400	0. 14106	0. 97700	0. 11845	1. 03000	0. 09914
0. 87200	0. 16694	0. 92500	0. 14060	0. 97800	0. 11806	1. 03100	0. 09880
0. 87300	0. 16640	0. 92600	0. 14015	0. 97900	0. 11767	1. 03200	0. 09847
0. 87400	0. 16587	0. 92700	0. 13969	0. 98000	0. 11727	1. 03300	0. 09814
0. 87500	0. 16534	0. 92800	0. 13923	0. 98100	0. 11688	1. 03400	0. 09780
0. 87600	0. 16481	0. 92900	0. 13878	0. 98200	0. 11650	1. 03500	0. 09747
0. 87700	0. 16428	0. 93000	0. 13832	0. 98300	0. 11611	1. 03600	0. 09714
0. 87800	0. 16375	0. 93100	0. 13787	0. 98400	0. 11572	1. 03700	0. 09681
0. 87900	0. 16322	0. 93200	0. 13742	0. 98500	0. 11534	1. 03800	0. 09648
0. 88000	0. 16270	0. 93300	0. 13697	0. 98600	0. 11495	1. 03900	0. 09616
0. 88100	0. 16218	0. 93400	0. 13652	0. 98700	0. 11457	1. 04000	0. 09583
0. 88200	0. 16166	0. 93500	0. 13608	0. 98800	0. 11419	1. 04100	0. 09550
0. 88300	0. 16114	0. 93600	0. 13563	0. 98900	0. 11381	1. 04200	0. 09518
0. 88400	0. 16062	0. 93700	0. 13519	0. 99000	0. 11343	1. 04300	0. 09486
0. 88500	0. 16010	0. 93800	0. 13474	0. 99100	0. 11305	1. 04400	0. 09453
0. 88600	0. 15958	0. 93900	0. 13430	0. 99200	0. 11267	1. 04500	0. 09421
0. 88700	0. 15907	0. 94000	0. 13386	0. 99300	0. 11229	1. 04600	0. 09389
0. 88800	0. 15856	0. 94100	0. 13342	0. 99400	0. 11192	1. 04700	0. 09357
0. 88900	0. 15805	0. 94200	0. 13298	0. 99500	0. 11155	1. 04800	0. 09325
0. 89000	0. 15754	0. 94300	0. 13255	0. 99600	0. 11117	1. 04900	0. 09293
0. 89100	0. 15703	0. 94400	0. 13211	0. 99700	0. 11080	1. 05000	0. 09262
0. 89200	0. 15652	0. 94500	0. 13168	0. 99800	0. 11043	1. 05100	0. 09230
0. 89300	0. 15602	0. 94600	0. 13125	0. 99900	0. 11006	1. 05200	0. 09199
0. 89400	0. 15551	0. 94700	0. 13082	1. 00000	0. 10969	1. 05300	0. 09167
0. 89500	0. 15501	0. 94800	0. 13038	1. 00100	0. 10932	1. 05400	0. 09136
0. 89600	0. 15451	0. 94900	0. 12996	1. 00200	0. 10896	1. 05500	0. 09105
0. 89700	0. 15401	0. 95000	0. 12953	1. 00300	0. 10859	1. 05600	0. 09074
0. 89800	0. 15351	0. 95100	0. 12910	1. 00400	0. 10823	1. 05700	0. 09043
0. 89900	0. 15302	0. 95200	0. 12868	1. 00500	0. 10787	1. 05800	0. 09012
0. 90000	0. 15252	0. 95300	0. 12825	1. 00600	0. 10750	1. 05900	0. 08981
0. 90100	0. 15203	0. 95400	0. 12783	1. 00700	0. 10714	1. 06000	0. 08950
0. 90200	0. 15154	0. 95500	0. 12741	1. 00800	0. 10678	1. 06100	0. 08920
0. 90300	0. 15104	0. 95600	0. 12699	1. 00900	0. 10643	1. 06200	0. 08889
0. 90400	0. 15056	0. 95700	0. 12657	1. 01000	0. 10607	1. 06300	0. 08859
0. 90500	0. 15007	0. 95800	0. 12615	1. 01100	0. 10571	1. 06400	0. 08828
0. 90600	0. 14958	0. 95900	0. 12574	1. 01200	0. 10536	1. 06500	0. 08798

TABLE 1.—*Values of the integral* $\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$
1. 06600	0. 08768	1. 11900	0. 07296	1. 17200	0. 06049	1. 22500	0. 04996
1. 06700	0. 08738	1. 12000	0. 07270	1. 17300	0. 06027	1. 22600	0. 04978
1. 06800	0. 08708	1. 12100	0. 07245	1. 17400	0. 06006	1. 22700	0. 04960
1. 06900	0. 08678	1. 12200	0. 07220	1. 17500	0. 05984	1. 22800	0. 04942
1. 07000	0. 08648	1. 12300	0. 07194	1. 17600	0. 05963	1. 22900	0. 04924
1. 07100	0. 08619	1. 12400	0. 07169	1. 17700	0. 05942	1. 23000	0. 04906
1. 07200	0. 08589	1. 12500	0. 07144	1. 17800	0. 05921	1. 23100	0. 04888
1. 07300	0. 08559	1. 12600	0. 07119	1. 17900	0. 05899	1. 23200	0. 04870
1. 07400	0. 08530	1. 12700	0. 07094	1. 18000	0. 05878	1. 23300	0. 04853
1. 07500	0. 08501	1. 12800	0. 07069	1. 18100	0. 05857	1. 23400	0. 04835
1. 07600	0. 08471	1. 12900	0. 07044	1. 18200	0. 05836	1. 23500	0. 04817
1. 07700	0. 08442	1. 13000	0. 07020	1. 18300	0. 05815	1. 23600	0. 04800
1. 07800	0. 08413	1. 13100	0. 06995	1. 18400	0. 05795	1. 23700	0. 04782
1. 07900	0. 08384	1. 13200	0. 06970	1. 18500	0. 05774	1. 23800	0. 04765
1. 08000	0. 08355	1. 13300	0. 06946	1. 18600	0. 05753	1. 23900	0. 04747
1. 08100	0. 08327	1. 13400	0. 06922	1. 18700	0. 05733	1. 24000	0. 04730
1. 08200	0. 08298	1. 13500	0. 06897	1. 18800	0. 05712	1. 24100	0. 04713
1. 08300	0. 08269	1. 13600	0. 06873	1. 18900	0. 05692	1. 24200	0. 04695
1. 08400	0. 08241	1. 13700	0. 06849	1. 19000	0. 05671	1. 24300	0. 04678
1. 08500	0. 08212	1. 13800	0. 06825	1. 19100	0. 05651	1. 24400	0. 04661
1. 08600	0. 08184	1. 13900	0. 06801	1. 19200	0. 05630	1. 24500	0. 04644
1. 08700	0. 08156	1. 14000	0. 06777	1. 19300	0. 05610	1. 24600	0. 04627
1. 08800	0. 08127	1. 14100	0. 06753	1. 19400	0. 05590	1. 24700	0. 04610
1. 08900	0. 08099	1. 14200	0. 06729	1. 19500	0. 05570	1. 24800	0. 04593
1. 09000	0. 08071	1. 14300	0. 06705	1. 19600	0. 05550	1. 24900	0. 04576
1. 09100	0. 08043	1. 14400	0. 06682	1. 19700	0. 05530	1. 25000	0. 04559
1. 09200	0. 08016	1. 14500	0. 06658	1. 19800	0. 05510	1. 25100	0. 04543
1. 09300	0. 07988	1. 14600	0. 06635	1. 19900	0. 05490	1. 25200	0. 04526
1. 09400	0. 07960	1. 14700	0. 06611	1. 20000	0. 05470	1. 25300	0. 04509
1. 09500	0. 07933	1. 14800	0. 06588	1. 20100	0. 05451	1. 25400	0. 04493
1. 09600	0. 07905	1. 14900	0. 06565	1. 20200	0. 05431	1. 25500	0. 04476
1. 09700	0. 07878	1. 15000	0. 06541	1. 20300	0. 05411	1. 25600	0. 04460
1. 09800	0. 07850	1. 15100	0. 06518	1. 20400	0. 05392	1. 25700	0. 04443
1. 09900	0. 07823	1. 15200	0. 06495	1. 20500	0. 05373	1. 25800	0. 04427
1. 10000	0. 07796	1. 15300	0. 06472	1. 20600	0. 05353	1. 25900	0. 04411
1. 10100	0. 07769	1. 15400	0. 06449	1. 20700	0. 05334	1. 26000	0. 04394
1. 10200	0. 07742	1. 15500	0. 06426	1. 20800	0. 05315	1. 26100	0. 04378
1. 10300	0. 07715	1. 15600	0. 06404	1. 20900	0. 05295	1. 26200	0. 04362
1. 10400	0. 07688	1. 15700	0. 06381	1. 21000	0. 05276	1. 26300	0. 04346
1. 10500	0. 07662	1. 15800	0. 06358	1. 21100	0. 05257	1. 26400	0. 04330
1. 10600	0. 07635	1. 15900	0. 06336	1. 21200	0. 05238	1. 26500	0. 04314
1. 10700	0. 07608	1. 16000	0. 06313	1. 21300	0. 05219	1. 26600	0. 04298
1. 10800	0. 07582	1. 16100	0. 06291	1. 21400	0. 05200	1. 26700	0. 04282
1. 10900	0. 07555	1. 16200	0. 06268	1. 21500	0. 05181	1. 26800	0. 04266
1. 11000	0. 07529	1. 16300	0. 06246	1. 21600	0. 05163	1. 26900	0. 04251
1. 11100	0. 07503	1. 16400	0. 06224	1. 21700	0. 05144	1. 27000	0. 04235
1. 11200	0. 07477	1. 16500	0. 06202	1. 21800	0. 05125	1. 27100	0. 04219
1. 11300	0. 07451	1. 16600	0. 06180	1. 21900	0. 05107	1. 27200	0. 04204
1. 11400	0. 07425	1. 16700	0. 06158	1. 22000	0. 05088	1. 27300	0. 04188
1. 11500	0. 07399	1. 16800	0. 06136	1. 22100	0. 05070	1. 27400	0. 04173
1. 11600	0. 07373	1. 16900	0. 06114	1. 22200	0. 05051	1. 27500	0. 04157
1. 11700	0. 07347	1. 17000	0. 06092	1. 22300	0. 05033	1. 27600	0. 04142
1. 11800	0. 07322	1. 17100	0. 06071	1. 22400	0. 05015	1. 27700	0. 04126

TABLE 1.—*Values of the integral* $\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$
1. 27800	0. 04111	1. 33100	0. 03369	1. 38400	0. 02750	1. 43700	0. 02236
1. 27900	0. 04096	1. 33200	0. 03356	1. 38500	0. 02739	1. 43800	0. 02227
1. 28000	0. 04081	1. 33300	0. 03344	1. 38600	0. 02729	1. 43900	0. 02218
1. 28100	0. 04065	1. 33400	0. 03331	1. 38700	0. 02718	1. 44000	0. 02209
1. 28200	0. 04050	1. 33500	0. 03318	1. 38800	0. 02708	1. 44100	0. 02200
1. 28300	0. 04035	1. 33600	0. 03306	1. 38900	0. 02697	1. 44200	0. 02192
1. 28400	0. 04020	1. 33700	0. 03293	1. 39000	0. 02687	1. 44300	0. 02183
1. 28500	0. 04005	1. 33800	0. 03281	1. 39100	0. 02676	1. 44400	0. 02175
1. 28600	0. 03990	1. 33900	0. 03268	1. 39200	0. 02666	1. 44500	0. 02166
1. 28700	0. 03975	1. 34000	0. 03256	1. 39300	0. 02656	1. 44600	0. 02157
1. 28800	0. 03961	1. 34100	0. 03244	1. 39400	0. 02646	1. 44700	0. 02149
1. 28900	0. 03946	1. 34200	0. 03231	1. 39500	0. 02635	1. 44800	0. 02140
1. 29000	0. 03931	1. 34300	0. 03219	1. 39600	0. 02625	1. 44900	0. 02132
1. 29100	0. 03917	1. 34400	0. 03207	1. 39700	0. 02615	1. 45000	0. 02123
1. 29200	0. 03902	1. 34500	0. 03194	1. 39800	0. 02605	1. 45100	0. 02115
1. 29300	0. 03887	1. 34600	0. 03182	1. 39900	0. 02595	1. 45200	0. 02107
1. 29400	0. 03873	1. 34700	0. 03170	1. 40000	0. 02584	1. 45300	0. 02098
1. 29500	0. 03858	1. 34800	0. 03158	1. 40100	0. 02574	1. 45400	0. 02090
1. 29600	0. 03844	1. 34900	0. 03146	1. 40200	0. 02564	1. 45500	0. 02082
1. 29700	0. 03830	1. 35000	0. 03134	1. 40300	0. 02554	1. 45600	0. 02073
1. 29800	0. 03815	1. 35100	0. 03122	1. 40400	0. 02545	1. 45700	0. 02065
1. 29900	0. 03801	1. 35200	0. 03110	1. 40500	0. 02535	1. 45800	0. 02057
1. 30000	0. 03787	1. 35300	0. 03098	1. 40600	0. 02525	1. 45900	0. 02049
1. 30100	0. 03773	1. 35400	0. 03087	1. 40700	0. 02515	1. 46000	0. 02041
1. 30200	0. 03759	1. 35500	0. 03075	1. 40800	0. 02505	1. 46100	0. 02033
1. 30300	0. 03745	1. 35600	0. 03063	1. 40900	0. 02495	1. 46200	0. 02024
1. 30400	0. 03730	1. 35700	0. 03051	1. 41000	0. 02486	1. 46300	0. 02016
1. 30500	0. 03717	1. 35800	0. 03040	1. 41100	0. 02476	1. 46400	0. 02008
1. 30600	0. 03703	1. 35900	0. 03028	1. 41200	0. 02466	1. 46500	0. 02000
1. 30700	0. 03689	1. 36000	0. 03016	1. 41300	0. 02457	1. 46600	0. 01992
1. 30800	0. 03675	1. 36100	0. 03005	1. 41400	0. 02447	1. 46700	0. 01985
1. 30900	0. 03661	1. 36200	0. 02993	1. 41500	0. 02437	1. 46800	0. 01977
1. 31000	0. 03647	1. 36300	0. 02982	1. 41600	0. 02428	1. 46900	0. 01969
1. 31100	0. 03634	1. 36400	0. 02970	1. 41700	0. 02418	1. 47000	0. 01961
1. 31200	0. 03620	1. 36500	0. 02959	1. 41800	0. 02409	1. 47100	0. 01953
1. 31300	0. 03606	1. 36600	0. 02948	1. 41900	0. 02400	1. 47200	0. 01945
1. 31400	0. 03593	1. 36700	0. 02936	1. 42000	0. 02390	1. 47300	0. 01937
1. 31500	0. 03579	1. 36800	0. 02925	1. 42100	0. 02381	1. 47400	0. 01930
1. 31600	0. 03566	1. 36900	0. 02914	1. 42200	0. 02372	1. 47500	0. 01922
1. 31700	0. 03552	1. 37000	0. 02903	1. 42300	0. 02362	1. 47600	0. 01914
1. 31800	0. 03539	1. 37100	0. 02892	1. 42400	0. 02353	1. 47700	0. 01907
1. 31900	0. 03526	1. 37200	0. 02880	1. 42500	0. 02344	1. 47800	0. 01899
1. 32000	0. 03512	1. 37300	0. 02869	1. 42600	0. 02335	1. 47900	0. 01891
1. 32100	0. 03499	1. 37400	0. 02858	1. 42700	0. 02325	1. 48000	0. 01884
1. 32200	0. 03486	1. 37500	0. 02847	1. 42800	0. 02316	1. 48100	0. 01876
1. 32300	0. 03473	1. 37600	0. 02836	1. 42900	0. 02307	1. 48200	0. 01869
1. 32400	0. 03460	1. 37700	0. 02825	1. 43000	0. 02298	1. 48300	0. 01861
1. 32500	0. 03447	1. 37800	0. 02815	1. 43100	0. 02289	1. 48400	0. 01854
1. 32600	0. 03434	1. 37900	0. 02804	1. 43200	0. 02280	1. 48500	0. 01846
1. 32700	0. 03421	1. 38000	0. 02793	1. 43300	0. 02271	1. 48600	0. 01839
1. 32800	0. 03408	1. 38100	0. 02782	1. 43400	0. 02262	1. 48700	0. 01832
1. 32900	0. 03395	1. 38200	0. 02771	1. 43500	0. 02253	1. 48800	0. 01824
1. 33000	0. 03382	1. 38300	0. 02761	1. 43600	0. 02244	1. 48900	0. 01817

TABLE 1.—*Values of the integral* $\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$
1.49000	0.01810	1.54300	0.01459	1.59600	0.01171	1.64900	0.00935
1.49100	0.01802	1.54400	0.01453	1.59700	0.01166	1.65000	0.00932
1.49200	0.01795	1.54500	0.01447	1.59800	0.01161	1.65100	0.00928
1.49300	0.01788	1.54600	0.01441	1.59900	0.01156	1.65200	0.00924
1.49400	0.01781	1.54700	0.01435	1.60000	0.01151	1.65300	0.00920
1.49500	0.01774	1.54800	0.01429	1.60100	0.01146	1.65400	0.00916
1.49600	0.01766	1.54900	0.01423	1.60200	0.01142	1.65500	0.00912
1.49700	0.01759	1.55000	0.01417	1.60300	0.01137	1.65600	0.00908
1.49800	0.01752	1.55100	0.01411	1.60400	0.01132	1.65700	0.00904
1.49900	0.01745	1.55200	0.01406	1.60500	0.01127	1.65800	0.00900
1.50000	0.01738	1.55300	0.01400	1.60600	0.01123	1.65900	0.00896
1.50100	0.01731	1.55400	0.01394	1.60700	0.01118	1.66000	0.00892
1.50200	0.01724	1.55500	0.01388	1.60800	0.01113	1.66100	0.00889
1.50300	0.01717	1.55600	0.01383	1.60900	0.01109	1.66200	0.00885
1.50400	0.01710	1.55700	0.01377	1.61000	0.01104	1.66300	0.00881
1.50500	0.01703	1.55800	0.01371	1.61100	0.01099	1.66400	0.00877
1.50600	0.01696	1.55900	0.01366	1.61200	0.01095	1.66500	0.00873
1.50700	0.01690	1.56000	0.01360	1.61300	0.01090	1.66600	0.00870
1.50800	0.01683	1.56100	0.01354	1.61400	0.01085	1.66700	0.00866
1.50900	0.01676	1.56200	0.01349	1.61500	0.01081	1.66800	0.00862
1.51000	0.01669	1.56300	0.01343	1.61600	0.01076	1.66900	0.00859
1.51100	0.01662	1.56400	0.01338	1.61700	0.01072	1.67000	0.00855
1.51200	0.01656	1.56500	0.01332	1.61800	0.01067	1.67100	0.00851
1.51300	0.01649	1.56600	0.01327	1.61900	0.01063	1.67200	0.00848
1.51400	0.01642	1.56700	0.01321	1.62000	0.01058	1.67300	0.00844
1.51500	0.01636	1.56800	0.01316	1.62100	0.01054	1.67400	0.00840
1.51600	0.01629	1.56900	0.01310	1.62200	0.01049	1.67500	0.00837
1.51700	0.01622	1.57000	0.01305	1.62300	0.01045	1.67600	0.00833
1.51800	0.01616	1.57100	0.01299	1.62400	0.01040	1.67700	0.00829
1.51900	0.01609	1.57200	0.01294	1.62500	0.01036	1.67800	0.00826
1.52000	0.01603	1.57300	0.01289	1.62600	0.01032	1.67900	0.00822
1.52100	0.01596	1.57400	0.01283	1.62700	0.01027	1.68000	0.00819
1.52200	0.01590	1.57500	0.01278	1.62800	0.01023	1.68100	0.00815
1.52300	0.01583	1.57600	0.01273	1.62900	0.01019	1.68200	0.00812
1.52400	0.01577	1.57700	0.01267	1.63000	0.01014	1.68300	0.00808
1.52500	0.01570	1.57800	0.01262	1.63100	0.01010	1.68400	0.00805
1.52600	0.01564	1.57900	0.01257	1.63200	0.01006	1.68500	0.00801
1.52700	0.01557	1.58000	0.01252	1.63300	0.01001	1.68600	0.00798
1.52800	0.01551	1.58100	0.01246	1.63400	0.00997	1.68700	0.00794
1.52900	0.01545	1.58200	0.01241	1.63500	0.00993	1.68800	0.00791
1.53000	0.01539	1.58300	0.01236	1.63600	0.00989	1.68900	0.00787
1.53100	0.01532	1.58400	0.01231	1.63700	0.00985	1.69000	0.00784
1.53200	0.01526	1.58500	0.01226	1.63800	0.00980	1.69100	0.00781
1.53300	0.01520	1.58600	0.01221	1.63900	0.00976	1.69200	0.00777
1.53400	0.01514	1.58700	0.01216	1.64000	0.00972	1.69300	0.00774
1.53500	0.01507	1.58800	0.01211	1.64100	0.00968	1.69400	0.00771
1.53600	0.01501	1.58900	0.01206	1.64200	0.00964	1.69500	0.00767
1.53700	0.01495	1.59000	0.01200	1.64300	0.00960	1.69600	0.00764
1.53800	0.01489	1.59100	0.01195	1.64400	0.00956	1.69700	0.00761
1.53900	0.01483	1.59200	0.01190	1.64500	0.00952	1.69800	0.00757
1.54000	0.01477	1.59300	0.01186	1.64600	0.00948	1.69900	0.00754
1.54100	0.01471	1.59400	0.01181	1.64700	0.00944	1.70000	0.00751
1.54200	0.01465	1.59500	0.01176	1.64800	0.00939	1.70100	0.00747

TABLE 1.—*Values of the integral* $\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$
1.70200	0.00744	1.75500	0.00589	1.80800	0.00465	1.86100	0.00365
1.70300	0.00741	1.75600	0.00587	1.80900	0.00463	1.86200	0.00363
1.70400	0.00738	1.75700	0.00584	1.81000	0.00461	1.86300	0.00361
1.70500	0.00735	1.75800	0.00582	1.81100	0.00458	1.86400	0.00360
1.70600	0.00731	1.75900	0.00579	1.81200	0.00456	1.86500	0.00358
1.70700	0.00728	1.76000	0.00576	1.81300	0.00454	1.86600	0.00356
1.70800	0.00725	1.76100	0.00574	1.81400	0.00452	1.86700	0.00355
1.70900	0.00722	1.76200	0.00571	1.81500	0.00450	1.86800	0.00353
1.71000	0.00719	1.76300	0.00569	1.81600	0.00448	1.86900	0.00351
1.71100	0.00716	1.76400	0.00566	1.81700	0.00446	1.87000	0.00350
1.71200	0.00712	1.76500	0.00564	1.81800	0.00444	1.87100	0.00348
1.71300	0.00709	1.76600	0.00561	1.81900	0.00442	1.87200	0.00347
1.71400	0.00706	1.76700	0.00559	1.82000	0.00440	1.87300	0.00345
1.71500	0.00703	1.76800	0.00556	1.82100	0.00438	1.87400	0.00343
1.71600	0.00700	1.76900	0.00554	1.82200	0.00436	1.87500	0.00342
1.71700	0.00697	1.77000	0.00551	1.82300	0.00434	1.87600	0.00340
1.71800	0.00694	1.77100	0.00549	1.82400	0.00432	1.87700	0.00339
1.71900	0.00691	1.77200	0.00546	1.82500	0.00430	1.87800	0.00337
1.72000	0.00688	1.77300	0.00544	1.82600	0.00428	1.87900	0.00336
1.72100	0.00685	1.77400	0.00542	1.82700	0.00426	1.88000	0.00334
1.72200	0.00682	1.77500	0.00539	1.82800	0.00424	1.88100	0.00332
1.72300	0.00679	1.77600	0.00537	1.82900	0.00422	1.88200	0.00331
1.72400	0.00776	1.77700	0.00534	1.83000	0.00420	1.88300	0.00329
1.72500	0.00673	1.77800	0.00532	1.83100	0.00419	1.88400	0.00328
1.72600	0.00670	1.77900	0.00530	1.83200	0.00417	1.88500	0.00326
1.72700	0.00667	1.78000	0.00527	1.83300	0.00415	1.88600	0.00325
1.72800	0.00664	1.78100	0.00525	1.83400	0.00413	1.88700	0.00323
1.72900	0.00661	1.78200	0.00522	1.83500	0.00411	1.88800	0.00322
1.73000	0.00658	1.78300	0.00520	1.83600	0.00409	1.88900	0.00320
1.73100	0.00655	1.78400	0.00518	1.83700	0.00407	1.89000	0.00319
1.73200	0.00653	1.78500	0.00515	1.83800	0.00405	1.89100	0.00317
1.73300	0.00650	1.78600	0.00513	1.83900	0.00404	1.89200	0.00316
1.73400	0.00647	1.78700	0.00511	1.84000	0.00402	1.89300	0.00314
1.73500	0.00644	1.78800	0.00509	1.84100	0.00400	1.89400	0.00313
1.73600	0.00641	1.78900	0.00506	1.84200	0.00398	1.89500	0.00311
1.73700	0.00638	1.79000	0.00504	1.84300	0.00396	1.89600	0.00310
1.73800	0.00636	1.79100	0.00502	1.84400	0.00394	1.89700	0.00309
1.73900	0.00633	1.79200	0.00500	1.84500	0.00393	1.89800	0.00307
1.74000	0.00630	1.79300	0.00497	1.84600	0.00391	1.89900	0.00306
1.74100	0.00627	1.79400	0.00495	1.84700	0.00389	1.90000	0.00304
1.74200	0.00624	1.79500	0.00493	1.84800	0.00387	1.90100	0.00303
1.74300	0.00622	1.79600	0.00491	1.84900	0.00385	1.90200	0.00301
1.74400	0.00619	1.79700	0.00488	1.85000	0.00384	1.90300	0.00300
1.74500	0.00616	1.79800	0.00486	1.85100	0.00382	1.90400	0.00299
1.74600	0.00613	1.79900	0.00484	1.85200	0.00380	1.90500	0.00297
1.74700	0.00611	1.80000	0.00482	1.85300	0.00378	1.90600	0.00296
1.74800	0.00608	1.80100	0.00480	1.85400	0.00377	1.90700	0.00294
1.74900	0.00605	1.80200	0.00477	1.85500	0.00375	1.90800	0.00293
1.75000	0.00603	1.80300	0.00475	1.85600	0.00373	1.90900	0.00292
1.75100	0.00600	1.80400	0.00473	1.85700	0.00371	1.91000	0.00290
1.75200	0.00597	1.80500	0.00471	1.85800	0.00370	1.91100	0.00289
1.75300	0.00595	1.80600	0.00469	1.85900	0.00368	1.91200	0.00288
1.75400	0.00592	1.80700	0.00467	1.86000	0.00366	1.91300	0.00286

TABLE 1.—*Values of the integral* $\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\, du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\, du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\, du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\, du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\, du$
1.91400	0.00285	1.96700	0.00222	2.02000	0.00171	2.07300	0.00132
1.91500	0.00284	1.96800	0.00220	2.02100	0.00171	2.07400	0.00131
1.91600	0.00282	1.96900	0.00219	2.02200	0.00170	2.07500	0.00131
1.91700	0.00281	1.97000	0.00218	2.02300	0.00169	2.07600	0.00130
1.91800	0.00280	1.97100	0.00217	2.02400	0.00168	2.07700	0.00129
1.91900	0.00278	1.97200	0.00216	2.02500	0.00167	2.07800	0.00129
1.92000	0.00277	1.97300	0.00215	2.02600	0.00166	2.07900	0.00128
1.92100	0.00276	1.97400	0.00214	2.02700	0.00166	2.08000	0.00128
1.92200	0.00274	1.97500	0.00213	2.02800	0.00165	2.08100	0.00127
1.92300	0.00273	1.97600	0.00212	2.02900	0.00164	2.08200	0.00126
1.92400	0.00272	1.97700	0.00211	2.03000	0.00163	2.08300	0.00126
1.92500	0.00271	1.97800	0.00210	2.03100	0.00162	2.08400	0.00125
1.92600	0.00269	1.97900	0.00209	2.03200	0.00162	2.08500	0.00124
1.92700	0.00268	1.98000	0.00208	2.03300	0.00161	2.08600	0.00124
1.92800	0.00267	1.98100	0.00207	2.03400	0.00160	2.08700	0.00123
1.92900	0.00265	1.98200	0.00206	2.03500	0.00159	2.08800	0.00123
1.93000	0.00264	1.98300	0.00205	2.03600	0.00159	2.08900	0.00122
1.93100	0.00263	1.98400	0.00204	2.03700	0.00158	2.09000	0.00121
1.93200	0.00262	1.98500	0.00203	2.03800	0.00157	2.09100	0.00121
1.93300	0.00260	1.98600	0.00202	2.03900	0.00156	2.09200	0.00120
1.93400	0.00259	1.98700	0.00201	2.04000	0.00155	2.09300	0.00120
1.93500	0.00258	1.98800	0.00200	2.04100	0.00155	2.09400	0.00119
1.93600	0.00257	1.98900	0.00199	2.04200	0.00154	2.09500	0.00118
1.93700	0.00256	1.99000	0.00198	2.04300	0.00153	2.09600	0.00118
1.93800	0.00254	1.99100	0.00197	2.04400	0.00152	2.09700	0.00117
1.93900	0.00253	1.99200	0.00196	2.04500	0.00152	2.09800	0.00117
1.94000	0.00252	1.99300	0.00195	2.04600	0.00151	2.09900	0.00116
1.94100	0.00251	1.99400	0.00195	2.04700	0.00150	2.10000	0.00115
1.94200	0.00250	1.99500	0.00194	2.04800	0.00149	2.10100	0.00115
1.94300	0.00248	1.99600	0.00193	2.04900	0.00149	2.10200	0.00114
1.94400	0.00247	1.99700	0.00192	2.05000	0.00148	2.10300	0.00114
1.94500	0.00246	1.99800	0.00191	2.05100	0.00147	2.10400	0.00113
1.94600	0.00245	1.99900	0.00190	2.05200	0.00147	2.10500	0.00113
1.94700	0.00244	2.00000	0.00189	2.05300	0.00146	2.10600	0.00112
1.94800	0.00243	2.00100	0.00188	2.05400	0.00145	2.10700	0.00111
1.94900	0.00241	2.00200	0.00187	2.05500	0.00144	2.10800	0.00111
1.95000	0.00240	2.00300	0.00186	2.05600	0.00144	2.10900	0.00110
1.95100	0.00239	2.00400	0.00185	2.05700	0.00143	2.11000	0.00110
1.95200	0.00238	2.00500	0.00184	2.05800	0.00142	2.11100	0.00109
1.95300	0.00237	2.00600	0.00184	2.05900	0.00142	2.11200	0.00109
1.95400	0.00236	2.00700	0.00183	2.06000	0.00141	2.11300	0.00108
1.95500	0.00235	2.00800	0.00182	2.06100	0.00140	2.11400	0.00108
1.95600	0.00233	2.00900	0.00181	2.06200	0.00139	2.11500	0.00107
1.95700	0.00232	2.01000	0.00180	2.06300	0.00139	2.11600	0.00106
1.95800	0.00231	2.01100	0.00179	2.06400	0.00138	2.11700	0.00106
1.95900	0.00230	2.01200	0.00178	2.06500	0.00137	2.11800	0.00105
1.96000	0.00229	2.01300	0.00177	2.06600	0.00137	2.11900	0.00105
1.96100	0.00228	2.01400	0.00177	2.06700	0.00136	2.12000	0.00104
1.96200	0.00227	2.01500	0.00176	2.06800	0.00135	2.12100	0.00104
1.96300	0.00226	2.01600	0.00175	2.06900	0.00135	2.12200	0.00103
1.96400	0.00225	2.01700	0.00174	2.07000	0.00134	2.12300	0.00103
1.96500	0.00224	2.01800	0.00173	2.07100	0.00133	2.12400	0.00102
1.96600	0.00223	2.01900	0.00172	2.07200	0.00133	2.12500	0.00102

TABLE 1.—*Values of the integral* $\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}du$
2. 12600	0. 00101	2. 17900	0. 00077	2. 23200	0. 00059	2. 28500	0. 00044
2. 12700	0. 00101	2. 18000	0. 00077	2. 23300	0. 00058	2. 28600	0. 00044
2. 12800	0. 00100	2. 18100	0. 00076	2. 23400	0. 00058	2. 28700	0. 00044
2. 12900	0. 00100	2. 18200	0. 00076	2. 23500	0. 00058	2. 28800	0. 00044
2. 13000	0. 00099	2. 18300	0. 00076	2. 23600	0. 00057	2. 28900	0. 00043
2. 13100	0. 00099	2. 18400	0. 00075	2. 23700	0. 00057	2. 29000	0. 00043
2. 13200	0. 00098	2. 18500	0. 00075	2. 23800	0. 00057	2. 29100	0. 00043
2. 13300	0. 00098	2. 18600	0. 00074	2. 23900	0. 00057	2. 29200	0. 00043
2. 13400	0. 00097	2. 18700	0. 00074	2. 24000	0. 00056	2. 29300	0. 00042
2. 13500	0. 00097	2. 18800	0. 00074	2. 24100	0. 00056	2. 29400	0. 00042
2. 13600	0. 00096	2. 18900	0. 00073	2. 24200	0. 00056	2. 29500	0. 00042
2. 13700	0. 00096	2. 19000	0. 00073	2. 24300	0. 00055	2. 29600	0. 00042
2. 13800	0. 00095	2. 19100	0. 00073	2. 24400	0. 00055	2. 29700	0. 00042
2. 13900	0. 00095	2. 19200	0. 00072	2. 24500	0. 00055	2. 29800	0. 00041
2. 14000	0. 00094	2. 19300	0. 00072	2. 24600	0. 00054	2. 29900	0. 00041
2. 14100	0. 00094	2. 19400	0. 00071	2. 24700	0. 00054	2. 30000	0. 00041
2. 14200	0. 00093	2. 19500	0. 00071	2. 24800	0. 00054	2. 30100	0. 00041
2. 14300	0. 00093	2. 19600	0. 00071	2. 24900	0. 00054	2. 30200	0. 00040
2. 14400	0. 00092	2. 19700	0. 00070	2. 25000	0. 00053	2. 30300	0. 00040
2. 14500	0. 00092	2. 19800	0. 00070	2. 25100	0. 00053	2. 30400	0. 00040
2. 14600	0. 00091	2. 19900	0. 00070	2. 25200	0. 00053	2. 30500	0. 00040
2. 14700	0. 00091	2. 20000	0. 00069	2. 25300	0. 00052	2. 30600	0. 00040
2. 14800	0. 00091	2. 20100	0. 00069	2. 25400	0. 00052	2. 30700	0. 00039
2. 14900	0. 00090	2. 20200	0. 00069	2. 25500	0. 00052	2. 30800	0. 00039
2. 15000	0. 00090	2. 20300	0. 00068	2. 25600	0. 00052	2. 30900	0. 00039
2. 15100	0. 00089	2. 20400	0. 00068	2. 25700	0. 00051	2. 31000	0. 00039
2. 15200	0. 00089	2. 20500	0. 00068	2. 25800	0. 00051	2. 31100	0. 00039
2. 15300	0. 00088	2. 20600	0. 00067	2. 25900	0. 00051	2. 31200	0. 00038
2. 15400	0. 00088	2. 20700	0. 00067	2. 26000	0. 00051	2. 31300	0. 00038
2. 15500	0. 00087	2. 20800	0. 00066	2. 26100	0. 00050	2. 31400	0. 00038
2. 15600	0. 00087	2. 20900	0. 00066	2. 26200	0. 00050	2. 31500	0. 00038
2. 15700	0. 00086	2. 21000	0. 00066	2. 26300	0. 00050	2. 31600	0. 00038
2. 15800	0. 00086	2. 21100	0. 00065	2. 26400	0. 00050	2. 31700	0. 00037
2. 15900	0. 00086	2. 21200	0. 00065	2. 26500	0. 00049	2. 31800	0. 00037
2. 16000	0. 00085	2. 21300	0. 00065	2. 26600	0. 00049	2. 31900	0. 00037
2. 16100	0. 00085	2. 21400	0. 00064	2. 26700	0. 00049	2. 32000	0. 00037
2. 16200	0. 00084	2. 21500	0. 00064	2. 26800	0. 00048	2. 32100	0. 00037
2. 16300	0. 00084	2. 21600	0. 00064	2. 26900	0. 00048	2. 32200	0. 00036
2. 16400	0. 00083	2. 21700	0. 00063	2. 27000	0. 00048	2. 32300	0. 00036
2. 16500	0. 00083	2. 21800	0. 00063	2. 27100	0. 00048	2. 32400	0. 00036
2. 16600	0. 00083	2. 21900	0. 00063	2. 27200	0. 00047	2. 32500	0. 00036
2. 16700	0. 00082	2. 22000	0. 00062	2. 27300	0. 00047	2. 32600	0. 00036
2. 16800	0. 00082	2. 22100	0. 00062	2. 27400	0. 00047	2. 32700	0. 00035
2. 16900	0. 00081	2. 22200	0. 00062	2. 27500	0. 00047	2. 32800	0. 00035
2. 17000	0. 00081	2. 22300	0. 00061	2. 27600	0. 00046	2. 32900	0. 00035
2. 17100	0. 00080	2. 22400	0. 00061	2. 27700	0. 00046	2. 33000	0. 00035
2. 17200	0. 00080	2. 22500	0. 00061	2. 27800	0. 00046	2. 33100	0. 00035
2. 17300	0. 00080	2. 22600	0. 00061	2. 27900	0. 00046	2. 33200	0. 00034
2. 17400	0. 00079	2. 22700	0. 00060	2. 28000	0. 00045	2. 33300	0. 00034
2. 17500	0. 00079	2. 22800	0. 00060	2. 28100	0. 00045	2. 33400	0. 00034
2. 17600	0. 00078	2. 22900	0. 00060	2. 28200	0. 00045	2. 33500	0. 00034
2. 17700	0. 00078	2. 23000	0. 00059	2. 28300	0. 00045	2. 33600	0. 00034
2. 17800	0. 00078	2. 23100	0. 00059	2. 28400	0. 00045	2. 33700	0. 00033

TABLE 1.—*Values of the integral* $\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}$ du *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$
2.33800	0.00033	2.39100	0.00025	2.44400	0.00019	2.49700	0.00014
2.33900	0.00033	2.39200	0.00025	2.44500	0.00018	2.49800	0.00014
2.34000	0.00033	2.39300	0.00025	2.44600	0.00018	2.49900	0.00014
2.34100	0.00033	2.39400	0.00025	2.44700	0.00018	2.50000	0.00014
2.34200	0.00033	2.39500	0.00024	2.44800	0.00018	2.50100	0.00013
2.34300	0.00032	2.39600	0.00024	2.44900	0.00018	2.50200	0.00013
2.34400	0.00032	2.39700	0.00024	2.45000	0.00018	2.50300	0.00013
2.34500	0.00032	2.39800	0.00024	2.45100	0.00018	2.50400	0.00013
2.34600	0.00032	2.39900	0.00024	2.45200	0.00018	2.50500	0.00013
2.34700	0.00032	2.40000	0.00024	2.45300	0.00018	2.50600	0.00013
2.34800	0.00032	2.40100	0.00024	2.45400	0.00018	2.50700	0.00013
2.34900	0.00031	2.40200	0.00023	2.45500	0.00017	2.50800	0.00013
2.35000	0.00031	2.40300	0.00023	2.45600	0.00017	2.50900	0.00013
2.35100	0.00031	2.40400	0.00023	2.45700	0.00017	2.51000	0.00013
2.35200	0.00031	2.40500	0.00023	2.45800	0.00017	2.51100	0.00013
2.35300	0.00031	2.40600	0.00023	2.45900	0.00017	2.51200	0.00013
2.35400	0.00031	2.40700	0.00023	2.46000	0.00017	2.51300	0.00013
2.35500	0.00030	2.40800	0.00023	2.46100	0.00017	2.51400	0.00012
2.35600	0.00030	2.40900	0.00023	2.46200	0.00017	2.51500	0.00012
2.35700	0.00030	2.41000	0.00022	2.46300	0.00017	2.51600	0.00012
2.35800	0.00030	2.41100	0.00022	2.46400	0.00017	2.51700	0.00012
2.35900	0.00030	2.41200	0.00022	2.46500	0.00016	2.51800	0.00012
2.36000	0.00030	2.41300	0.00022	2.46600	0.00016	2.51900	0.00012
2.36100	0.00029	2.41400	0.00022	2.46700	0.00016	2.52000	0.00012
2.36200	0.00029	2.41500	0.00022	2.46800	0.00016	2.52100	0.00012
2.36300	0.00029	2.41600	0.00022	2.46900	0.00016	2.52200	0.00012
2.36400	0.00029	2.41700	0.00022	2.47000	0.00016	2.52300	0.00012
2.36500	0.00029	2.41800	0.00021	2.47100	0.00016	2.52400	0.00012
2.36600	0.00029	2.41900	0.00021	2.47200	0.00016	2.52500	0.00012
2.36700	0.00028	2.42000	0.00021	2.47300	0.00016	2.52600	0.00012
2.36800	0.00028	2.42100	0.00021	2.47400	0.00016	2.52700	0.00012
2.36900	0.00028	2.42200	0.00021	2.47500	0.00016	2.52800	0.00011
2.37000	0.00028	2.42300	0.00021	2.47600	0.00015	2.52900	0.00011
2.37100	0.00028	2.42400	0.00021	2.47700	0.00015	2.53000	0.00011
2.37200	0.00028	2.42500	0.00021	2.47800	0.00015	2.53100	0.00011
2.37300	0.00028	2.42600	0.00021	2.47900	0.00015	2.53200	0.00011
2.37400	0.00027	2.42700	0.00020	2.48000	0.00015	2.53300	0.00011
2.37500	0.00027	2.42800	0.00020	2.48100	0.00015	2.53400	0.00011
2.37600	0.00027	2.42900	0.00020	2.48200	0.00015	2.53500	0.00011
2.37700	0.00027	2.43000	0.00020	2.48300	0.00015	2.53600	0.00011
2.37800	0.00027	2.43100	0.00020	2.48400	0.00015	2.53700	0.00011
2.37900	0.00027	2.43200	0.00020	2.48500	0.00015	2.53800	0.00011
2.38000	0.00026	2.43300	0.00020	2.48600	0.00015	2.53900	0.00011
2.38100	0.00026	2.43400	0.00020	2.48700	0.00015	2.54000	0.00011
2.38200	0.00026	2.43500	0.00020	2.48800	0.00014	2.54100	0.00011
2.38300	0.00026	2.43600	0.00019	2.48900	0.00014	2.54200	0.00011
2.38400	0.00026	2.43700	0.00019	2.49000	0.00014	2.54300	0.00011
2.38500	0.00026	2.43800	0.00019	2.49100	0.00014	2.54400	0.00010
2.38600	0.00026	2.43900	0.00019	2.49200	0.00014	2.54500	0.00010
2.38700	0.00025	2.44000	0.00019	2.49300	0.00014	2.54600	0.00010
2.38800	0.00025	2.44100	0.00019	2.49400	0.00014	2.54700	0.00010
2.38900	0.00025	2.44200	0.00019	2.49500	0.00014	2.54800	0.00010
2.39000	0.00025	2.44300	0.00019	2.49600	0.00014	2.54900	0.00010

TABLE 1.—*Values of the integral* $\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$
2.55000	0.00010	2.60300	0.00007	2.65600	0.00005	2.70900	0.00004
2.55100	0.00010	2.60400	0.00007	2.65700	0.00005	2.71000	0.00004
2.55200	0.00010	2.60500	0.00007	2.65800	0.00005	2.71100	0.00004
2.55300	0.00010	2.60600	0.00007	2.65900	0.00005	2.71200	0.00004
2.55400	0.00010	2.60700	0.00007	2.66000	0.00005	2.71300	0.00004
2.55500	0.00010	2.60800	0.00007	2.66100	0.00005	2.71400	0.00004
2.55600	0.00010	2.60900	0.00007	2.66200	0.00005	2.71500	0.00004
2.55700	0.00010	2.61000	0.00007	2.66300	0.00005	2.71600	0.00004
2.55800	0.00010	2.61100	0.00007	2.66400	0.00005	2.71700	0.00004
2.55900	0.00010	2.61200	0.00007	2.66500	0.00005	2.71800	0.00004
2.56000	0.00010	2.61300	0.00007	2.66600	0.00005	2.71900	0.00004
2.56100	0.00009	2.61400	0.00007	2.66700	0.00005	2.72000	0.00004
2.56200	0.00009	2.61500	0.00007	2.66800	0.00005	2.72100	0.00004
2.56300	0.00009	2.61600	0.00007	2.66900	0.00005	2.72200	0.00004
2.56400	0.00009	2.61700	0.00007	2.67000	0.00005	2.72300	0.00004
2.56500	0.00009	2.61800	0.00007	2.67100	0.00005	2.72400	0.00004
2.56600	0.00009	2.61900	0.00007	2.67200	0.00005	2.72500	0.00004
2.56700	0.00009	2.62000	0.00007	2.67300	0.00005	2.72600	0.00004
2.56800	0.00009	2.62100	0.00007	2.67400	0.00005	2.72700	0.00004
2.56900	0.00009	2.62200	0.00007	2.67500	0.00005	2.72800	0.00003
2.57000	0.00009	2.62300	0.00007	2.67600	0.00005	2.72900	0.00003
2.57100	0.00009	2.62400	0.00007	2.67700	0.00005	2.73000	0.00003
2.57200	0.00009	2.62500	0.00007	2.67800	0.00005	2.73100	0.00003
2.57300	0.00009	2.62600	0.00006	2.67900	0.00005	2.73200	0.00003
2.57400	0.00009	2.62700	0.00006	2.68000	0.00005	2.73300	0.00003
2.57500	0.00009	2.62800	0.00006	2.68100	0.00005	2.73400	0.00003
2.57600	0.00009	2.62900	0.00006	2.68200	0.00005	2.73500	0.00003
2.57700	0.00009	2.63000	0.00006	2.68300	0.00005	2.73600	0.00003
2.57800	0.00009	2.63100	0.00006	2.68400	0.00005	2.73700	0.00003
2.57900	0.00009	2.63200	0.00006	2.68500	0.00005	2.73800	0.00003
2.58000	0.00008	2.63300	0.00006	2.68600	0.00005	2.73900	0.00003
2.58100	0.00008	2.63400	0.00006	2.68700	0.00004	2.74000	0.00003
2.58200	0.00008	2.63500	0.00006	2.68800	0.00004	2.74100	0.00003
2.58300	0.00008	2.63600	0.00006	2.68900	0.00004	2.74200	0.00003
2.58400	0.00008	2.63700	0.00006	2.69000	0.00004	2.74300	0.00003
2.58500	0.00008	2.63800	0.00006	2.69100	0.00004	2.74400	0.00003
2.58600	0.00008	2.63900	0.00006	2.69200	0.00004	2.74500	0.00003
2.58700	0.00008	2.64000	0.00006	2.69300	0.00004	2.74600	0.00003
2.58800	0.00008	2.64100	0.00006	2.69400	0.00004	2.74700	0.00003
2.58900	0.00008	2.64200	0.00006	2.69500	0.00004	2.74800	0.00003
2.59000	0.00008	2.64300	0.00006	2.69600	0.00004	2.74900	0.00003
2.59100	0.00008	2.64400	0.00006	2.69700	0.00004	2.75000	0.00003
2.59200	0.00008	2.64500	0.00006	2.69800	0.00004	2.75100	0.00003
2.59300	0.00008	2.64600	0.00006	2.69900	0.00004	2.75200	0.00003
2.59400	0.00008	2.64700	0.00006	2.70000	0.00004	2.75300	0.00003
2.59500	0.00008	2.64800	0.00006	2.70100	0.00004	2.75400	0.00003
2.59600	0.00008	2.64900	0.00006	2.70200	0.00004	2.75500	0.00003
2.59700	0.00008	2.65000	0.00006	2.70300	0.00004	2.75600	0.00003
2.59800	0.00008	2.65100	0.00006	2.70400	0.00004	2.75700	0.00003
2.59900	0.00008	2.65200	0.00006	2.70500	0.00004	2.75800	0.00003
2.60000	0.00008	2.65300	0.00006	2.70600	0.00004	2.75900	0.00003
2.60100	0.00008	2.65400	0.00005	2.70700	0.00004	2.76000	0.00003
2.60200	0.00007	2.65500	0.00005	2.70800	0.00004	2.76100	0.00003

TABLE 1.—*Values of the integral* $\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u}\,du$
2.76200	0.00003	2.81500	0.00002	2.86800	0.00001	2.92100	0.00001
2.76300	0.00003	2.81600	0.00002	2.86900	0.00001	2.92200	0.00001
2.76400	0.00003	2.81700	0.00002	2.87000	0.00001	2.92300	0.00001
2.76500	0.00003	2.81800	0.00002	2.87100	0.00001	2.92400	0.00001
2.76600	0.00003	2.81900	0.00002	2.87200	0.00001	2.92500	0.00001
2.76700	0.00003	2.82000	0.00002	2.87300	0.00001	2.92600	0.00001
2.76800	0.00003	2.82100	0.00002	2.87400	0.00001	2.92700	0.00001
2.76900	0.00003	2.82200	0.00002	2.87500	0.00001	2.92800	0.00001
2.77000	0.00003	2.82300	0.00002	2.87600	0.00001	2.92900	0.00001
2.77100	0.00003	2.82400	0.00002	2.87700	0.00001	2.93000	0.00001
2.77200	0.00003	2.82500	0.00002	2.87800	0.00001	2.93100	0.00001
2.77300	0.00003	2.82600	0.00002	2.87900	0.00001	2.93200	0.00001
2.77400	0.00003	2.82700	0.00002	2.88000	0.00001	2.93300	0.00001
2.77500	0.00003	2.82800	0.00002	2.88100	0.00001	2.93400	0.00001
2.77600	0.00003	2.82900	0.00002	2.88200	0.00001	2.93500	0.00001
2.77700	0.00003	2.83000	0.00002	2.88300	0.00001	2.93600	0.00001
2.77800	0.00003	2.83100	0.00002	2.88400	0.00001	2.93700	0.00001
2.77900	0.00003	2.83200	0.00002	2.88500	0.00001	2.93800	0.00001
2.78000	0.00003	2.83300	0.00002	2.88600	0.00001	2.93900	0.00001
2.78100	0.00003	2.83400	0.00002	2.88700	0.00001	2.94000	0.00001
2.78200	0.00002	2.83500	0.00002	2.88800	0.00001	2.94100	0.00001
2.78300	0.00002	2.83600	0.00002	2.88900	0.00001	2.94200	0.00001
2.78400	0.00002	2.83700	0.00002	2.89000	0.00001	2.94300	0.00001
2.78500	0.00002	2.83800	0.00002	2.89100	0.00001	2.94400	0.00001
2.78600	0.00002	2.83900	0.00002	2.89200	0.00001	2.94500	0.00001
2.78700	0.00002	2.84000	0.00002	2.89300	0.00001	2.94600	0.00001
2.78800	0.00002	2.84100	0.00002	2.89400	0.00001	2.94700	0.00001
2.78900	0.00002	2.84200	0.00002	2.89500	0.00001	2.94800	0.00001
2.79000	0.00002	2.84300	0.00002	2.89600	0.00001	2.94900	0.00001
2.79100	0.00002	2.84400	0.00002	2.89700	0.00001	2.95000	0.00001
2.79200	0.00002	2.84500	0.00002	2.89800	0.00001	2.95100	0.00001
2.79300	0.00002	2.84600	0.00002	2.89900	0.00001	2.95200	0.00001
2.79400	0.00002	2.84700	0.00002	2.90000	0.00001	2.95300	0.00001
2.79500	0.00002	2.84800	0.00002	2.90100	0.00001	2.95400	0.00001
2.79600	0.00002	2.84900	0.00002	2.90200	0.00001	2.95500	0.00001
2.79700	0.00002	2.85000	0.00002	2.90300	0.00001	2.95600	0.00001
2.79800	0.00002	2.85100	0.00002	2.90400	0.00001	2.95700	0.00001
2.79900	0.00002	2.85200	0.00002	2.90500	0.00001	2.95800	0.00001
2.80000	0.00002	2.85300	0.00002	2.90600	0.00001	2.95900	0.00001
2.80100	0.00002	2.85400	0.00002	2.90700	0.00001	2.96000	0.00001
2.80200	0.00002	2.85500	0.00002	2.90800	0.00001	2.96100	0.00001
2.80300	0.00002	2.85600	0.00002	2.90900	0.00001	2.96200	0.00001
2.80400	0.00002	2.85700	0.00002	2.91000	0.00001	2.96300	0.00001
2.80500	0.00002	2.85800	0.00002	2.91100	0.00001	2.96400	0.00001
2.80600	0.00002	2.85900	0.00002	2.91200	0.00001	2.96500	0.00001
2.80700	0.00002	2.86000	0.00002	2.91300	0.00001	2.96600	0.00001
2.80800	0.00002	2.86100	0.00001	2.91400	0.00001	2.96700	0.00001
2.80900	0.00002	2.86200	0.00001	2.91500	0.00001	2.96800	0.00001
2.81000	0.00002	2.86300	0.00001	2.91600	0.00001	2.96900	0.00001
2.81100	0.00002	2.86400	0.00001	2.91700	0.00001	2.97000	0.00001
2.81200	0.00002	2.86500	0.00001	2.91800	0.00001	2.97100	0.00001
2.81300	0.00002	2.86600	0.00001	2.91900	0.00001	2.97200	0.00001
2.81400	0.00002	2.86700	0.00001	2.92000	0.00001	2.97300	0.00001

TABLE 1.—*Values of the integral* $\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$
2.97400	0.00001	2.98100	0.00001	2.98800	0.00001	2.99500	0.00001
2.97500	0.00001	2.98200	0.00001	2.98900	0.00001	2.99600	0.00001
2.97600	0.00001	2.98300	0.00001	2.99000	0.00001	2.99700	0.00001
2.97700	0.00001	2.98400	0.00001	2.99100	0.00001	2.99800	0.00001
2.97800	0.00001	2.98500	0.00001	2.99200	0.00001	2.99900	0.00001
2.97900	0.00001	2.98600	0.00001	2.99300	0.00001	3.00000	0.00001
2.98000	0.00001	2.98700	0.00001	2.99400	0.00001		

Solution. Since 1 cubic foot per second is equivalent to 448.8 gallons per minute,

$$Q=\frac{500}{448.8}=1.1141 \text{ ft}^3 \text{ per sec},$$

$$\alpha=\frac{KD}{V}=\frac{(0.0020)(70)}{0.2}=0.70 \text{ ft}^2 \text{ per sec}$$

$$t=(72)(3{,}600)=259{,}200 \text{ seconds.}$$

The given data are now in consistent units, since only foot and second units are involved.

$$\frac{Q}{2\pi KD}=\frac{1.1141}{(6.2832)(0.0020)(70)}=\frac{1.1141}{0.87965}=1.2665 \text{ feet},$$

$$\sqrt{4\alpha t}=\sqrt{(4)(0.70)(259{,}200)}=\sqrt{725{,}760}=851.9 \text{ feet},$$

$$\frac{r_1}{\sqrt{4\alpha t}}=\frac{50}{851.9}=0.0587,$$

$$\frac{r_2}{\sqrt{4\alpha t}}=\frac{100}{851.9}=0.1174.$$

From Table 1, for

$$\frac{r_1}{\sqrt{4\alpha t}}=0.0587, \int_{\frac{r_1}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du=2.5484;$$

$$\frac{r_2}{\sqrt{4\alpha t}}=0.1174, \int_{\frac{r_2}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du=1.8611.$$

Then the computed drawdowns are

$$\text{at } r_1=50 \text{ feet}, s_1=\frac{Q}{2\pi KD}(2.5484)=3.23 \text{ feet};$$

$$\text{at } r_2=100 \text{ feet}, s_2=\frac{Q}{2\pi KD}(1.8611)=2.36 \text{ feet}.$$

These are the required drawdowns.

Readings are sometimes made in observation wells adjacent to a pumped well of this sort for the purpose of finding the permeability of the aquifer. Many authorities state that the well should be pumped long enough for the drawdowns to stabilize before taking readings for such purposes. Strictly speaking, the drawdowns never stabilize, in this case, as the level of the water table continues to sink as long as the well is pumped. A better criterion for determining when the observed drawdowns can be used for permeability determinations is, therefore, needed.

When simultaneous readings of the drawdowns s_1 and s_2 are taken in observation wells located at distances r_1 and r_2 from the pumped well, with $r_2>r_1$, the permeability may be computed from the expression

$$K=\frac{Q}{2\pi D}\frac{\log_e\left(\frac{r_2}{r_1}\right)}{(s_1-s_2)}, \tag{12}$$

provided that:

$$\frac{r_2^2-r_1^2}{4\alpha t}$$

is small compared to

$$2\log_e\left(\frac{r_2}{r_1}\right). \tag{13}$$

When observations are made at the times t_1 and t_2, with $t_2>t_1$, of the drawdowns s_1 and s_2 in the same well, the permeability may be computed from the relation

$$K=\frac{Q}{4\pi D}\frac{\log_e\left(\frac{t_2}{t_1}\right)}{(s_2-s_1)}, \tag{14}$$

provided that:

$$\left[\frac{r^2}{4\alpha t_1}-\frac{r^2}{4\alpha t_2}\right]$$

is small compared to

$$2\log_e\left(\frac{t_2}{t_1}\right). \tag{15}$$

These values are deduced from the series (11) by noting the relationships among the terms when x is small compared to $-\log_e x$.

Example 2

Estimate the permeability K by using the drawdowns computed in the previous example as observed drawdowns.

Solution. Since the data from two observation wells are to be used, Formula (12) is appropriate.

It will first be desirable to test the suitability of these data. Since simultaneous readings from two wells are to be used, Formula (13) is appropriate. Then

$$\frac{r_2^2-r_1^2}{4\alpha t}=\frac{100^2-50^2}{(4)(0.70)(259{,}200)}=\frac{7{,}500}{725{,}760}=0.01033,$$

$$\frac{r_2}{r_1}=2.00,\ \log_e\left(\frac{r_2}{r_1}\right)=0.69315,$$

$$2\log_e\left(\frac{r_2}{r_1}\right)=1.3863.$$

Then since the first of these values, 0.01033, is small compared to the second, 1.3863, note Formula (13), it is concluded that the conditions favor a close determination of K. From Formula (12)

$$\frac{Q}{2\pi D}=\frac{1.1141}{(2)(3.1416)(70)}=\frac{1.1141}{439.8}=0.002533,$$

$$s_1-s_2=3.224-2.357=0.867,$$

and

$$K=\frac{(0.002533)(0.69315)}{0.867}=0.00202 \text{ ft/sec.}$$

The advantage of using the computed drawdowns is now apparent, since the value of $K=0.0020$ used in the first example should be recovered. This expectation is very nearly realized.

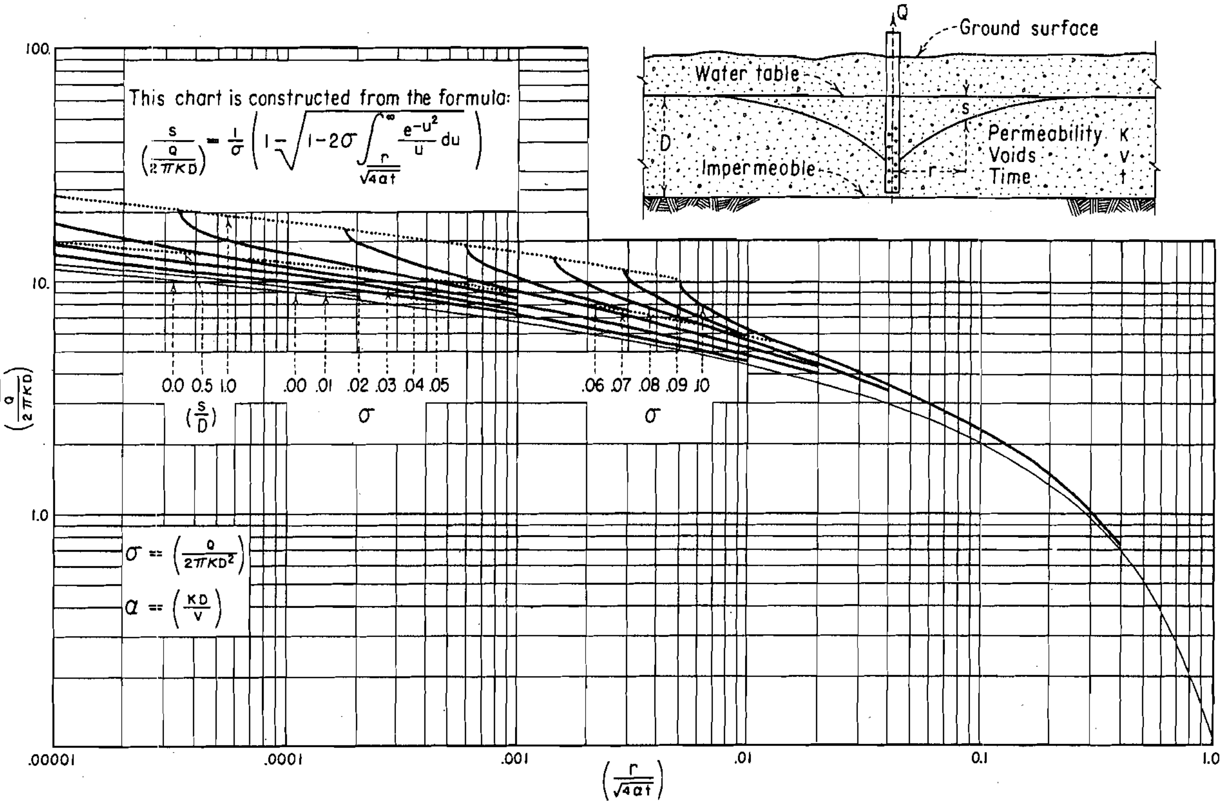


FIGURE 2.—*Drawdowns around a pumped well in an unconfined aquifer where the drawdown is not small when compared to the original saturated depth.*

Pumped well—unconfined aquifer—large drawdowns. The case where the drawdowns s are not small when compared with the original saturated depth D has been treated by Glover and Bittinger (Reference 11). Based upon considerations described in this reference, they obtain a second approximation which accounts for the effect of drawdown on the area available for the flow of ground water. The results of their investigation are shown on Figure 2. Numerical values are given in Table 2 which was computed for this monograph. In addition to the quantity $(Q/2\pi KD)$ which appears in the first approximation solution of Formula (9), a new parameter $\sigma=(Q/2\pi KD^2)$ appears. The quantity $2\pi KD^2$ is the quantity of water which would flow across a cylindrical area of radius D and height D under the action of a unit gradient. The parameter σ, therefore, relates the pumping rate to the capacity of the aquifer to transmit ground water. The ratio (s/D) is also shown on this chart. The second approximation is given by the formula

$$\frac{s}{\left(\frac{Q}{2\pi KD}\right)}=\frac{1}{\sigma}\left(1-\sqrt{1-2\sigma\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u}\,du}\right). \qquad (16)$$

Formulas 9 and 16 are developed upon a basis which implies that the flow of the well is all taken from storage. It should be expected, therefore, that the drawdown at the well will ultimately reach the base of the aquifer. When this happens, the original pumping rate can no longer be maintained because the aquifer cannot supply water as fast as the pump takes it out. In Formula (16) this point is reached if, with $r=a$, the integral attains such a value as will make the quantity under the radical equal to zero. When this happens it will be found that $s=D$ at $r=a$.

To avoid overstepping this limitation Formula (16) should not be used if

$$2\sigma\int_{\frac{a}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u}\,du>1.$$

Example 3

This second approximation may be used to estimate the drawdowns near the well under the conditions of Example 1. The computation may be arranged as shown below.

Computation of drawdowns near a well by use of Formula (16) based on data of example 1

Radius (feet)	$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u}du$	s**	s***
1	0.001174	*6.4587	8.18	8.72
2	0.002348	*5.7656	7.30	7.73
3	0.003521	*5.3604	6.79	7.15
4	0.004695	*5.0727	6.42	6.75
5	0.005869	*4.8495	6.14	6.44
10	0.01174	*4.1562	5.26	5.48
20	0.02348	3.4633	4.39	4.53
30	0.03521	3.0584	3.87	3.99
40	0.04695	2.7712	3.51	3.60
50	0.05869	2.5486	3.23	3.31
100	0.1174	1.8611	2.36	2.40

$\sigma=\frac{Q}{2\pi KD^2}=0.018092$ $\quad 2\sigma=0.036184$

$\frac{Q}{2\pi KD}\cdot\frac{1}{\sigma}=70=D$ $\quad \frac{1}{\sigma}=55.74$

* Computed from the series of Formula (11), $\log_e 1{,}000=6.90776$.
** First approximation.
*** Second approximation.

The drawdowns s near the well are sensitive to the pumping rate. An increase of the pumping rate to 1,000 gallons per minute would give a drawdown at $r=1.0$ foot which is 16.36 feet by the first approximation but 18.91 by the second approximation. A pumping rate of 2,000 gallons per minute would cause a drawdown of 32.7 feet by the first approximation but 52.1 feet by the second approximation.

The Development of Boulton

An analysis of the drawdowns produced in an unconfined aquifer by pumping at a constant rate, based upon the Laplace formulation, is described in Reference (2). The drawdown s is given by an expression of the form

$$s=\frac{Q}{2\pi KD}\,V_0(\rho,\tau) \qquad (17)$$

where

$$\rho=\frac{r}{D}$$

$$\tau=\frac{Kt}{VD} \qquad (18)$$

TABLE 2.—*Values of* $\frac{s}{\left(\frac{Q}{2\pi KD}\right)}=\frac{1}{\sigma}\left(1-\sqrt{1-2\sigma\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u}du}\right)$ *for given values of the parameters* $\frac{r}{\sqrt{4\alpha t}}$ *and* σ

$\frac{r}{\sqrt{4\alpha t}}$	$\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u}du$	σ									
		0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
0.00001	11.22430	11.93672	12.88437	14.28538	17.01344	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00002	10.53120	11.15316	11.96212	13.10881	15.07830	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00003	10.12570	10.69792	11.43278	12.45117	14.10435	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00004	9.83800	10.37634	11.06158	11.99687	13.46310	17.45441	0.00000	0.00000	0.00000	0.00000	0.00000
0.00005	9.61480	10.12764	10.77602	11.65098	12.98917	16.07469	0.00000	0 00000	0.00000	0.00000	0.00000
0.00006	9.43260	9.92514	10.54445	11.37266	12.61573	15.23596	0.00000	0.00000	0.00000	0.00000	0.00000
0.00007	9.27840	9.75411	10.34952	11.13984	12.30827	14.62747	0.00000	0.00000	0.00000	0.00000	0.00000
0.00008	9.14490	9.60630	10.18153	10.94022	12.04797	14.15158	0.00000	0.00000	0.00000	0.00000	0.00000
0.00009	9.02710	9.47608	10.03388	10.76555	11.82255	13.76173	0.00000	0.00000	0.00000	0.00000	0.00000
0.00010	8.92170	9.35972	9.90224	10.61041	11.62408	13.43250	0.00000	0.00000	0.00000	0.00000	0.00000
0.00020	8.22860	8.59825	9.04710	9.61545	10.38596	11.58239	14.79821	0.00000	0.00000	0.00000	0.00000
0.00030	7.82310	8.15567	8.55497	9.05224	9.70800	10.66854	12.54261	0.00000	0.00000	0.00000	0.00000
0.00040	7.53540	7.84296	8.20933	8.66045	9.24468	10.07105	11.50936	0.00000	0.00000	0.00000	0.00000
0.00050	7.31230	7.60119	7.94325	8.36085	8.89456	9.63139	10.83276	0.00000	0.00000	0.00000	0.00000
0.00060	7.13000	7.40410	7.72707	8.11869	8.61403	9.28552	10.33333	0.00000	0.00000	0.00000	0.00000
0.00070	6.97580	7.23772	7.54508	7.91566	8.38043	9.00145	9.93977	13.67962	0.00000	0.00000	0.00000
0.00080	6.84230	7.09391	7.38814	7.74119	8 18081	8.76131	9.61676	12.10098	0.00000	0.00000	0.00000
0.00090	6.72450	6.96721	7.25014	7.58821	8.00661	8.55360	9.34356	11.35529	0.00000	0.00000	0.00000
0.00100	6.61920	6.85409	7.12716	7.45223	7.85240	8.37107	9.10771	10.82839	0.00000	0.00000	0.00000
0.00200	5.92600	6.11283	6.32620	6.57432	6.86991	7.23442	7.70874	10.41768	0.00000	0.00000	0.00000
0.00300	5.52050	5.68192	5.86441	6.07388	6.31912	6.61418	6.98363	8.38932	9.65395	0.00000	0.00000
0.00400	5.23290	5.37748	5.53979	5.72443	5.93812	6.19116	6.50065	7.47740	8.22946	10.22849	0.00000
0.00500	5.00970	5.14189	5.28948	5.45626	5.64761	5.87158	6.14108	6.89854	7.45743	8.43340	0.00000
0.00600	4.82740	4.94990	5.08608	5.23912	5.41352	5.61584	5.85628	6.47883	6.93156	7.62827	0.00000
0.00700	4.67320	4.78781	4.91474	5.05676	5.21768	5.40301	5.62110	6.15208	6.53636	7.08852	8.14204
0.00800	4.53970	4.64770	4.76693	4.89982	5.04968	5.22123	5.42146	5.88561	6.22146	6.68303	7.44343
0.00900	4.42200	4.52434	4.63701	4.76217	4.90273	5.06279	5.24835	5.66156	5.96107	6.35984	6.96586
0.01000	4.21660	4.30945	4.41118	4.52353	4.64883	4.79026	4.95238	5.46875	5.73982	6.09213	6.60000
0.02000	3.62360	3.69174	3.76538	3.84540	3.93296	4.02952	4.13705	5.14200	5.37013	5.65633	6.04171
0.03000	3.21840	3.27192	3.32923	3.39086	3.45748	3.52990	3.60918	4.25824	4.39691	4.55883	4.75328
0.04000	2.93110	2.97536	3.02245	3.07272	3.12661	3.18465	3.24748	3.69669	3.79425	3.90439	4.03074
0.05000	2.70840	2.74610	2.78601	2.82839	2.87354	2.92182	2.97368	3.31594	3.39107	3.47427	3.56742
0.06000	2.52660	2.55935	2.59388	2.63038	2.66907	2.71023	2.75416	3.02965	3.09043	3.15685	3.23006
0.07000	2.37310	2.40194	2.43225	2.46418	2.49788	2.53357	2.57147	2.80124	2.85194	2.90683	2.96665
0.08000	2.24030	2,26597	2.29287	2.32111	2.35082	2.38216	2.41531	2.61186	2.65507	2.70151	2.75168
0.09000	2.12340	2.14643	2.17051	2.19571	2.22216	2.24995	2.27924	2.45046	2.48788	2.52785	2.57073
0.10000	2.01900	2.03980	2.06149	2.08415	2.10786	2.13271	2.15881	2.31019	2.34298	2.37783	2.41501
0.20000	1.34060	1.34970	1.35907	1.36870	1.37861	1.38882	1.39934	2.18629	2.21530	2.24600	2.27860

0.30000	0.95940	0.96404	0.96878	0.97361	0.97855	0.98358	0.98827	0.99397	0.99934	1.00483	1.01045
0.40000	0.70460	0.70710	0.70963	0.71220	0.71481	0.71746	0.72015	0.72288	0.72566	0.72848	0.73134
0.50000	0.52210	0.52347	0.52485	0.52625	0.52766	0.52909	0.53054	0.53200	0.53348	0.53497	0.53649
0.60000	0.38720	0.38795	0.38871	0.38947	0.39024	0.39102	0.39180	0.39259	0.39339	0.39419	0.39500
0.70000	0.28600	0.28641	0.28682	0.28723	0.28765	0.28807	0.28849	0.28892	0.28934	0.28977	0.29021
0.80000	0.20980	0.21002	0.21024	0.21046	0.21068	0.21091	0.21113	0.21136	0.21159	0.21181	0.21204
0.90000	0.15250	0.15261	0.15273	0.15285	0.15296	0.15308	0.15320	0.15332	0.15344	0.15356	0.15368
1.00000	0.10970	0.10976	0.10982	0.10988	0.10994	0.11000	0.11006	0.11012	0.11018	0.11024	0.11030

and V_0 is a function developed in the paper. A table of values of $V_0(\rho, \tau)$ is given for values of τ between 0.05 and 5.00 and of ρ between 0.2 and 1.5. Some simplifications introduced make the function $V_0(\rho, \tau)$ approximate and correction curves are presented to improve the treatment in certain ranges of the variable τ. For $\tau > 5$ his solution agrees with the exponential integral within 3 percent. It is this writer's understanding that this result would also hold for Formula (9). For values of τ between 0.05 and 5, certain restrictions on Q and ρ should be observed. When τ is less than 0.05 the correction curves may be used.

With the conditions of Example 1, it will be found that $\tau = 37.0$. This is in the range where $\tau > 5$ and the solution should agree closely with that given by the first approximation, Formula (9).

This is the only case known to this writer where a transient condition has been treated under the Laplace formulation. The close agreement obtained between formulas developed under the Laplace and Dupuit-Forchheimer idealizations for this case is reassuring.

Determination of Aquifer Properties

A convenient means of deriving aquifer constants from test data makes use of a characteristic of logarithmic scales. With such scales a shift represents multiplication, and this is the principle used in the construction of slide rules. Suppose a curve is plotted on graph paper which has logarithmic scales in both directions. If the ordinates of this curve are all multiplied by some constant and the curve is replotted, it will be found to be shifted vertically on the chart but will be unchanged in shape. A horizontal shift will be observed if all of the abscissas are multiplied by some constant. The important feature to be borne in mind is that the shape of the curve remains unchanged.

To put this characteristic to use for determination of aquifer properties, Equation (16) is first plotted on a sheet of transparent logarithmic graph paper. For this plot the ordinate will be $\dfrac{s}{\left(\dfrac{Q}{2\pi KD}\right)}$ and the abscissa will be $\dfrac{r}{\sqrt{4\alpha t}}$. Both of these are dimensionless quantities. For reference purposes, this chart will be called the master chart. An index point is selected on the master chart. It will be convenient to use the point where the ordinate and abscissa are both unity. An example of such a chart is shown on Figure 2.

The test data can now be plotted on a similar sheet of graph paper. In this case the ordinate can be s and the abscissa $\dfrac{r}{\sqrt{t}}$. The master chart is now laid over this plot and adjusted, while keeping the axis parallel, until the plotted points coincide with one of the curves of the master chart. The index point of the master chart will now fall on some point on the chart showing the observations. On this chart read the value of $s = s_1$ and $\dfrac{r}{\sqrt{t}} = \dfrac{r_1}{\sqrt{t_1}}$ for the point on which the index falls. For this point:

$$\frac{s_1}{\left(\dfrac{Q}{2\pi KD}\right)} = 1 \text{ or } KD = \frac{Q}{2\pi s_1}$$

and

$$\frac{r_1}{\sqrt{4\alpha t_1}} = 1 \text{ or } \alpha = \frac{r_1^2}{4t_1}.$$

Since the flow of the well Q is a known quantity, the transmissibility KD and the diffusion constant α can be determined. If the observed drawdowns are large enough, the values of δ may also be found. The curve for $\delta = 0$ on the master chart is a plot of Formula (9). If the observed drawdowns s are small compared to D the plotted points from all observation wells at all times should fall on this curve. The process of making a determination of aquifer properties by this method is shown in the following example.

Example 4

It will be of interest to use the second approximation drawdowns for radii of 1, 10, 50, and 100 feet as assumed observed drawdowns for the purpose of illustrating the use of the graphical method of determining the aquifer properties, since the aquifer properties from which they were obtained are already known.

From Examples 1 and 3 and Table 1 with $t=259{,}200$ seconds, or 72 hours,

r (*feet*)	$\frac{r}{\sqrt{t}}$	s (*feet*)
1	0.00196	8.72
10	0.01964	5.48
50	0.09821	3.31
100	0.19642	2.40

$$\sqrt{t}=509.117 \qquad \frac{1}{\sqrt{t}}=0.00196419.$$

When these points are plotted with the drawdown s in feet against $r/\sqrt{t}$ and the adjustment is made on the master chart in the manner described, the index of the master chart is found to fall on the point

$$s=1.26 \qquad \frac{r}{\sqrt{t}}=1.70$$

then

$$\frac{s}{\left(\frac{Q}{2\pi KD}\right)}=\frac{1.26}{\left(\frac{1.1141}{6.2832KD}\right)}=1$$

and

$$KD=\frac{1.1141}{(6.2832)(1.26)}=0.1407 \text{ ft}^2/\text{sec}$$

with $D=70$ ft

$$K=\frac{0.1407}{70}=0.00201 \text{ ft/sec}$$

also with

$$\frac{r}{\sqrt{t}}=1.70$$

$$\frac{r}{\sqrt{4\alpha t}}=\frac{1.70}{\sqrt{4\alpha}}=1$$

$$\alpha=\frac{1.70^2}{4}=0.722 \text{ ft}^2/\text{sec}$$

$$V=\frac{KD}{\alpha}=0.195.$$

The values which should have been recovered are: $KD=0.1400$ ft^2/sec, $K=0.0020$ ft/sec, $\alpha=0.70$ ft^2/sec, and $V=0.20$. In the process described, the difference of units is absorbed into the factor represented by the scale shift.

The Flowing Artesian Well

When a well penetrates a confined aquifer of thickness D and permeability K which is under sufficient pressure to raise water above the top of the casing, a flowing artesian well can be obtained. To treat this case a solution of Equation (5) must be sought which satisfies the conditions

$$s=0 \text{ when } t=0 \text{ for } r>a$$
$$s=s_0 \text{ when } r=a \text{ for } t>0. \qquad (19)$$

A solution for the case of the infinitely extended aquifer, as required to meet these conditions, is not available, but it is possible to construct a chart for the infinite case by using the solution for a finite case having an outer boundary at the radius b. The solution for this finite case is

$$s=s_0\left[1-\sum_{n=1}^{n=\infty} A_n U_0(\beta_n r)e^{-\alpha\beta_n^2 t}\right] \qquad (20)$$

where

$$U_0(\beta_n r)=J_0(\beta_n r)Y_0(\beta_n a)-J_0(\beta_n a)Y_0(\beta_n r) \qquad (21)$$

and

$$A_n=\frac{\pi}{\left[1-\frac{J_0^2(\beta_n a)}{J_0^2(\beta_n b)}\right]} \qquad {}^*(22)$$

The symbol β_n represents the roots of

$$U_0'(\beta_n b)=0. \qquad (23)$$

This arrangement adapts the solution to represent the case where an impermeable barrier exists at the radius $r=b$. If a ratio of b to a is chosen it will be found that a large number of terms are required to compute the pattern for values of t near zero and also that, after the lapse of a certain amount of time, the value of s at $r=b$ will begin to depart from zero. If a series of values of the b/a ratio is chosen, the use of an excessive number of terms can be avoided and, when s begins to depart from zero at the radius b, a new b/a ratio can be chosen and the process repeated. In this way the computations can be extended to as great an outer radius as desired. A chart prepared for the infinite case, based in part on computations made by this process, is included as Figure 3. For values of the parameter $\frac{\sqrt{4\alpha t}}{a}>1{,}000$, the curves were computed by an approximate method to be described later.

*A form developed by W. T. Moody.

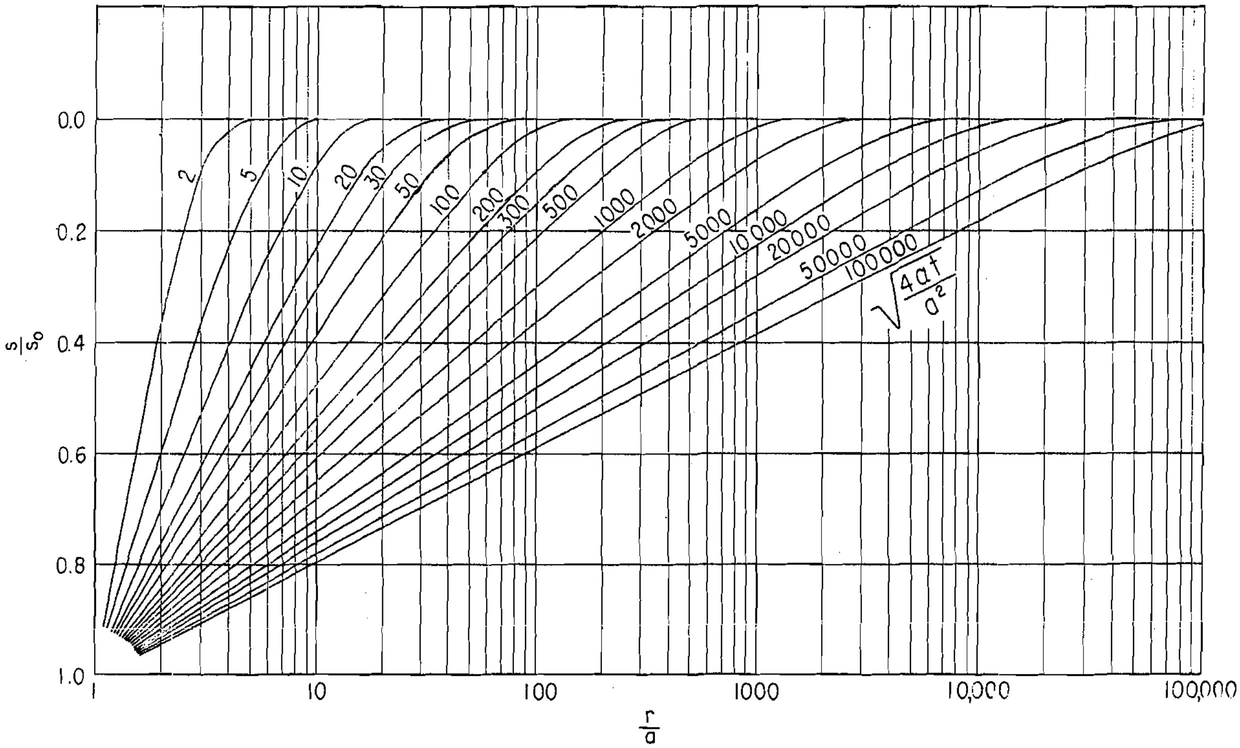


FIGURE 3.—*Pressure reduction caused by a flowing artesian well.*

The flow of an artesian well diminishes with time. The variation of flow with time has been investigated for the artesian case. Table 3, which is based upon the data of References (18), (19), and (20), contains values of the function $G\left(\frac{\sqrt{4\alpha t}}{a}\right)$. The flow of the well is given in terms of this function by an expression of the form,

$$Q=2\pi KDs_0G\left(\frac{\sqrt{4\alpha t}}{a}\right). \tag{24}$$

A series of the tabular values were checked by use of the formula

$$G\left(\frac{\sqrt{4\alpha t}}{a}\right)=\sum_{n=1}^{n=\infty}(\beta_n a)A_nU_0'(\beta_n a)e^{-\left(\frac{\beta_n a}{2}\right)^2\frac{4\alpha t}{a^2}}. \tag{25}$$

Possession of this outflow function permits the construction of a simple approximate formula for obtaining values of s for values of the parameter $\frac{\sqrt{4\alpha t}}{a}$ greater than 1,000. This formula is

$$s=s_0G\left(\frac{\sqrt{4\alpha t}}{a}\right)\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u}du. \tag{26}$$

The curves of Figure 3 for values of $\frac{\sqrt{4\alpha t}}{a}$ greater than 1,000 were computed by use of this relation.

Aquifer with a semipermeable upper confining bed. This development relates to the case of a well of infinitesimal radius drawing water at the rate Q from an aquifer of thickness D, permeability K, and coefficient of storage V. The aquifer has an upper confining bed of thickness m and permeability p, with the permeability p being small compared to the permeability K. Before pumping begins, the water table lies above the upper confining bed as shown in Figure 4, and the water pressures are continuous through the upper confining bed and the aquifer. After pumping begins, the pressure in the aquifer is diminished by the amount s, and a gradient is created which drives water vertically downward through the upper confining bed, and horizontally through the aquifer. Initially, water to supply the well comes in part from storage and in part from seepage through the upper confining bed. The drawdowns eventually stabilize when the pressures in the aquifer become sufficiently low to supply the entire flow of the well by seepage

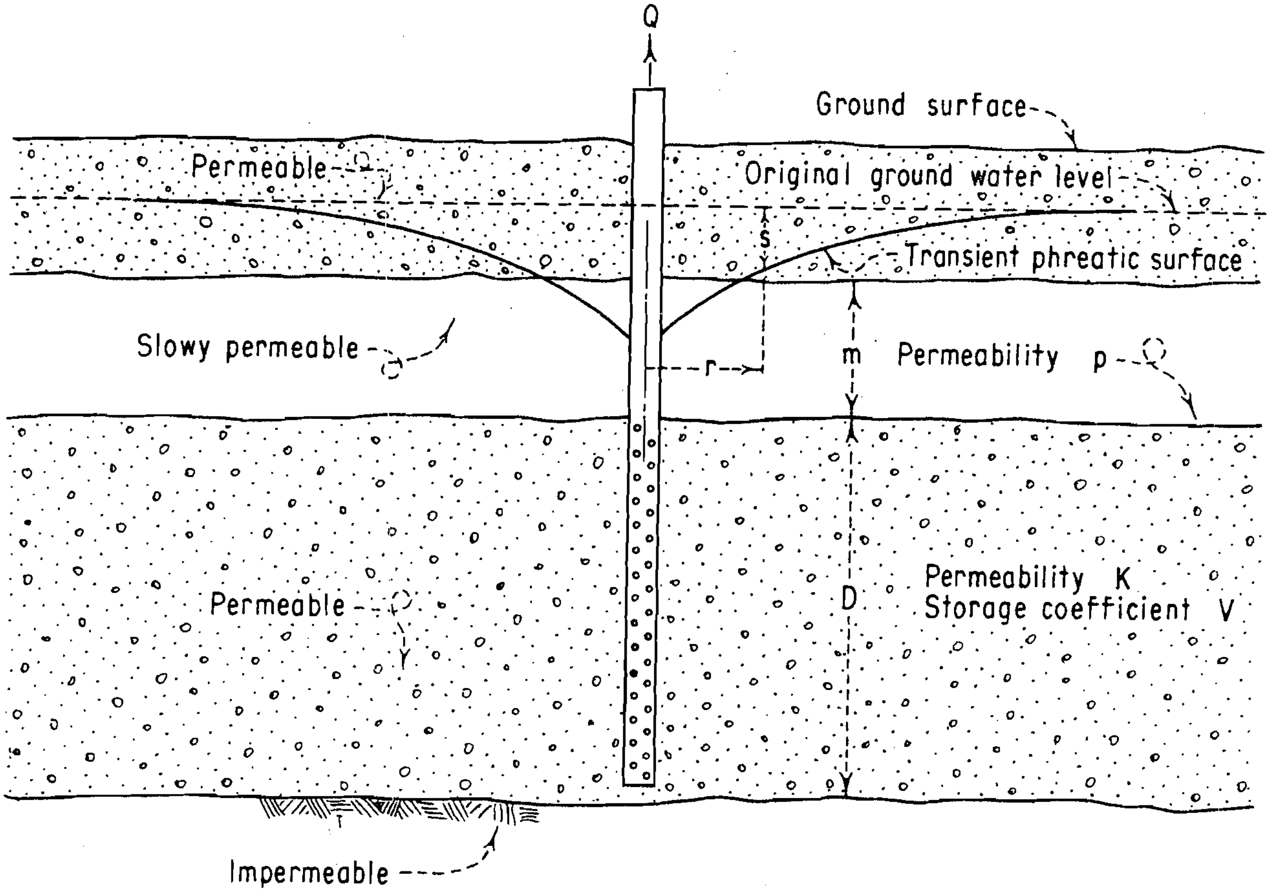


FIGURE 4.—*Well with a semipermeable confining bed.*

through the upper confining bed. This case possesses a definite terminal state. It occurs whenever the upper confining bed is sufficiently permeable to contribute significantly to the flow of the well. It appears to occur frequently in glaciated regions where aquifers of sand are overlain by beds of glacial till.

This case has been treated by Jacob, Reference (17). The continuity condition is

$$\frac{\partial s}{\partial t}=\frac{KD}{V}\left(\frac{\partial^2 s}{\partial r^2}+\frac{1}{r}\frac{\partial s}{\partial r}\right)-\frac{p}{mV}s. \tag{27}$$

The solution given by Jacob for the finite case where the drawdown s is maintained at zero at the radius b can be put in the form

$$\mu=\left[K_0(\rho)=\frac{K_0(\rho_e)}{I_0(\rho_e)}I_0(\rho)\right]-\sum_{n=1}^{n=\infty}\frac{2J_0(\beta_n\rho)e^{-(1+\beta_n^2)\eta}}{\rho_e^2J_1^2(\beta_n\rho_e)(1+\beta_n^2)} \tag{28}$$

where $J_0(\beta_n\rho)$ represents the zero order Bessel functions of the parameter $(\beta_n\rho)$ of the first kind.

$I_0(\rho)$ and $K_0(\rho)$ represent the zero order modified Bessel functions of the parameter (ρ) of the first and second kinds, respectively, and:

$$\rho=r\sqrt{\frac{p}{mKD}} \qquad \rho_e=b\sqrt{\frac{p}{mKD}}$$

$$\mu=\frac{s}{\left(\dfrac{Q}{2\pi KD}\right)} \tag{29}$$

β_n are the roots of $J_0(\beta_n\rho_e)=0$

$$\eta=\left(\frac{p}{mV}\right)t.$$

The first term in the right-hand member of Equation (28) represents the final steady state drawdown. In the infinite case, this reduces to

$$\mu=K_0(\rho). \tag{30}$$

TABLE 3.—*Values of* $G\left(\frac{\sqrt{4\alpha t}}{a}\right)$

$\frac{\sqrt{4\alpha t}}{a}$	$G\left(\frac{\sqrt{4\alpha t}}{a}\right)$	$\frac{\sqrt{4\alpha t}}{a}$	$G\left(\frac{\sqrt{4\alpha t}}{a}\right)$	$\frac{\sqrt{4\alpha t}}{a}$	$G\left(\frac{\sqrt{4\alpha t}}{a}\right)$	$\frac{\sqrt{4\alpha t}}{a}$	$G\left(\frac{\sqrt{4\alpha t}}{a}\right)$
0.001	1,128.88	0.055	21.02	0.19	6.427	0.73	2.006
0.002	564.69	0.056	20.65	0.20	6.129	0.74	1.984
0.003	376.63	0.057	20.30	0.21	5.860	0.75	1.964
0.004	282.60	0.058	19.95	0.22	5.615	0.76	1.944
0.005	226.18	0.059	19.62	0.23	5.391	0.77	1.924
0.006	188.56	0.060	19.30	0.24	5.186	0.78	1.905
0.007	161.70	0.061	18.99	0.25	4.998	0.79	1.886
0.008	141.55	0.062	18.70	0.26	4.824	0.80	1.868
0.009	125.88	0.063	18.41	0.27	4.662	0.81	1.850
0.010	113.34	0.064	18.13	0.28	4.513	0.82	1.833
0.011	103.08	0.065	17.86	0.29	4.373	0.83	1.816
0.012	94.53	0.066	17.59	0.30	4.243	0.84	1.799
0.013	87.30	0.067	17.34	0.31	4.121	0.85	1.783
0.014	81.10	0.068	17.09	0.32	4.007	0.86	1.767
0.015	75.73	0.069	16.85	0.33	3.899	0.87	1.752
0.016	71.02	0.070	16.61	0.34	3.798	0.88	1.736
0.017	66.88	0.071	16.39	0.35	3.703	0.89	1.722
0.018	63.19	0.072	16.17	0.36	3.613	0.90	1.707
0.019	59.88	0.073	15.95	0.37	3.528	0.91	1.693
0.020	56.92	0.074	15.74	0.38	3.447	0.92	1.679
0.021	54.23	0.075	15.54	0.39	3.370	0.93	1.665
0.022	51.79	0.076	15.34	0.40	3.298	0.94	1.652
0.023	49.56	0.077	15.14	0.41	3.227	0.95	1.639
0.024	47.52	0.078	14.96	0.42	3.161	0.96	1.626
0.025	45.64	0.079	14.78	0.43	3.098	0.97	1.614
0.026	43.90	0.080	14.60	0.44	3.038	0.98	1.602
0.027	42.29	0.081	14.42	0.45	2.980	0.99	1.590
0.028	40.80	0.082	14.25	0.46	2.926	1.00	1.578
0.029	39.41	0.083	14.09	0.47	2.873	1.01	1.566
0.030	38.11	0.084	13.93	0.48	2.822	1.02	1.555
0.031	36.90	0.085	13.77	0.49	2.774	1.03	1.544
0.032	35.76	0.086	13.61	0.50	2.728	1.04	1.533
0.033	34.69	0.087	13.46	0.51	2.683	1.05	1.523
0.034	33.69	0.088	13.31	0.52	2.640	1.06	1.512
0.035	32.74	0.089	13.17	0.53	2.599	1.07	1.502
0.036	31.84	0.090	13.03	0.54	2.559	1.08	1.492
0.037	31.00	0.091	12.89	0.55	2.520	1.09	1.482
0.038	30.19	0.092	12.76	0.56	2.483	*1.10*	1.472
0.039	29.43	0.093	12.62	0.57	2.447	1.20	1.383
0.040	28.71	0.094	12.49	0.58	2.412	1.30	1.307
0.041	28.02	0.095	12.37	0.59	2.379	1.40	1.242
0.042	27.37	0.096	12.24	0.60	2.346	1.50	1.185
0.043	26.74	0.097	12.12	0.61	2.316	1.60	1.136
0.044	26.14	0.098	12.00	0.62	2.285	1.70	1.091
0.045	25.58	0.099	11.89	0.63	2.256	1.80	1.052
0.046	25.03	*0.100*	11.78	0.64	2.227	1.90	1.016
0.047	24.51	0.11	10.751	0.65	2.200	2.00	0.984
0.048	24.01	0.12	9.895	0.66	2.173	2.10	0.954
0.049	23.53	0.13	9.171	0.67	2.147	2.20	0.928
0.050	23.07	0.14	8.551	0.68	2.122	2.30	0.903
0.051	22.63	0.15	8.013	0.69	2.098	2.40	0.880
0.052	22.20	0.16	7.542	0.70	2.073	2.50	0.860
0.053	21.79	0.17	7.126	0.71	2.050	2.60	0.840
0.054	21.40	0.18	6.757	0.72	2.028	2.70	0.822

TABLE 3.—*Values of* $G\left(\frac{\sqrt{4\alpha t}}{a}\right)$—Continued

$\frac{\sqrt{4\alpha t}}{a}$	$G\left(\frac{\sqrt{4\alpha t}}{a}\right)$	$\frac{\sqrt{4\alpha t}}{a}$	$G\left(\frac{\sqrt{4\alpha t}}{a}\right)$	$\frac{\sqrt{4\alpha t}}{a}$	$G\left(\frac{\sqrt{4\alpha t}}{a}\right)$	$\frac{\sqrt{4\alpha t}}{a}$	$G\left(\frac{\sqrt{4\alpha t}}{a}\right)$
2.80	0.805	7.6	0.493	34	0.295	370	0.175
2.90	0.800	7.7	0.491	35	0.292	380	0.174
3.00	0.775	7.8	0.488	36	0.290	390	0.174
3.10	0.762	7.9	0.486	37	0.288	400	0.173
3.20	0.748	8.0	0.483	38	0.286	410	0.172
3.30	0.735	8.1	0.480	39	0.284	420	0.172
3.40	0.723	8.2	0.478	40	0.282	430	0.171
3.50	0.712	8.3	0.476	41	0.280	440	0.171
3.60	0.701	8.4	0.473	42	0.278	450	0.170
3.70	0.691	8.5	0.471	43	0.277	460	0.169
3.80	0.681	8.6	0.469	44	0.275	470	0.169
3.90	0.672	8.7	0.466	45	0.274	480	0.168
4.00	*0.664*	8.8	*0.464*	46	0.272	490	0.168
4.10	0.656	8.9	0.462	47	0.270	*500*	0.167
4.20	0.648	9.0	0.460	48	0.269	600	0.162
4.30	0.640	9.1	0.458	49	0.267	700	0.158
4.40	0.633	9.2	0.456	*50*	0.266	800	0.155
4.50	0.626	9.3	0.454	60	0.256	900	0.152
4.60	0.619	9.4	0.452	70	0.247	*1,000*	0.150
4.70	0.613	9.5	0.450	80	0.239	2,000	0.136
4.80	0.607	9.6	0.448	90	0.232	3,000	0.129
4.90	0.602	9.7	0.446	100	0.226	4,000	0.124
5.00	0.596	9.8	0.444	110	0.221	5,000	0.121
5.10	0.590	9.9	0.443	120	0.217	6,000	0.118
5.20	0.585	*10.0*	0.441	130	0.213	7,000	0.116
5.30	0.580	11	0.428	140	0.210	8,000	0.114
5.40	0.574	12	0.416	150	0.207	9,000	0.113
5.50	0.570	13	0.404	160	0.204	*10,000*	0.112
5.60	0.565	14	0.394	170	0.202	20,000	0.104
5.70	0.560	15	0.384	180	0.200	30,000	0.099
5.80	0.556	16	0.375	190	0.198	40,000	0.097
5.90	0.551	17	0.367	200	0.196	50,000	0.095
6.00	0.547	18	0.359	210	0.194	60,000	0.093
6.1	0.543	19	0.352	220	0.192	70,000	0.092
6.2	0.539	20	0.346	230	0.191	80,000	0.091
6.3	0.535	21	*0.340*	240	*0.189*	90,000	0.090
6.4	0.531	22	0.335	250	0.188	*100,000*	0.089
6.5	0.528	23	0.330	260	0.187	200,000	0.084
6.6	0.524	24	0.326	270	0.185	300,000	0.081
6.7	0.521	25	0.322	280	0.184	400,000	0.080
6.8	0.517	26	0.318	290	0.183	500,000	0.078
6.9	0.514	27	0.314	300	0.182	600,000	0.077
7.0	0.511	28	0.311	310	0.181	700,000	0.076
7.1	0.508	29	0.308	320	0.180	800,000	0.075
7.2	0.505	30	0.306	330	0.179	900,000	0.074
7.3	0.502	31	0.303	340	0.178	10^6	0.074
7.4	0.499	32	0.300	350	0.177		
7.5	0.496	33	0.297	360	0.176		

The infinite case can be treated by the computation procedure described for the case of the flowing artesian well. A chart prepared in this way for the present case is shown in Figure 5.

When many wells are located in an area and have been pumped long enough to have reached the final steady state, as given by Formula (30), their zones of influence overlap and it becomes of interest to determine the influence of the distributed pumping on the drawdown at an individual well. We may idealize this situation by assuming that the pumping is distributed with complete uniformity over the area and that the individual well is located at the center of a circular tract of outer radius b. With a distributed pumping of amount f, per unit of area, the drawdown due to the distributed pumping is given by

$$s=\frac{fm}{p}\left[1-\rho_e K_1(\rho_e)\right] \tag{31}$$

where, in this case also:

$$\rho_e=b\sqrt{\frac{p}{mKD}} \tag{32}$$

and $K_1(\rho)$ represents the modified Bessel function of the first order and second kind. Values of this function can be obtained from tables or from Figure 6.

Then the value of s given by Equation (31) represents the amount which should be added to the drawdowns computed for an individual well to account for the effect of other wells in the area.

Flow in One Direction Only

In this section the flows of ground water in one direction only will be considered. A number of important cases are of this type. These will include, among others, the cases of flows to parallel drains, return flows from irrigated tracts, canal leakage, and bank storage.

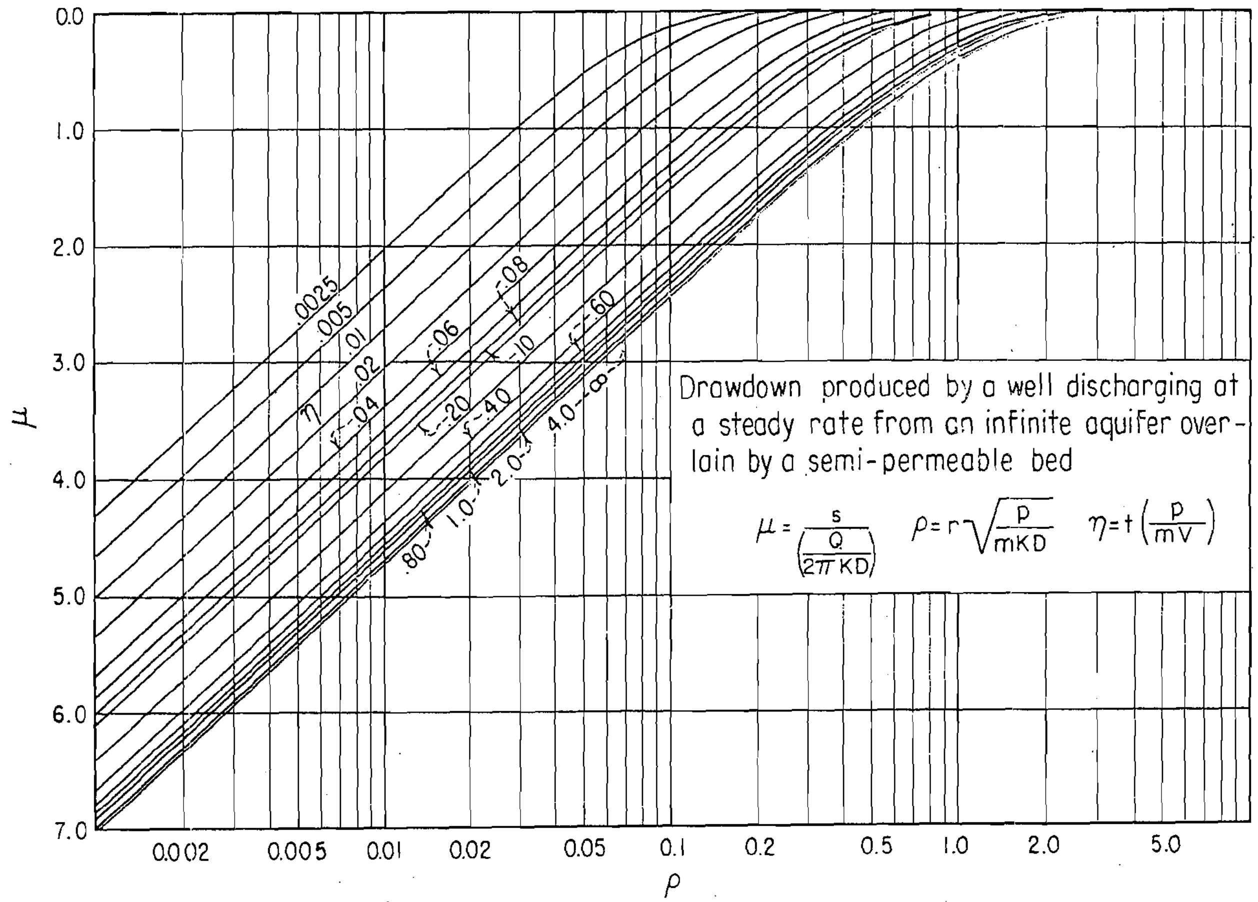


FIGURE 5.—*Drawdown caused by pumping a well with a semipermeable confining bed.*

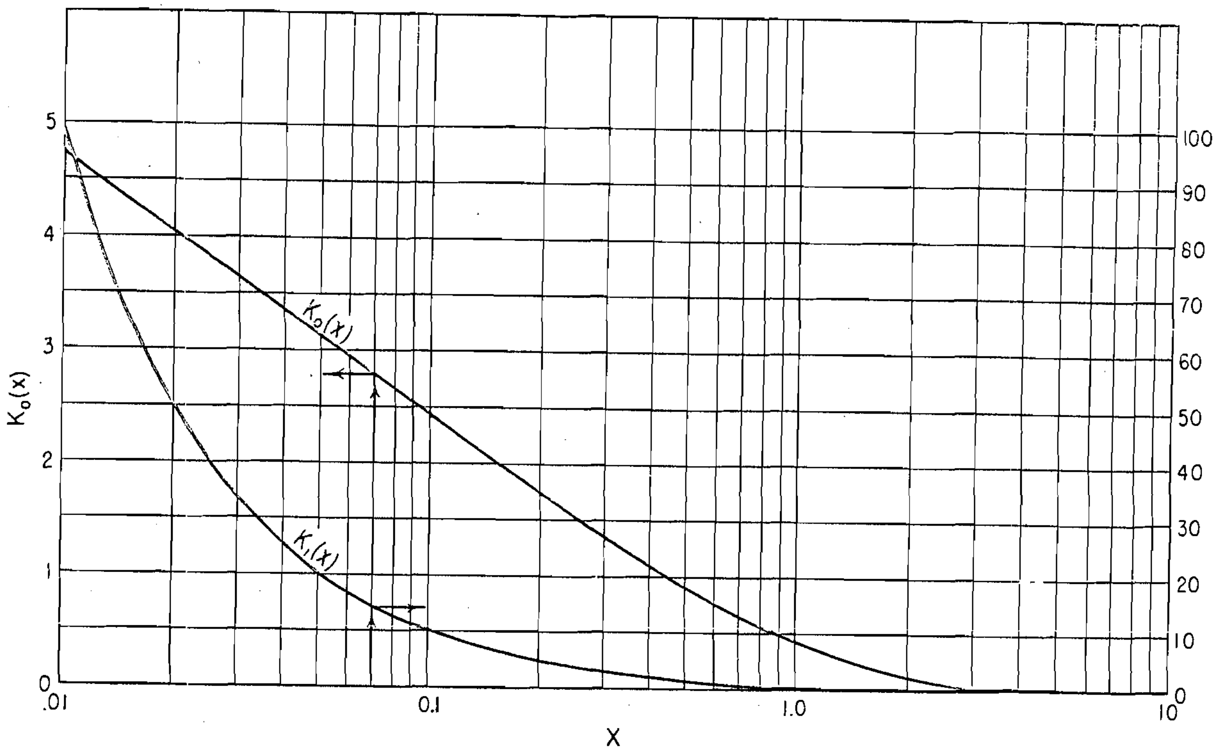


FIGURE 6.—*Values of* $K_0(x)$ *and* $K_1(x)$.

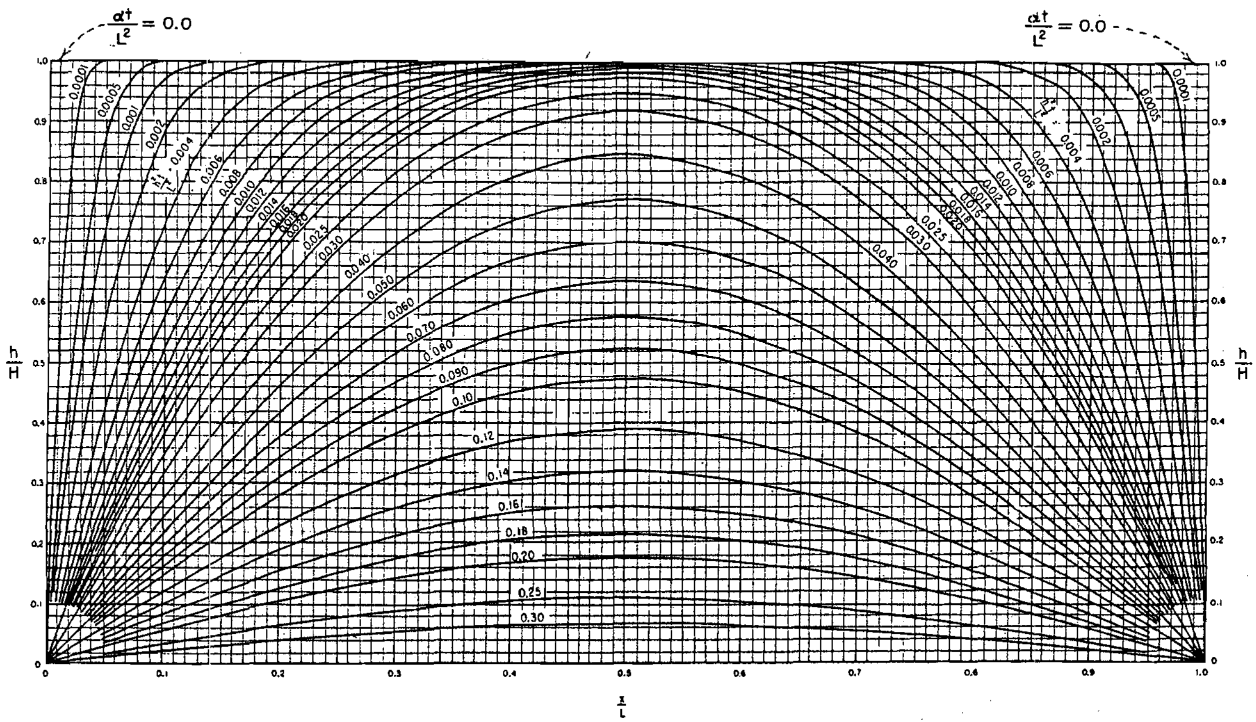


FIGURE 7.—*Water table configurations produced by parallel drains.*

The solutions presented in the following paragraphs are based upon the Dupuit-Forchheimer idealization.

Parallel drains. A solution of Equation (3) which satisfies the conditions:

$$h=0 \text{ when } x=0 \text{ for } t>0$$

$$h=0 \text{ when } x=L \text{ for } t>0$$

$$h=H \text{ for } 0<x<L \text{ when } t=0 \tag{33}$$

is

$$h=H\frac{4}{\pi}\sum_{n=1,3,5,\text{ etc.}}^{n=\infty}\frac{e^{-\frac{n^2\pi^2\alpha t}{L^2}}}{n}\sin\frac{n\pi}{L}x. \tag{34}$$

A plot of this function is shown in Figure 7. Because of the limitations imposed by the Dupuit-Forchheimer idealization, the solution will be appropriate if H is small compared to d and if d is small compared to L. The notation is shown on Figure 8.

The part p_1 of the drainable water which still remains in the ground at the time t can be obtained by integrating the above expression with respect to x. Then

$$p_1=\frac{\int_0^L h\,dx}{HL}. \tag{35}$$

The result of this integration is

$$p_1=\frac{8}{\pi^2}\sum_{n=1,3,5}^{n=\infty}\frac{e^{-\frac{n^2\pi^2\alpha t}{L^2}}}{n^2}. \tag{36}$$

A plot of this function is shown on Figure 8.

Because of the limitations on the solution of Expression (34) it will be of interest to consider a development for the limiting case where the drains are placed on the barrier. For this development the origin is placed at one of the drains. The continuity condition is

$$\frac{\partial}{\partial x}\left(Kh\frac{\partial h}{\partial x}\right)=V\frac{\partial h}{\partial t}. \tag{37}$$

Let H represent the value of h at $x=L/2$ when $t=0$, and let

$$U=\frac{h}{H}$$

$$\xi=\frac{x}{L}$$

$$\eta_1=\frac{KH}{VL^2}t. \tag{38}$$

Then the differential Equation (35) takes the form

$$\frac{\partial}{\partial\xi}\left(U\frac{\partial U}{\partial\xi}\right)=\frac{\partial U}{\partial\eta}. \tag{39}$$

A solution of this equation is

$$U=WY \tag{40}$$

where W is determined from the relation

$$\xi=C\int_0^W\frac{\omega\,d\omega}{\sqrt{1-\omega^3}} \tag{41}$$

where

$$C=\frac{\Gamma(7/6)}{\sqrt{\pi}\,\Gamma(5/3)}=0.5798 \tag{42}$$

and Y is given by*

$$Y\cong\frac{1}{\frac{9}{2}\left(\frac{\alpha t}{L^2}\right)+1}. \tag{43}$$

A plot of this solution is shown on Figure 9.

A comparison of computations made on the basis of Expressions (34) and (38) indicates that if

$$D_a=\left(d+\frac{H}{2}\right) \tag{44}$$

is used to compute a diffusion constant of the form

$$\alpha=\left(\frac{KD_a}{V}\right) \tag{45}$$

Expression (34) may be used as a reasonable approximation for all depths of drains.

The Method of Brooks

To obtain an improved treatment for cases where H is not small compared to d, R. Brooks (5) has used the Poincare-Lighthill-Kuo method to treat the nonlinear form of Equation (3). He has also obtained a second approximation of the type described by Haushild and Kruse (14). This second approximation is convenient to use because it is expressed in terms of the first approximation. If h_a represents the first approximation and h_b the second approximation and the notation is otherwise as used herein, this second approximation takes the form

$$h_b=-d+\sqrt{d^2+2dh_a+\left(\frac{H}{2}\right)^2}. \tag{46}$$

*It has been brought to the attention of this writer by Marinus Maasland that the solution described above had been obtained previously by J. Boussinesq (3).

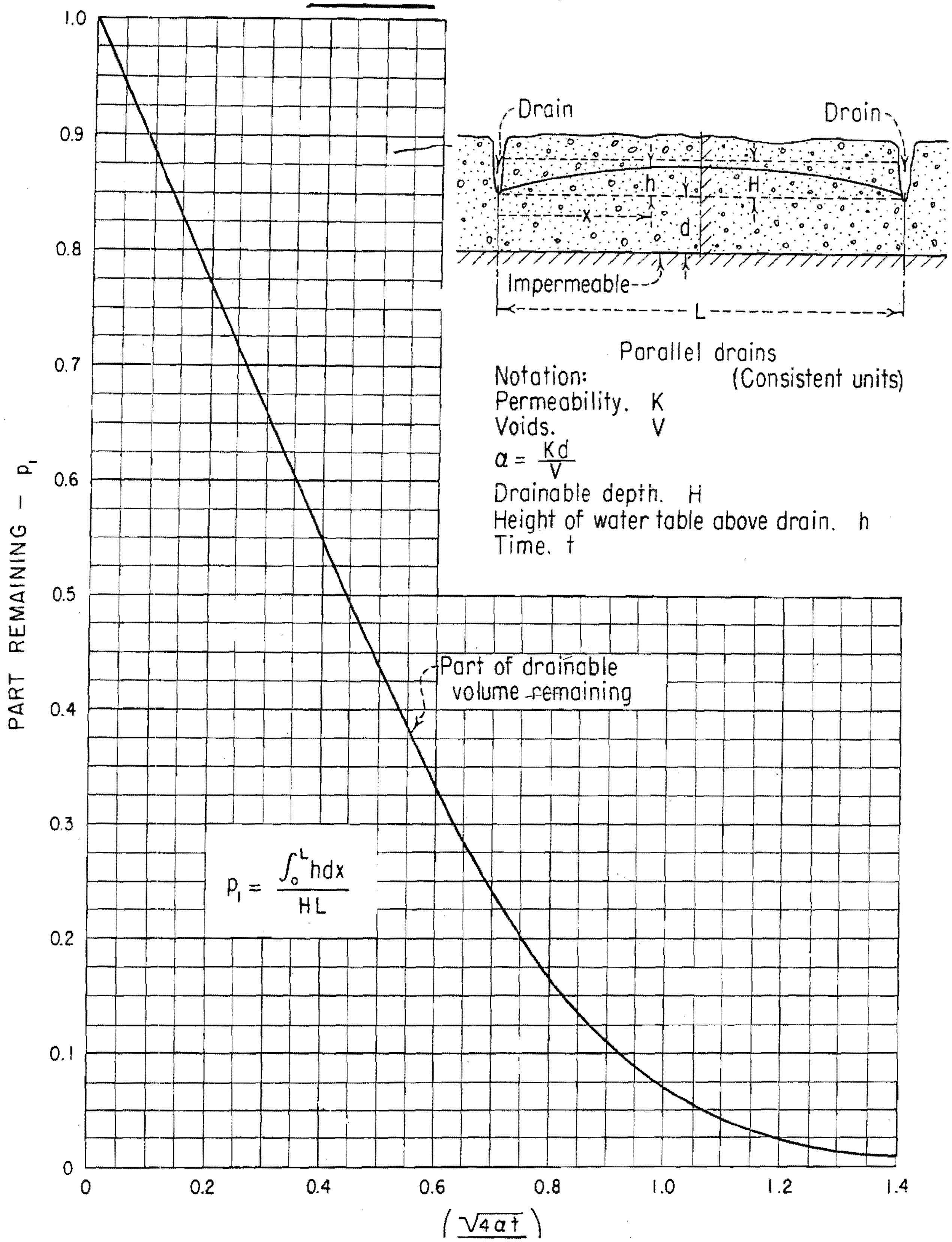


FIGURE 8.—*Part of drainable volume remaining.*

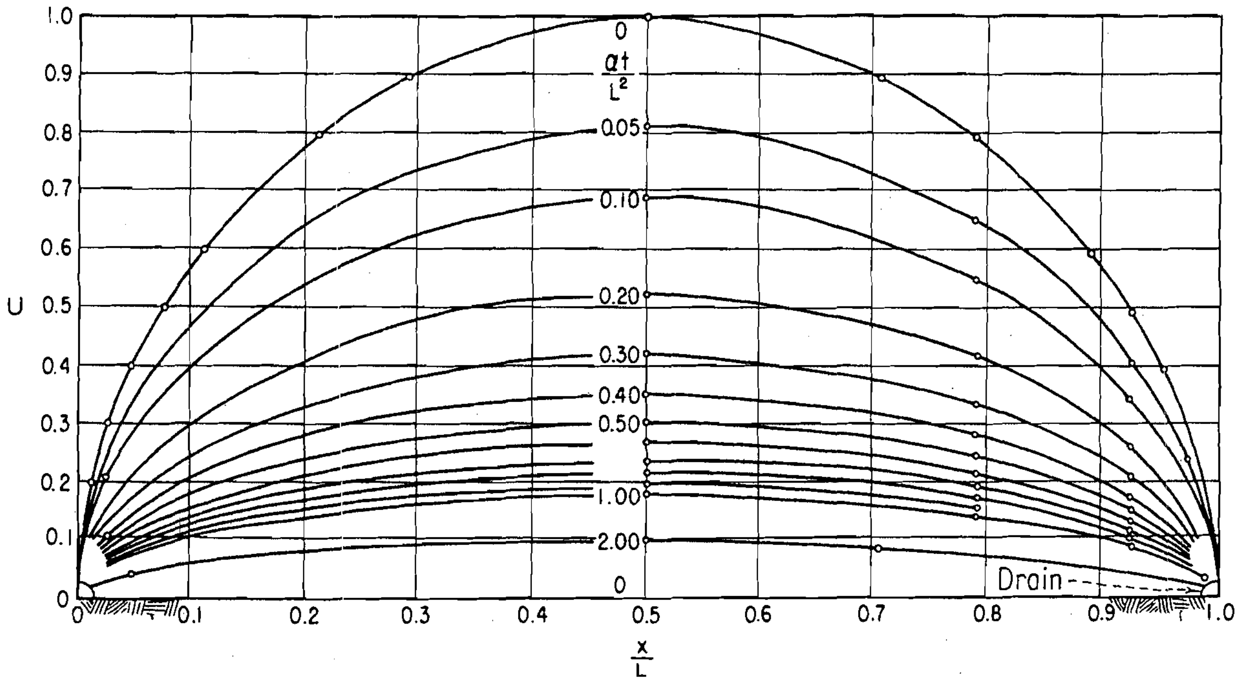


FIGURE 9.—*Water table configurations produced if the drains are on the barrier.*

First approximation $\frac{h_1}{H}$ *and second approximation* $\frac{h_2}{H}$ *as computed from formulas (34) and (46). Position of the water table midway between drains.*

Time (days)	$\frac{\alpha t}{L^2}$	$\frac{h_1}{H}$*	$\frac{h_a}{H}=\left(\frac{h_1}{H}-\frac{1}{2}\right)$	$\frac{h_b}{H}$**	$\frac{h_2}{H}=\left(\frac{h_b}{H}+\frac{1}{2}\right)$
0	0	1. 000	0. 500	0. 500	1. 000
1	0. 01728	0. 984	0. 484	0. 489	0. 989
2	0. 03456	0. 885	0. 385	0. 421	0. 921
3	0. 05184	0. 755	0. 255	0. 327	0. 827
4	0. 06912	0. 640	0. 140	0. 237	0. 737
5	0. 08640	0. 545	0. 045	0. 158	0. 658
6	0. 10368	0. 460	−0. 040	0. 082	0. 582
7	0. 12096	0. 382	−0. 118	0. 007	0. 507
8	0. 13824	0. 330	−0. 170	−0. 046	0. 454
9	0. 15552	0. 279	−0. 221	−0. 101	0. 399
10	0. 17280	0. 230	−0. 270	−0. 157	0. 343
11	0. 19008	0. 192	−0. 308	−0. 204	0. 296
12	0. 20736	0. 168	−0. 332	−0. 234	0. 266
13	0. 22464	0. 150	−0. 350	−0. 258	0. 242
14	0. 24192	0. 120	−0. 380	−0. 300	0. 200
15	0. 25920	0. 105	−0. 395	−0. 322	0. 178

For one day of $t_1=86{,}400$ seconds $\frac{\alpha t_1}{L^2}=0.017280$.

*Values of h_1/H_0 read from fig. 2.
**Computed from formula

$$\frac{h_b}{H}=\frac{d}{H}\left[-1+\sqrt{1+\frac{2h_a}{H}\cdot\frac{H}{d}+\frac{H^2}{4d^2}}\right] \text{ with } \frac{d}{H}=1.00.$$

In Brook's development the origin is taken at a point $\left(\frac{H}{2}\right)$ above the level of the drains. This position of the origin should be kept clearly in mind when Formula (46) is used.

As an example of the use of these formulas, the drawdowns at the point midway between drains will be computed for a case where $H=d$.

Example

Compute the rate of sinking of the water table midway between drains if the following conditions prevail:

$H=10$ feet
$d=10$ feet
$L=500$ feet
$K=0.0005$ foot/second
$V=0.15$

$$D_a=\left(d+\frac{H}{2}\right)=15 \text{ feet}$$

$$\alpha=\frac{K\cdot D_a}{V}=\frac{(0.0005)(15)}{0.15}$$

$$=0.05 \text{ foot}^2/\text{second.}$$

The computed results are shown on page 40.

A comparison of observed values of (h/H) and values computed from the second approximation is shown by Brooks (5) for the ratio $(H/d)=1.08$. A good correlation is shown, indicating that the second approximation gives good results for values of (H/d) in the range between zero and one.

Drain Spacing Formula

In the series of Formula (34) it will be found that the higher order terms die away rapidly, leaving the first term to represent nearly all of the computed value after the drainage has proceeded only a short time. This characteristic was employed in a paper by Dumm (8) to develop a drain spacing formula. When only the first term of the series remains, the resulting expression may be solved for the drain spacing L in the form

$$L=\pi\sqrt{\frac{KD_a t_i}{V\cdot\log_e\left(\frac{4}{\pi}\frac{y_0}{y_c}\right)}} \qquad (47)$$

where:

- L represents the drain spacing
- K permeability
- t_i time between irrigations
- y_0 vertical distance between the draintile and the maximum permissible water-table level
- y_c difference in elevation between the draintile and the water table midway between drains at the end of the period between irrigations
- d depth of the impermeable layer below the level of the draintile

$$D_a=\left(d+\frac{y_0}{2}\right).$$

With the permissible height of the water table above the level of the drains and the drainable depth contributed by deep percolation from each irrigation known, the ratio (y_0/y_c) at the point midway between drains can be fixed. This value can then be used in Formula (47) to make a direct determination of the drain spacing.

The spacing obtained in this way will be such that the deep percolation from each irrigation will be drained away by the time the next irrigation is applied. The spacings given by this formula have been compared with the field performance of drainage systems by Lee D. Dumm and he finds that, in some cases at least, the winter drainout may make sufficient storage space available so that the drains are never required to dispose of the entire drainable increment between irrigations at any time during the summer irrigation period. In such cases adequate drainage may be provided by drains installed at wider spacings than are indicated by Formula (47).

Computation of Flow to the Drains

The rate of flow of ground water to drains can be estimated by differentiating Equation (34) with respect to x to find the gradient and applying these to the transmissibility. The result obtained by this process is shown on Figure 11. A peculiar difficulty arises here. Formula (34) is developed on the Dupuit-Forschheimer basis, which is valid so long as the gradients are small compared to unity. At the drain, at $x=0$ or $x=L$, and at times near zero, the gradients approach infinity. It is obvious that the limitations imposed by the

Dupuit-Forschheimer idealization have been exceeded and that the very high flow rates implied by this formula for the early times do not need to be taken seriously. There is generally a local resistance at the drain, due to convergence of flow, and this factor alone will limit the initial flow to moderate rates.

On the basis that the flow q_1 converges radially through a quadrant from an outer radius d to an inner radius a the local resistance can be approximated by the expression

$$h_1 = \frac{2q_1}{\pi K} \log_e \left(\frac{d}{a}\right) \tag{48}$$

where:

- h_1 represents the head required to drive the flow q through the restricted area adjacent to the drain
- a represents the effective drain radius
- q_1 represents the flow to a unit length of drain from one side
- d the distance from the drain to the barrier and
- K the permeability.

This limitation of the flow can be recognized by truncating the top of the graph and moving the remainder to the right a sufficient distance so that all of the flow is accounted for. Some modifications of this type are shown on Figure 11. These curves are constructed on the basis that the limiting flow F_L prevails for a time t_L such that the total flow out to the time t_L would be the same as would have entered the drain if no local restrictions were present when the flow had dropped to F_L.

The effect of a local resistance which will limit the flow to a unit length of a drain, from each side, to the finite value F_L, when the drain is inundated to a depth H at the beginning of a drainage cycle, may be accounted for, approximately, in the following way:

The flow to a drain, F, in the absence of a local resistance, would be given by Formula (36) and would plot as shown by the solid line of Figure 10. Let it now be supposed that a local resistance limits the flow to the finite value F_L. The total amount of water to be drained away will be properly represented if a time t_L is determined so that the flow quantity $F_L t_L$ is the same as that which would have drained out in the time t_e if the local resistance had been absent, and the solid curve is shifted to the right to the position of the dashed curve. The shaded area of Figure 10 represents the volume of water discharged up to the time t_1 in the absence of a local resistance.

The early stages of a drainage cycle may be idealized as a semi-infinite case to which Formula (58) applies. This idealization will be appropriate until the effect of the drain begins to be felt at the point midway between drains. With this idealization, the volume discharged to the drain is given by Formula (61) and the relation described above can be expressed in the form

$$F_L t_L = HV\sqrt{\frac{4\alpha t_e}{\pi}}.$$

The right-hand member of this expression is an evaluation of the volume of flow represented by the shaded area of Figure 10. By making use of the relation

$$\alpha = \frac{KD}{V} \text{ or } V = \frac{KD}{\alpha}$$

and by multiplying the numerator and denominator of the right-hand member by L^2 followed by a multiplication of both sides by 4, the expression can be put in the form

$$4F_L t_L = \frac{4KDHL^2}{\alpha\sqrt{\pi}L} \frac{\sqrt{4\alpha t_e}}{L}$$

or, by rearrangement

$$\left(\frac{4\alpha t_L}{L^2}\right) = \frac{4}{\sqrt{\pi}} \frac{\left(\frac{\sqrt{4\alpha t_e}}{L}\right)}{\frac{F_L}{\left(\frac{KDH}{L}\right)}}. \tag{49}$$

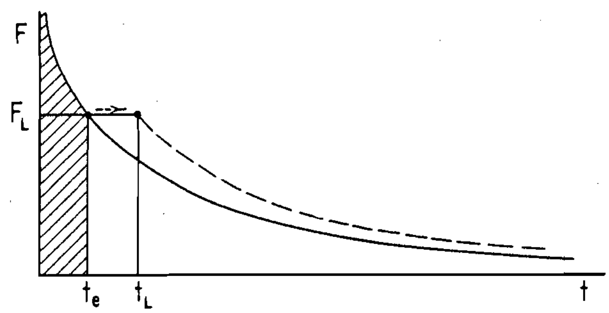


FIGURE 10.—*Flow to a drain from one side.*

As an example of the use of this relation choose an ordinate $F\Big/\frac{KDH}{L}=5.0$ and read from the chart of Figure 11.

$$\left(\frac{4\alpha t_e}{L^2}\right)=0.052.$$

Then

$$\frac{\sqrt{4\alpha t_e}}{L}=\sqrt{0.052}=0.228$$

and

$$\left(\frac{4\alpha t_L}{L^2}\right)=\frac{4}{\sqrt{\pi}}\frac{\left(\frac{\sqrt{4\alpha t_e}}{L}\right)}{\frac{F}{\left(\frac{KDH}{L}\right)}}=\frac{(2.25676)(0.228)}{5}=0.103.$$

If the curve of Figure 10 is now shifted to the right until it passes through the point $\frac{F}{\left(\frac{KDH}{L}\right)}=5.0$ and $\frac{4\alpha t}{2}=0.103$, an approximation to the actual drainage performance will be obtained and the total volume of water to be drained away will be properly accounted for.

It remains to determine whether the limitations of the idealization used have been exceeded. The effect of the drain will begin to be felt at the point midway between drains when, approximately

$$\frac{\left(\frac{L}{2}\right)}{\sqrt{4\alpha t}}\geq 2 \text{ or } \frac{4\alpha t_e}{L^2}=\frac{1}{16}=0.0625.$$

The value read for $\frac{4\alpha t_e}{L^2}$ was 0.052. This is less than 0.0625 and it is concluded that the above computation is valid.

It can be concluded, therefore, that the actual drainage performance will be represented by $\frac{F}{\left(\frac{KDH}{L}\right)}$ out to $\left(\frac{4\alpha t}{L^2}\right)=0.103$ and by the shifted curve, as represented by the dashed line, for all greater values of $\frac{4\alpha t}{L^2}$.

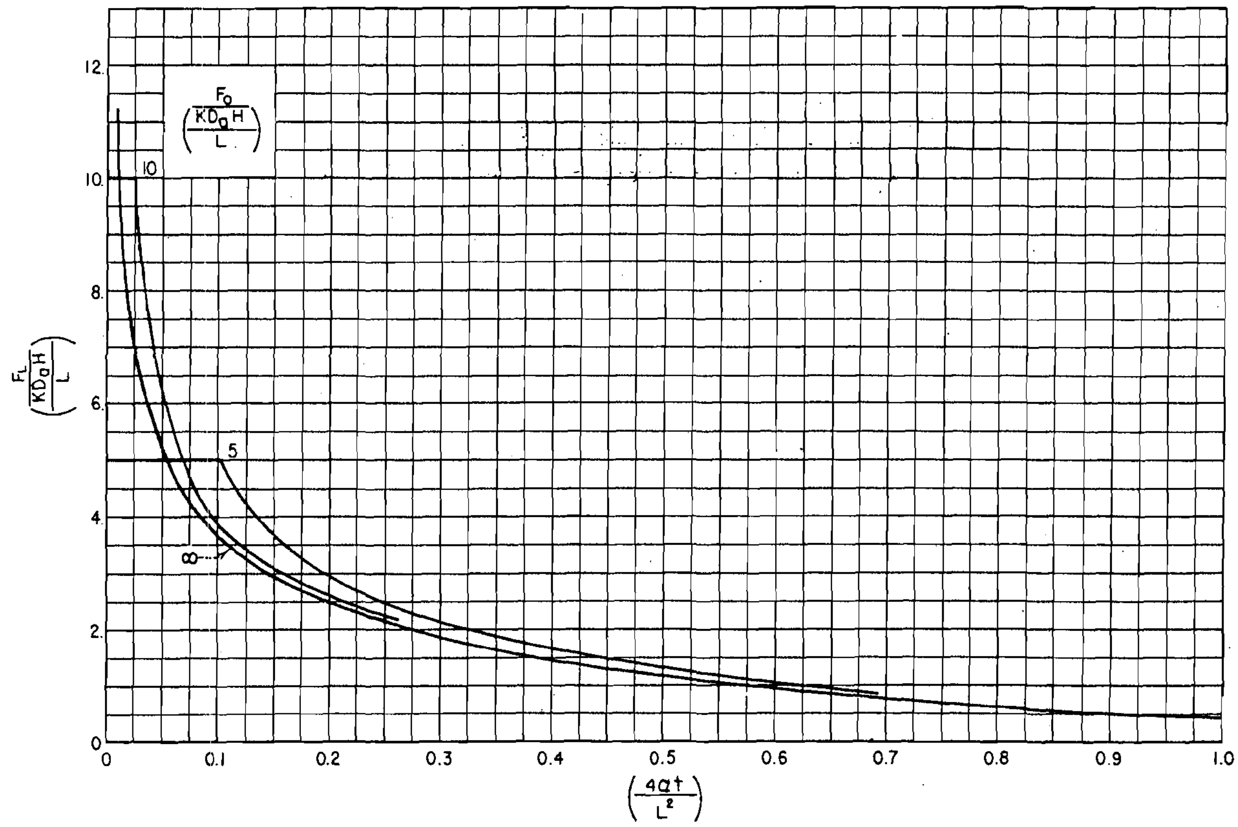


FIGURE 11.—*Flow to a drain from one side as limited by a local resistance due to convergence.*

The Dumm, Tapp, Moody procedure, described in the next section, ignores the high initial rates which may prevail for a brief period following each irrigation. Drainage systems so designed may have the drainage rates temporarily limited by the drain capacity but this will not prevent an effective performance, as a reference to Figure 11 will show, since the $\frac{F_L}{\left(\frac{KD_aH}{L}\right)}$ values of real systems will generally be of the order of 30.

The Methods of Dumm, Tapp, and Moody

The methods of these investigations were developed to provide an expeditious procedure for estimating drain spacings and for estimating the flows to be carried by drains. It was realized that cases would occur where the drains would have to be placed near the barrier and it was desired to make the treatment of such cases as effective as possible. The following considerations provide the fundamental basis for these developments.

(1) While the first irrigation in a new area or the first irrigation of the season in an older area that has been previously irrigated may yield a nearly uniform drainable depth, such as is postulated in the development of Formula (34), application of a succession of irrigations will develop a ground-water mound with a rounded profile. The height of this mound will be greatest at the point midway between the drains.

(2) A study of a number of observed profiles indicates that the shape of the mound can be well represented by the fourth-degree parabola.

$$y_{x0}=8y_{c0}\left(\frac{x}{L}-\frac{3x^2}{L^2}+\frac{4x^3}{L^3}-\frac{2x^4}{L^4}\right). \tag{50}$$

A solution of Equation (3) which has this configuration at time zero is

$$y_{xt}=\frac{192y_{c0}}{\pi^5}\sum_{m=0}^{\infty}\frac{(2m+1)^2\pi^2-8}{(2m+1)^5}e^{-\frac{(2m+1)^2\pi^2\alpha t}{L^2}}\sin\frac{(2m+1)\pi x}{L}. \tag{51}$$

At the point midway between drains, this takes the form

$$\frac{y_{ct}}{y_{c0}}=\frac{192}{\pi^3}\sum_{n=1,3,5,\ldots}^{\infty}(-1)^{\frac{n-1}{2}}\frac{\left(n^2-\frac{8}{\pi^2}\right)}{n^5}e^{-\frac{\pi^2n^2\alpha t}{L^2}}. \tag{52}$$

A plot of this profile is shown on Figure 12.

(3) A flow to a drain from one side can be estimated approximately from the gradients derived from Formula (51). After sufficient time has elapsed so that only the first term remains, this is found to be

$$q=\frac{\pi KDy_{ct}}{L} \tag{53}$$

where y_{ct} represents the value of y at $x=L/2$ at the time t.

(4) Computations are based on the depth of water over the drains at the point midway between the drains (y_{ct}) and on the basis that the two drains are at the same elevation.

(5) Each computation of the sinking of the water table during an interval between irrigations is made on the basis of a D_a value appropriate at the beginning of the interval. The winter drain-out period may be split into a few intervals, when the drains are near the barrier, in order to permit corrections to be made in the D_a value even though no irrigation or precipitation may be applied.

(6) The principle of superposition is not used. Each computation makes a new beginning.

(7) Drain spacings are determined by cut-and-try procedure. By use of Formula (47), or from other considerations, a trial drain spacing is selected. This is then checked by the computation procedure described above, carrying the computations over a period of several years, if necessary, to evaluate the buildup of the ground-water mound by the contributions from previous irrigations. The drain spacing which will hold the rise of the

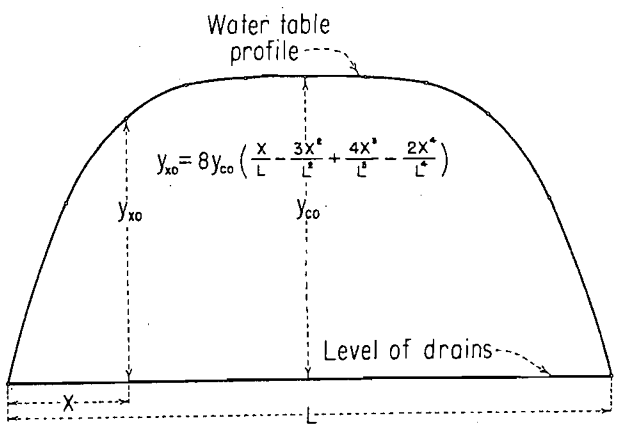


FIGURE 12.—*Water table profile used as a basis for the Dumm, Tapp, and Moody procedure.*

water table within the prescribed limits at the end of the last of a succession of irrigation seasons is accepted.

(8) When the drain is placed on or very near the barrier, Formula (41) as represented by Chart 9 is used. The flow to the drain from one side is then taken as

$$q=\frac{2Kh_{ct}^{2*}}{L}. \qquad (54)$$

(9) Where $\frac{d}{y_{c0}} \leq 0.1$ the computations are made on the basis that the drain is on the barrier.

(10) If the depth to the barrier is greater than $L/4$ the computations are made on the basis that $d=L/4$.

A worked example is shown below.

Example

A field in need of drainage has a barrier 28 feet below the surface of the ground. Drains are to be installed at a depth of 8 feet, and it is desired to keep the water table at least 4 feet below the ground surface at all times. The permeability of the soil is 5 inches per hour and the voids ratio appropriate for drainage is 0.15. Deep percolation from each irrigation is estimated to be 1 inch of water. An early season snowmelt will add a similar amount. It is assumed that increments of recharge will occur on the following dates:

Source of recharge	Date	Drainable increment (inches)	Time (days)	Interval (hours)
Snowmelt	Apr. 22	6.67		
First irrigation	June 6	6.67	45	1,080
Second irrigation	July 1	6.67	25	600
Third irrigation	July 21	6.67	20	480
Fourth irrigation	Aug. 4	6.67	14	336
Fifth irrigation	Aug. 18	6.67	14	336
Sixth irrigation	Sept. 1	6.67	14	336
Totals		46.69	132	3,168

Inch and hour units will be used as a basis for the computations to illustrate the use of consistent units. Then

$K=5$ inches per hour

$d=240$ inches

$V=0.15$ (dimensionless).

The depth of ground water produced by each increment reaching the water table is $\frac{1}{0.15}=6.67$ inches. After the drainage system has been in operation for a period of years a proper drain spacing would permit the ground water to rise to the allowable limit at the time of the last irrigation of the season. By starting with such a ground-water position the computations can be limited to a single year. The following computation is made by the methods of Dumm, Tapp, and Moody.

A trial spacing of 1,200 feet proves to be too short and a trial spacing of 1,600 feet proves to be too wide. By plotting the computed water-table levels on cross section paper and making a linear interpolation, it is indicated that a spacing of about 1,400 feet should be suitable. The computation for this spacing is shown below. The computed position of the water table after the September 1 irrigation is found to be 49.37 inches. The desired value is 48 inches.

$L=16{,}800$ inches (1,400 feet)

$L^2=282{,}240{,}000$ inches^2

$\frac{K}{V}=33.333.$

The computation for the 1,400-foot spacing will be checked by superposition to illustrate the operation of this method, and to bring out certain limitations of these procedures. In the previous computation a new D_a value was computed at each step and the computation proceeded one step at a time. In the computation to follow the D_a value will be taken as 240 inches or 20 feet. The corresponding α value will be

$$\alpha=\frac{(5)(240)}{0.15}=8{,}000 \text{ in}^2/\text{hour}.$$

The effect contributed by each increment at the end of the irrigation season will be computed and the results added. The (h/H) values are read from Figure 7 for $(x/L)=0.5$.

Check by superposition.

$L=16{,}800$ inches$=1{,}400$ feet

$L^2=282{,}240{,}000$

$\alpha=8{,}000$ in.^{2} per hr

$\alpha/L^2=(28.345)(10)^{-6}.$ (1/hr)

*A value consistent with Boussinesq's development would be $q=\frac{(1.7247)Kh_{ct}^2}{L}$. The formula with the Factor 2 has an empirical basis but accords well with the available observations. It may be conceded that, in practice, the axis of the drain is generally above the barrier.

Time	Time interval (hours)	Added increment (inches)	y_{co} (inches)	$D_e=\left(d+\frac{y_{co}}{2}\right)$ (inches)	$\alpha=\left(\frac{KD_e}{V}\right)$	$\left(\frac{\alpha t}{L^2}\right)$	$\frac{y_{ct}}{y_{co}}$	y_{ct}
September 1, 1960	5, 592	--------	48. 00	264	8, 800	0. 1744	0. 212	10. 17
April 22, 1961	1, 080	6. 67	16. 84	248	8, 281	0. 0317	0. 840	14. 14
June 6, 1961	600	6. 67	20. 82	250	8, 347	0. 0177	0. 934	19. 44
July 1, 1961	480	6. 67	26. 11	253	8, 435	0. 0143	0. 956	24. 96
July 21, 1961	336	6. 67	31. 63	256	8, 527	0. 0102	0. 980	31. 00
August 4, 1961	336	6. 67	37. 66	259	8, 628	0. 0102	0. 980	36. 90
August 18, 1961	336	6. 67	43. 58	262	8, 726	0. 0104	0. 980	42. 70
September 1, 1961	--------	6. 67	49. 37	------------	----------	--------	--------	--------
Totals	8, 760	46. 69	--------	------------	----------	--------	--------	--------

Time	Time interval (hours)	$\left(\frac{\alpha t}{L^2}\right)$	$\left(\frac{h}{H}\right)$	H (inches)	h
September 1, 1960	8, 760	0. 2483	0. 120	48. 00	5. 76
April 22, 1961	3, 168	0. 0898	0. 530	6. 67	3. 54
June 6, 1961	2, 088	0. 0592	0. 710	6. 67	4. 74
July 1, 1961	1, 488	0. 0422	0. 835	6. 67	5. 57
July 21, 1961	1, 008	0. 0286	0. 925	6. 67	6. 17
August 4, 1961	672	0. 0190	0. 980	6. 67	6. 54
August 18, 1961	336	0. 0095	1. 000	6. 67	6. 67
September 1, 1961	0	0	1. 000	6. 67	6. 67
Total	----------	----------	----------	----------	45. 66

Comparison

A comparison of these results indicates a difference of $49.37-45.66=3.71$ inches in the computed height of the ground water at the end of the irrigation season. The analytical work for the superposition method is based upon the assumptions that the added increments of ground water are uniform in depth over the drain spacing L and that the drain holds the water table to its own level at the drain. Since the superposition computation is based upon a saturated depth of only 240 inches (20 feet) available for the flow of ground water and the 48-inch depth initial increment is treated as uniform over the width L between drains, the computed rise of 45.66 inches should be an overestimate unless some fault can be found with the assumptions upon which the method is based. The assumption of a uniform ground-water increment arising from deep percolation from an application of irrigation water seems reasonable enough but the boundary condition may be questioned on the basis that there may be, and generally is, a local resistance to flow near the drain which is not accounted for in the analytical development leading to Formula (34). This local resistance comes from the convergence of flow near the drain. On this basis it may be conceded that the computed rise is not necessarily too high.

The Dumm, Tapp, and Moody procedure may introduce some errors because of difficulty with initial conditions. If the fourth-degree parabola does truly represent the ground-water profiles, then no trouble can actually arise from this source. But it is found that if the winter drainout period is broken into a number of steps to permit a more accurate representation of the saturated depth, the computed depths increase with the number of steps. A moderate change of this sort should be expected but the magnitude of the computed increase indicates that another factor is present. The difficulty can be traced to the initial conditions. The curve of y_{ct} versus time is flat near $t=0$ and it can be seen that if the number of steps in the winter drainout period were progressively increased, the computations would soon begin to indicate that there was no drainout at all. The process of starting over with each irrigation may introduce a small error of this kind into each computation because of an imperfect representation of the initial conditions. Replacement of the fourth-degree parabola with a sinusoidal variation, as represented by the first term of the series of Formula (34), could be expected to cure the trouble encountered in the winter drainout period but, if the computations were based upon the depth midway between drains the amount of irrigation water to be drained away would be underestimated and the computed rises would be too low. The fourth-degree parabola, therefore, appears as a compromise between conflicting requirements. Its general adequacy is supported by comparisons with field observations on the performance of operating drainage systems. The superposition procedure works well in this case, but it would run into difficulties if the drains were near the barrier.

Canal Seepage

If a canal leaks at the rate q_1 per unit length, the height h of the mound built up at the time t will be given by the expression:

$$h=\frac{q_1 x}{2\pi KD}\sqrt{\pi}\int_{\frac{x}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}\,du \qquad (55)$$

where x represents the distance from the canal, which is assumed to have a straight alinement. At $x=0$ this expression becomes indeterminate and is to be replaced by the relation:

$$h_0=\frac{q_1\sqrt{4\pi\alpha t}}{2\pi KD}. \qquad (56)$$

The leakage is assumed to flow away from the canal on both sides so that the flow $\left(\frac{q_1}{2}\right)$ goes each way. Formula (55) is a solution of Equation (3) and is subject to the limitations imposed by the simplification introduced in the development of this differential equation.

The integral appearing in this equation can be expressed in terms of tabulated functions in the form:

$$\sqrt{\pi}\int_{\frac{x}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}\,du=$$

$$\pi\left[\frac{1}{\sqrt{\pi}}\frac{e^{-\frac{x^2}{4\alpha t}}}{\left(\frac{x}{\sqrt{4\alpha t}}\right)}-1+\frac{2}{\sqrt{\pi}}\int_0^{\frac{x}{\sqrt{4\alpha t}}}e^{-u^2}du\right] \qquad (57)$$

Values of the function are given on Table 4.

When the canal is in a river valley, the leakage which reaches the water table may ultimately find its way back to the river. The pattern of the return flow will be given by Equation (66) since the leakage from each element of length of the canal can be considered as a source which would behave as a recharge well. This formula is appropriate if all of the leakage does return to the river. If the canal parallels the river for some distance L_c then the total leakage in the reach will be $Q=q_1L_c$. This is the Q value to be used in Equation (66).

Example

A canal leaks at the rate of 1-cubic-foot-per-second per mile of length and parallels a river at a distance of 2 miles. The aquifer properties are:

$K=0.0005$ foot per second
$D=70$ feet $\quad KD=0.035$
$V=0.17$
$\alpha=\frac{KD}{V}=0.206$ foot^2 per second.

Estimate the height of the mound under the canal, the height at a distance of 1 mile from the canal, and the return flow to the river if the canal parallels the river for 15 miles and has been in operation for 6 months.

$q_1=\frac{1}{5{,}280}=0.0001894$ ft^2 per sec
$L_c=(5{,}280)(15)=79{,}200$ feet
$Q=q_1\ L_c=(0.0001894)(79{,}200)=15$ ft^3 per sec
$t=15{,}768{,}000$ seconds.

At the canal, where $x=0$,

$$h_0=\frac{q_1\sqrt{4\pi\alpha t}}{2\pi KD}=\frac{(0.0001894)\sqrt{4\pi(0.206)(15{,}768{,}000)}}{2\pi(0.035)1.77245}$$

$$=5.51 \text{ feet.}$$

This is the rise at the center of the canal. At $x=5{,}280$ feet

$$\frac{x}{\sqrt{4\alpha t}}=\frac{5{,}280}{\sqrt{(4)(0.206)(15{,}768{,}000)}}=\frac{5{,}280}{3{,}605}=1.46.$$

From Table 4

$$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}\,du=0.02168$$

and

$$h=\frac{q_1 x}{2\pi KD}\,(0.02168)=\frac{(1)(5{,}280)(0.02168)}{(5{,}280)(2)\pi(0.035)}$$

$$=0.098 \text{ feet.}$$

This is the rise of the water table 1 mile from the canal if the river is absent.

The values computed in this way represent the heights of the mound if the aquifer extended to great distances on either side of the canal. The presence of the river can be accounted for by the use of an image. In this case it may be idealized as a pumped drain paralleling the river at a distance of 2 miles on the side opposite to the canal and having an inflow rate equal to the seepage rate of the canal. With this arrangement, the level of the water table at the river will be represented as unchanged. This will include in the computations a recognition of the ability of the river to control water-table levels along its course.

The point 1 mile from the canal is 3 miles from the drain. Then for the image, $\frac{x}{\sqrt{4\alpha t}}=(3)(1.46)=4.38$. A reference to Table 4 will show that the effect of the image will be negligibly small at this time. This will be true for the point under the canal also. The estimated heights therefore remain at 5.51 feet and 0.098 foot at the canal and 1 mile from the canal, respectively. The return flow to the river is from Formula (66) on the basis that all of the leakage returns to the river.

$$q=-Q\left[1-\frac{2}{\sqrt{\pi}}\int_0^{\frac{x_1}{\sqrt{4\alpha t}}} e^{-u^2}du\right]$$

$$=-(15)(0.03895)=-0.584 \text{ ft}^3/\text{sec.}$$

The minus sign indicates that the flow is toward the river.

Bank Storage

When a reservoir is filled there is a flow of water into the banks and when the reservoir is emptied some of the water stored in the banks returns again to the reservoir. Similar changes accompany rising and falling stream stages.

A solution of Equation (3) subject to the conditions,

when $x=0$ $h=0$ for $t>0$,

when $t=0$ $h=H$ for $x>0$, is

$$h=H\frac{2}{\sqrt{\pi}}\int_0^{\frac{x}{\sqrt{4\alpha t}}} e^{-u^2}du. \qquad (58)$$

The integral which appears here is the tabulated "Probability Integral." The notation is as shown on Figure 13.

The flow F toward the reservoir at x is:

$$F=\frac{2HKD}{\sqrt{\pi}}\,\frac{e^{-\frac{x^2}{4\alpha t}}}{\sqrt{4\alpha t}}. \qquad (59)$$

The flow out of the bank at $x=0$ at the time t is:

$$F_0=\frac{HKD}{\sqrt{\pi\alpha t}}. \qquad (60)$$

The total flow from the bank into the reservoir up to the time t is:

$$R=HV\sqrt{\frac{4\alpha t}{\pi}}. \qquad (61)$$

When the reservoir goes through a yearly cycle of filling and emptying, there is an amount of water which flows into the banks when the reservoir level is high and returns again when the level is low. If a reservoir goes through a regular cycle of filling and emptying year after year the

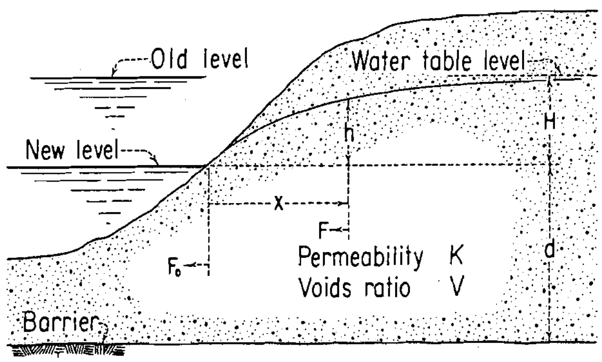


FIGURE 13.—*Bank storage conditions.*

TABLE 4.—*Values of* $\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$

$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$
0. 00010	17721. 4	0. 00062	2855. 7	0. 00240	735. 39	0. 00760	230. 09
0. 00011	16110. 1	0. 00063	2810. 3	0. 00250	705. 84	0. 00770	227. 06
0. 00012	14767. 3	0. 00064	2766. 3	0. 00260	678. 58	0. 00780	224. 11
0. 00013	13631. 1	0. 00065	2723. 7	0. 00270	653. 33	0. 00790	221. 23
0. 00014	12657. 2	0. 00066	2682. 4	0. 00280	629. 88	0. 00800	218. 43
0. 00015	11813. 2	0. 00067	2642. 3	0. 00290	608. 05	0. 00810	215. 69
0. 00016	11074. 7	0. 00068	2603. 4	0. 00300	587. 68	0. 00820	213. 03
0. 00017	10423. 1	0. 00069	2565. 6	0. 00310	568. 62	0. 00830	210. 42
0. 00018	9843. 8	0. 00070	2528. 9	0. 00320	550. 76	0. 00840	207. 88
0. 00019	9325. 6	0. 00071	2493. 3	0. 00330	533. 97	0. 00850	205. 40
0. 00020	8859. 1	0. 00072	2458. 6	0. 00340	518. 17	0. 00860	202. 97
0. 00021	8437. 1	0. 00073	2424. 9	0. 00350	503. 28	0. 00870	200. 60
0. 00022	8053. 5	0. 00074	2392. 1	0. 00360	489. 21	0. 00880	198. 29
0. 00023	7703. 2	0. 00075	2360. 1	0. 00370	475. 91	0. 00890	196. 03
0. 00024	7382. 1	0. 00076	2329. 0	0. 00380	463. 30	0. 00900	193. 81
0. 00025	7086. 7	0. 00077	2298. 7	0. 00390	451. 34	0. 00910	191. 65
0. 00026	6814. 0	0. 00078	2269. 2	0. 00400	439. 98	0. 00920	189. 53
0. 00027	6561. 5	0. 00079	2240. 5	0. 00410	429. 17	0. 00930	187. 46
0. 00028	6327. 1	0. 00080	2212. 4	0. 00420	418. 88	0. 00940	185. 43
0. 00029	6108. 8	0. 00081	2185. 1	0. 00430	409. 06	0. 00950	183. 45
0. 00030	5905. 0	0. 00082	2158. 4	0. 00440	399. 70	0. 00960	181. 51
0. 00031	5714. 5	0. 00083	2132. 3	0. 00450	390. 75	0. 00970	179. 60
0. 00032	5535. 8	0. 00084	2106. 9	0. 00460	382. 18	0. 00980	177. 74
0. 00033	5367. 9	0. 00085	2082. 1	0. 00470	373. 98	0. 00990	175. 91
0. 00034	5210. 0	0. 00086	2057. 9	0. 00480	366. 13	0. 01000	174. 12
0. 00035	5061. 0	0. 00087	2034. 2	0. 00490	358. 59	0. 01100	158. 010
0. 00036	4920. 3	0. 00088	2011. 0	0. 00500	351. 36	0. 01200	144. 584
0. 00037	4787. 3	0. 00089	1988. 4	0. 00510	344. 41	0. 01300	133. 224
0. 00038	4661. 2	0. 00090	1966. 3	0. 00520	337. 72	0. 01400	123. 487
0. 00039	4541. 6	0. 00091	1944. 6	0. 00530	331. 29	0. 01500	115. 049
0. 00040	4428. 0	0. 00092	1923. 4	0. 00540	325. 10	0. 01600	107. 665
0. 00041	4319. 9	0. 00093	1902. 7	0. 00550	319. 13	0. 01700	101. 151
0. 00042	4217. 0	0. 00094	1882. 4	0. 00560	313. 38	0. 01800	95. 360
0. 00043	4118. 8	0. 00095	1862. 6	0. 00570	307. 83	0. 01900	90. 179
0. 00044	4025. 2	0. 00096	1843. 2	0. 00580	302. 46	0. 02000	85. 517
0. 00045	3935. 6	0. 00097	1824. 1	0. 00590	297. 28	0. 02100	81. 298
0. 00046	3850. 0	0. 00098	1805. 5	0. 00600	292. 28	0. 02200	77. 463
0. 00047	3768. 0	0. 00099	1787. 2	0. 00610	287. 44	0. 02300	73. 962
0. 00048	3689. 5	0. 00100	1769. 3	0. 00620	282. 75	0. 02400	70. 753
0. 00049	3614. 1	0. 00110	1608. 18	0. 00630	278. 21	0. 02500	67. 801
0. 00050	3541. 8	0. 00120	1473. 91	0. 00640	273. 82	0. 02600	65. 076
0. 00051	3472. 3	0. 00130	1360. 29	0. 00650	269. 56	0. 02700	62. 553
0. 00052	3405. 4	0. 00140	1262. 90	0. 00660	265. 42	0. 02800	60. 210
0. 00053	3341. 1	0. 00150	1178. 50	0. 00670	261. 42	0. 02900	58. 029
0. 00054	3279. 2	0. 00160	1104. 64	0. 00680	257. 53	0. 03000	55. 993
0. 00055	3219. 5	0. 00170	1039. 48	0. 00690	253. 75	0. 03100	54. 089
0. 00056	3162. 0	0. 00180	981. 56	0. 00700	250. 08	0. 03200	52. 304
0. 00057	3106. 4	0. 00190	929. 73	0. 00710	246. 51	0. 03300	50. 628
0. 00058	3052. 8	0. 00200	883. 09	0. 00720	243. 05	0. 03400	49. 050
0. 00059	3001. 0	0. 00210	840. 89	0. 00730	239. 67	0. 03500	47. 562
0. 00060	2950. 9	0. 00220	802. 52	0. 00740	236. 39	0. 03600	46. 157
0. 00061	2902. 5	0. 00230	767. 49	0. 00750	233. 20	0. 03700	44. 828

TABLE 4.—*Values of* $\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$
0. 03800	43. 569	0. 09000	16. 712	0. 14200	9. 5913	0. 19400	6. 3365
0. 03900	42. 375	0. 09100	16. 497	0. 14300	9. 5058	0. 19500	6. 2914
0. 04000	41. 241	0. 09200	16. 287	0. 14400	9. 4215	0. 19600	6. 2467
0. 04100	40. 162	0. 09300	16. 082	0. 14500	9. 3383	0. 19700	6. 2026
0. 04200	39. 134	0. 09400	15. 881	0. 14600	9. 2564	0. 19800	6. 1589
0. 04300	38. 154	0. 09500	15. 684	0. 14700	9. 1755	0. 19900	6. 1156
0. 04400	37. 219	0. 09600	15. 491	0. 14800	9. 0958	0. 20000	6. 0728
0. 04500	36. 326	0. 09700	15. 303	0. 14900	9. 0172	0. 20100	6. 0305
0. 04600	35. 472	0. 09800	15. 118	0. 15000	8. 9396	0. 20200	5. 9886
0. 04700	34. 653	0. 09900	14. 937	0. 15100	8. 8631	0. 20300	5. 9471
0. 04800	33. 870	0. 10000	14. 760	0. 15200	8. 7877	0. 20400	5. 9060
0. 04900	33. 118	0. 10100	14. 5862	0. 15300	8. 7132	0. 20500	5. 8654
0. 05000	32. 396	0. 10200	14. 4159	0. 15400	8. 6397	0. 20600	5. 8251
0. 05100	31. 703	0. 10300	14. 2489	0. 15500	8. 5672	0. 20700	5. 7853
0. 05200	31. 036	0. 10400	14. 0852	0. 15600	8. 4957	0. 20800	5. 7459
0. 05300	30. 395	0. 10500	13. 9247	0. 15700	8. 4251	0. 20900	5. 7068
0. 05400	29. 777	0. 10600	13. 7672	0. 15800	8. 3554	0. 21000	5. 6682
0. 05500	29. 182	0. 10700	13. 6127	0. 15900	8. 2866	0. 21100	5. 6299
0. 05600	28. 609	0. 10800	13. 4611	0. 16000	8. 2186	0. 21200	5. 5920
0. 05700	28. 055	0. 10900	13. 3123	0. 16100	8. 1516	0. 21300	5. 5545
0. 05800	27. 521	0. 11000	13. 1662	0. 16200	8. 0854	0. 21400	5. 5173
0. 05900	27. 005	0. 11100	13. 0228	0. 16300	8. 0200	0. 21500	5. 4805
0. 06000	26. 506	0. 11200	12. 8820	0. 16400	7. 9554	0. 21600	5. 4441
0. 06100	26. 023	0. 11300	12. 7437	0. 16500	7. 8917	0. 21700	5. 4080
0. 06200	25. 556	0. 11400	12. 6079	0. 16600	7. 8287	0. 21800	5. 3723
0. 06300	25. 104	0. 11500	12. 4744	0. 16700	7. 7665	0. 21900	5. 3369
0. 06400	24. 666	0. 11600	12. 3433	0. 16800	7. 7051	0. 22000	5. 3018
0. 06500	24. 242	0. 11700	12. 2145	0. 16900	7. 6444	0. 22100	5. 2671
0. 06600	23. 831	0. 11800	12. 0879	0. 17000	7. 5845	0. 22200	5. 2327
0. 06700	23. 432	0. 11900	11. 9634	0. 17100	7. 5253	0. 22300	5. 1986
0. 06800	23. 044	0. 12000	11. 8410	0. 17200	7. 4667	0. 22400	5. 1649
0. 06900	22. 668	0. 12100	11. 7207	0. 17300	7. 4089	0. 22500	5. 1315
0. 07000	22. 303	0. 12200	11. 6024	0. 17400	7. 3518	0. 22600	5. 0983
0. 07100	21. 948	0. 12300	11. 4861	0. 17500	7. 2953	0. 22700	5. 0655
0. 07200	21. 603	0. 12400	11. 3716	0. 17600	7. 2395	0. 22800	5. 0330
0. 07300	21. 268	0. 12500	11. 2590	0. 17700	7. 1844	0. 22900	5. 0008
0. 07400	20. 942	0. 12600	11. 1482	0. 17800	7. 1299	0. 23000	4. 9688
0. 07500	20. 624	0. 12700	11. 0392	0. 17900	7. 0760	0. 23100	4. 9372
0. 07600	20. 315	0. 12800	10. 9320	0. 18000	7. 0227	0. 23200	4. 9059
0. 07700	20. 014	0. 12900	10. 8264	0. 18100	6. 9700	0. 23300	4. 8748
0. 07800	19. 720	0. 13000	10. 7224	0. 18200	6. 9180	0. 23400	4. 8440
0. 07900	19. 434	0. 13100	10. 6201	0. 18300	6. 8665	0. 23500	4. 8135
0. 08000	19. 156	0. 13200	10. 5194	0. 18400	6. 8156	0. 23600	4. 7833
0. 08100	18. 884	0. 13300	10. 4202	0. 18500	6. 7653	0. 23700	4. 7533
0. 08200	18. 619	0. 13400	10. 3225	0. 18600	6. 7155	0. 23800	4. 7236
0. 08300	18. 360	0. 13500	10. 2263	0. 18700	6. 6663	0. 23900	4. 6942
0. 08400	18. 108	0. 13600	10. 1315	0. 18800	6. 6176	0. 24000	4. 6650
0. 08500	17. 861	0. 13700	10. 0381	0. 18900	6, 5695	0. 24100	4. 6361
0. 08600	17. 621	0. 13800	9. 9461	0. 19000	6. 5219	0. 24200	4. 6074
0. 08700	17. 385	0. 13900	9. 8555	0. 19100	6. 4748	0. 24300	4. 5790
0. 08800	17. 156	0. 14000	9. 7661	0. 19200	6. 4282	0. 24400	4. 5508
0. 08900	16. 931	0. 14100	9. 6781	0. 19300	6. 3821	0. 24500	4. 5229

TABLE 4.—*Values of* $\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}\,du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}\,du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}\,du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}\,du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}\,du$
0.24600	4.4952	0.29800	3.3267	0.35000	2.5306	0.40200	1.9614
0.24700	4.4677	0.29900	3.3086	0.35100	2.5178	0.40300	1.9521
0.24800	4.4405	0.30000	3.2905	0.35200	2.5051	0.40400	1.9429
0.24900	4.4135	0.30100	3.2725	0.35300	2.4925	0.40500	1.9337
0.25000	4.3868	0.30200	3.2547	0.35400	2.4800	0.40600	1.9245
0.25100	4.3602	0.30300	3.2371	0.35500	2.4676	0.40700	1.9154
0.25200	4.3339	0.30400	3.2195	0.35600	2.4552	0.40800	1.9064
0.25300	4.3079	0.30500	3.2021	0.35700	2.4429	0.40900	1.8974
0.25400	4.2820	0.30600	3.1848	0.35800	2.4307	0.41000	1.8885
0.25500	4.2563	0.30700	3.1676	0.35900	2.4186	0.41100	1.8796
0.25600	4.2309	0.30800	3.1506	0.36000	2.4065	0.41200	1.8708
0.25700	4.2057	0.30900	3.1336	0.36100	2.3946	0.41300	1.8620
0.25800	4.1807	0.31000	3.1168	0.36200	2.3827	0.41400	1.8532
0.25900	4.1559	0.31100	3.1001	0.36300	2.3708	0.41500	1.8445
0.26000	4.1313	0.31200	3.0836	0.36400	2.3591	0.41600	1.8359
0.26100	4,1068	0.31300	3.0671	0.36500	2.3474	0.41700	1.8273
0.26200	4.0826	0.31400	3.0507	0.36600	2.3358	0.41800	1.8188
0.26300	4.0586	0.31500	3.0345	0.36700	2.3243	0.41900	1.8103
0.26400	4.0348	0.31600	3.0184	0.36800	2.3128	0.42000	1.8018
0.26500	4.0112	0.31700	3.0024	0.36900	2.3014	0.42100	1.7934
0.26600	3.9878	0.31800	2.9865	0.37000	2.2901	0.42200	1.7851
0.26700	3.9645	0.31900	2.9707	0.37100	2.2788	0.42300	1.7768
0.26800	3.9415	0.32000	2.9550	0.37200	2.2676	0.42400	1.7685
0.26900	3.9186	0.32100	2.9395	0.37300	2.2565	0.42500	1.7603
0.27000	3.8959	0.32200	2.9240	0.37400	2.2455	0.42600	1.7521
0.27100	3.8734	0.32300	2.9086	0.37500	2.2345	0.42700	1.7440
0.27200	3.8510	0.32400	2.8934	0.37600	2.2236	0.42800	1.7359
0.27300	3.8289	0.32500	2.8782	0.37700	2.2127	0.42900	1.7279
0.27400	3.8069	0.32600	2.8632	0.37800	2.2019	0.43000	1.7199
0.27500	3.7851	0.32700	2.8482	0.37900	2.1912	0.43100	1.7120
0.27600	3.7634	0.32800	2.8334	0.38000	2.1805	0.43200	1.7041
0.27700	3.7419	0.32900	2.8186	0.38100	2.1699	0.43300	1.6962
0.27800	3.7206	0.33000	2.8040	0.38200	2.1594	0.43400	1.6884
0.27900	3.6995	0.33100	2.7895	0.38300	2,1489	0.43500	1.6806
0.28000	3.6785	0.33200	2.7750	0.38400	2.1385	0.43600	1.6729
0.28100	3.6577	0.33300	2.7606	0.38500	2.1282	0.43700	1.6652
0.28200	3.6370	0.33400	2.7464	0.38600	2.1179	0.43800	1.6575
0.28300	3.6165	0.33500	2.7322	0.38700	2.1077	0.43900	1.6499
0.28400	3.5962	0.33600	2.7182	0.38800	2.0975	0.44000	1.6424
0.28500	3.5760	0.33700	2.7042	0.38900	2.0875	0.44100	1.6348
0.28600	3.5559	0.33800	2.6903	0.39000	2.0774	0.44200	1.6274
0.28700	3.5360	0.33900	2.6765	0.39100	2.0674	0.44300	1.6199
0.28800	3.5163	0.34000	2.6628	0.39200	2.0575	0.44400	1.6125
0.28900	3.4967	0.34100	2.6492	0.39300	2.0476	0.44500	1.6052
0.29000	3.4772	0.34200	2.6357	0.39400	2.0378	0.44600	1.5978
0.29100	3.4579	0.34300	2.6222	0.39500	2.0281	0.44700	1.5905
0.29200	3.4388	0.34400	2.6089	0.39600	2.0184	0.44800	1.5833
0.29300	3.4198	0.34500	2.5956	0.39700	2.0088	0.44900	1.5761
0.29400	3.4009	0.34600	2.5824	0.39800	1.9992	0.45000	1.5689
0.29500	3.3821	0.34700	2.5693	0.39900	1.9897	0.45100	1.5618
0.29600	3.3635	0.34800	2.5563	0.40000	1.9802	0.45200	1.5547
0.29700	3.3451	0.34900	2.5434	0.40100	1.9708	0.45300	1.5477

TABLE 4.—*Values of* $\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$
0.45400	1.5406	0.50600	1.2217	0.55800	0.97558	0.61000	0.78289
0.45500	1.5337	0.50700	1.2164	0.55900	0.97141	0.61100	0.77961
0.45600	1.5267	0.50800	1.2111	0.56000	0.96727	0.61200	0.77635
0.45700	1.5198	0.50900	1.2058	0.56100	0.96316	0.61300	0.77310
0.45800	1.5129	0.51000	1.2005	0.56200	0.95906	0.61400	0.76987
0.45900	1.5061	0.51100	1.1953	0.56300	0.95497	0.61500	0.76665
0.46000	1.4993	0.51200	1.1901	0.56400	0.95091	0.61600	0.76345
0.46100	1.4926	0.51300	1.1849	0.56500	0.94687	0.61700	0.76026
0.46200	1.4858	0.51400	1.1797	0.56600	0.94284	0.61800	0.75709
0.46300	1.4791	0.51500	1.1746	0.56700	0.93883	0.61900	0.75393
0.46400	1.4725	0.51600	1.1695	0.56800	0.93484	0.62000	0.75078
0.46500	1.4659	0.51700	1.1644	0.56900	0.93087	0.62100	0.74765
0.46600	1.4593	0.51800	1.1593	0.57000	0.92692	0.62200	0.74453
0.46700	1.4527	0.51900	1.1543	0.57100	0.92299	0.62300	0.74143
0.46800	1.4462	0.52000	1.1493	0.57200	0.91908	0.62400	0.73833
0.46900	1.4397	0.52100	1.1443	0.57300	0.91518	0.62500	0.73526
0.47000	1.4333	0.52200	1.1393	0.57400	0.91130	0.62600	0.73219
0.47100	1.4269	0.52300	1.1344	0.57500	0.90744	0.62700	0.72915
0.47200	1.4205	0.52400	1.1294	0.57600	0.90360	0.62800	0.72611
0.47300	1.4141	0.52500	1.1245	0.57700	0.89977	0.62900	0.72309
0.47400	1.4078	0.52600	1.1197	0.57800	0.89596	0.63000	0.72007
0.47500	1.4015	0.52700	1.1148	0.57900	0.89218	0.63100	0.71708
0.47600	1.3953	0.52800	1.1100	0.58000	0.88840	0.63200	0.71410
0.47700	1.3890	0.52900	1.1052	0.58100	0.88465	0.63300	0.71113
0.47800	1.3829	0.53000	1.1004	0.58200	0.88091	0.63400	0.70817
0.47900	1.3767	0.53100	1.0957	0.58300	0.87719	0.63500	0.70523
0.48000	1.3706	0.53200	1.0910	0.58400	0.87349	0.63600	0.70230
0.48100	1.3645	0.53300	1.0862	0.58500	0.86980	0.63700	0.69938
0.48200	1.3584	0.53400	1.0816	0.58600	0.86613	0.63800	0.69647
0.48300	1.3524	0.53500	1.0769	0.58700	0.86248	0.63900	0.69358
0.48400	1.3464	0.53600	1.0723	0.58800	0.85884	0.64000	0.69070
0.48500	1.3404	0.53700	1.0676	0.58900	0.85522	0.64100	0.68784
0.48600	1.3345	0.53800	1.0630	0.59000	0.85162	0.64200	0.68498
0.48700	1.3286	0.53900	1.0585	0.59100	0.84803	0.64300	0.68214
0.48800	1.3227	0.54000	1.0539	0.59200	0.84446	0.64400	0.67931
0.48900	1.3168	0.54100	1.0494	0.59300	0.84091	0.64500	0.67650
0.49000	1.3110	0.54200	1.0449	0.59400	0.83737	0.64600	0.67369
0.49100	1.3052	0.54300	1.0404	0.59500	0.83385	0.64700	0.67090
0.49200	1.2995	0.54400	1.0359	0.59600	0.83034	0.64800	0.66812
0.49300	1.2937	0.54500	1.0315	0.59700	0.82685	0.64900	0.66535
0.49400	1.2880	0.54600	1.0271	0.59800	0.82337	0.65000	0.66260
0.49500	1.2823	0.54700	1.0227	0.59900	0.81991	0.65100	0.65985
0.49600	1.2767	0.54800	1.0183	0.60000	0.81647	0.65200	0.65712
0.49700	1.2711	0.54900	1.0139	0.60100	0.81305	0.65300	0.65440
0.49800	1.2655	0.55000	1.0096	0.60200	0.80964	0.65400	0.65169
0.49900	1.2599	0.55100	1.00525	0.60300	0.80624	0.65500	0.64900
0.50000	1.2544	0.55200	1.00095	0.60400	0.80286	0.65600	0.64631
0.50100	1.2489	0.55300	0.99667	0.60500	0.79949	0.65700	0.64364
0.50200	1.2434	0.55400	0.99241	0.60600	0.79614	0.65800	0.64098
0.50300	1.2379	0.55500	0.98817	0.60700	0.79281	0.65900	0.63833
0.50400	1.2325	0.55600	0.98396	0.60800	0.78949	0.66000	0.63569
0.50500	1.2271	0.55700	0.97976	0.60900	0.78618	0.66100	0.63307

TABLE 4.—*Values of* $\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}\,du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}\,du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}\,du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}\,du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}\,du$
0.66200	0.63045	0.71500	0.50679	0.76800	0.40798	0.82100	0.32865
0.66300	0.62785	0.71600	0.54072	0.76900	0.40632	0.82200	0.32732
0.66400	0.62525	0.71700	0.50265	0.77000	0.40466	0.82300	0.32598
0.66500	0.62267	0.71800	0.50060	0.77100	0.40301	0.82400	0.32466
0.66600	0.62010	0.71900	0.49855	0.77200	0.40137	0.82500	0.32333
0.66700	0.61754	0.72000	0.49651	0.77300	0.39974	0.82600	0.32202
0.66800	0.61500	0.72100	0.49447	0.77400	0.39811	0.82700	0.32071
0.66900	0.61246	0.72200	0.49245	0.77500	0.39649	0.82800	0.31940
0.67000	0.60993	0.72300	0.49044	0.77600	0.39487	0.82900	0.31810
0.67100	0.60742	0.72400	0.48843	0.77700	0.39326	0.83000	0.31681
0.67200	0.60492	0.72500	0.48643	0.77800	0.39166	0.83100	0.31552
0.67300	0.60242	0.72600	0.48444	0.77900	0.39007	0.83200	0.31424
0.67400	0.59994	0.72700	0.48247	0.78000	0.38848	0.83300	0.31296
0.67500	0.59747	0.72800	0.48049	0.78100	0.38690	0.83400	0.31168
0.67600	0.59501	0.72900	0.47853	0.78200	0.38532	0.83500	0.31042
0.67700	0.59255	0.73000	0.47657	0.78300	0.38375	0.83600	0.30915
0.67800	0.59012	0.73100	0.47462	0.78400	0.38219	0.83700	0.30789
0.67900	0.58768	0.73200	0.47268	0.78500	0.38063	0.83800	0.30664
0.68000	0.58527	0.73300	0.47075	0.78600	0.37908	0.83900	0.30539
0.68100	0.58286	0.73400	0.46883	0.78700	0.37754	0.84000	0.30415
0.68200	0.58046	0.73500	0.46691	0.78800	0.37600	0.84100	0.30291
0.68300	0.57807	0.73600	0.46500	0.78900	0.37447	0.84200	0.30168
0.68400	0.57569	0.73700	0.46310	0.79000	0.37294	0.84300	0.30045
0.68500	0.57333	0.73800	0.46121	0.79100	0.37143	0.84400	0.29923
0.68600	0.57097	0.73900	0.45933	0.79200	0.36991	0.84500	0.29801
0.68700	0.56862	0.74000	0.45745	0.79300	0.36841	0.84600	0.29680
0.68800	0.56628	0.74100	0.45559	0.79400	0.36691	0.84700	0.29559
0.68900	0.56396	0.74200	0.45373	0.79500	0.36541	0.84800	0.29439
0.69000	0.56164	0.74300	0.45187	0.79600	0.36393	0.84900	0.29319
0.69100	0.55933	0.74400	0.45003	0.79700	0.36244	0.85000	0.29199
0.69200	0.55703	0.74500	0.44819	0.79800	0.36097	0.85100	0.29080
0.69300	0.55474	0.74600	0.44636	0.79900	0.35950	0.85200	0.28962
0.69400	0.55246	0.74700	0.44454	0.80000	0.35804	0.85300	0.28844
0.69500	0.55020	0.74800	0.44273	0.80100	0.35658	0.85400	0.28727
0.69600	0.54794	0.74900	0.44092	0.80200	0.35513	0.85500	0.28610
0.69700	0.54569	0.75000	0.43912	0.80300	0.35368	0.85600	0.28493
0.69800	0.54345	0.75100	0.43733	0.80400	0.35224	0.85700	0.28377
0.69900	0.54122	0.75200	0.43554	0.80500	0.35081	0.85800	0.28262
0.70000	0.53900	0.75300	0.43377	0.80600	0.34938	0.85900	0.28147
0.70100	0.53679	0.75400	0.43200	0.80700	0.34796	0.86000	0.28032
0.70200	0.53459	0.75500	0.43023	0.80800	0.34654	0.86100	0.27918
0.70300	0.53239	0.75600	0.42848	0.80900	0.34513	0.86200	0.27804
0.70400	0.53021	0.75700	0.42673	0.81000	0.34373	0.86300	0.27691
0.70500	0.52804	0.75800	0.42499	0.81100	0.34233	0.86400	0.27578
0.70600	0.52587	0.75900	0.42326	0.81200	0.34094	0.86500	0.27466
0.70700	0.52371	0.76000	0.42153	0.81300	0.33955	0.86600	0.27354
0.70800	0.52157	0.76100	0.41982	0.81400	0.33817	0.86700	0.27242
0.70900	0.51943	0.76200	0.41811	0.81500	0.33679	0.86800	0.27132
0.71000	0.51730	0.76300	0.41640	0.81600	0.33542	0.86900	0.27021
0.71100	0.51518	0.76400	0.41470	0.81700	0.33406	0.87000	0.26911
0.71200	0.51307	0.76500	0.41301	0.81800	0.33270	0.87100	0.26801
0.71300	0.51097	0.76600	0.41133	0.81900	0.33134	0.87200	0.26692
0.71400	0.50888	0.76700	0.40965	0.82000	0.32999	0.87300	0.26583

TABLE 4.—*Values of* $\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$
0.87400	0.26475	0.92600	0.21404	0.97800	0.17288	1.03000	0.13945
0.87500	0.26367	0.92700	0.21316	0.97900	0.17216	1.03100	0.13887
0.87600	0.26260	0.92800	0.21229	0.98000	0.17146	1.03200	0.13829
0.87700	0.26153	0.92900	0.21142	0.98100	0.17075	1.03300	0.13772
0.87800	0.26046	0.93000	0.21056	0.98200	0.17005	1.03400	0.13715
0.87900	0.25940	0.93100	0.20970	0.98300	0.16935	1.03500	0.13658
0.88000	0.25834	0.93200	0.20884	0.98400	0.16865	1.03600	0.13602
0.88100	0.25729	0.93300	0.20798	0.98500	0.16796	1.03700	0.13546
0.88200	0.25624	0.93400	0.20713	0.98600	0.16727	1.03800	0.13489
0.88300	0.25520	0.93500	0.20629	0.98700	0.16658	1.03900	0.13433
0.88400	0.25416	0.93600	0.20544	0.98800	0.16590	1.04000	0.13378
0.88500	0.25312	0.93700	0.20460	0.98900	0.16521	1.04100	0.13322
0.88600	0.25209	0.93800	0.20376	0.99000	0.16453	1.04200	0.13267
0.88700	0.25106	0.03900	0.20293	0.99100	0.16386	1.04300	0.13212
0.88800	0.25004	0.94000	0.20210	0.99200	0.16318	1.04400	0.13157
0.88900	0.24902	0.94100	0.20127	0.99300	0.16251	1.04500	0.13103
0.89000	0.24800	0.94200	0.20045	0.99400	0.16184	1.04600	0.13048
0.89100	0.24699	0.94300	0.19963	0.99500	0.16117	1.04700	0.12994
0.89200	0.24598	0.94400	0.19881	0.99600	0.16051	1.04800	0.12940
0.89300	0.24498	0.94500	0.19799	0.99700	0.15985	1.04900	0.12887
0.89400	0.24398	0.94600	0.19718	0.99800	0.15919	1.05000	0.12833
0.89500	0.24298	0.94700	0.19638	0.99900	0.15853	1.05100	0.12780
0.89600	0.24199	0.94800	0.19557	1.00000	0.15788	1.05200	0.12727
0.89700	0.24101	0.94900	0.19477	1.00100	0.15723	1.05300	0.12674
0.89800	0.24002	0.95000	0.19397	1.00200	0.15658	1.05400	0.12621
0.89900	0.23904	0.95100	0.19318	1.00300	0.15593	1.05500	0.12569
0.90000	0.23807	0.95200	0.19239	1.00400	0.15529	1.05600	0.12517
0.90100	0.23710	0.95300	0.19160	1.00500	0.15465	1.05700	0.12465
0.90200	0.23613	0.95400	0.19081	1.00600	0.15401	1.05800	0.12413
0.90300	0.23517	0.95500	0.19003	1.00700	0.15338	1.05900	0.12361
0.90400	0.23421	0.95600	0.18925	1.00800	0.15275	1.06000	0.12310
0.90500	0.23325	0.95700	0.18847	1.00900	0.15212	1.06100	0.12259
0.90600	0.23230	0.95800	0.18770	1.01000	0.15149	1.06200	0.12208
0.90700	0.23135	0.95900	0.18693	1.01100	0.15086	1.06300	0.12157
0.90800	0.23040	0.96000	0.18616	1.01200	0.15024	1.06400	0.12106
0.90900	0.22946	0.96100	0.18540	1.01300	0.14962	1.06500	0.12056
0.91000	0.22853	0.96200	0.18464	1.01400	0.14900	1.06600	0.12006
0.91100	0.22759	0.96300	0.18388	1.01500	0.14839	1.06700	0.11956
0.91200	0.22667	0.96400	0.18313	1.01600	0.14777	1.06800	0.11906
0.91300	0.22574	0.96500	0.18238	1.01700	0.14716	1.06900	0.11857
0.91400	0.22482	0.96600	0.18163	1.01800	0.14656	1.07000	0.11807
0.91500	0.22390	0.96700	0.18088	1.01900	0.14595	1.07100	0.11758
0.91600	0.22298	0.96800	0.18014	1.02000	0.14535	1.07200	0.11709
0.91700	0.22207	0.96900	0.17940	1.02100	0.14475	1.07300	0.11660
0.91800	0.22117	0.97000	0.17866	1.02200	0.14415	1.07400	0.11612
0.91900	0.22026	0.97100	0.17793	1.02300	0.14355	1.07500	0.11563
0.92000	0.21936	0.97200	0.17720	1.02400	0.14296	1.07600	0.11515
0.92100	0.21847	0.97300	0.17647	1.02500	0.14237	1.07700	0.11467
0.92200	0.21757	0.97400	0.17575	1.02600	0.14178	1.07800	0.11419
0.92300	0.21668	0.97500	0.17502	1.02700	0.14119	1.07900	0.11372
0.92400	0.21580	0.97600	0.17430	1.02800	0.14061	1.08000	0.11324
0.92500	0.21492	0.97700	0.17359	1.02900	0.14003	1.08100	0.11277

TABLE 4.—*Values of* $\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u^2}\, du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u^2} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u^2} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u^2} du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u^2} du$
1.08200	0.11230	1.13400	0.09026	1.18600	0.07240	1.23800	0.05793
1.08300	0.11183	1.13500	0.08988	1.18700	0.07209	1.23900	0.05768
1.08400	0.11136	1.13600	0.08951	1.18800	0.07178	1.24000	0.05743
1.08500	0.11090	1.13700	0.08913	1.18900	0.07148	1.24100	0.05718
1.08600	0.11044	1.13800	0.08875	1.19000	0.07117	1.24200	0.05694
1.08700	0.10997	1.13900	0.08838	1.19100	0.07087	1.24300	0.05669
1.08800	0.10951	1.14000	0.08801	1.19200	0.07057	1.24400	0.05645
1.08900	0.10906	1.14100	0.08763	1.19300	0.07027	1.24500	0.05621
1.09000	0.10860	1.14200	0.08727	1.19400	0.06997	1.24600	0.05596
1.09100	0.10815	1.14300	0.08690	1.19500	0.06967	1.24700	0.05572
1.09200	0.10770	1.14400	0.08653	1.19600	0.06937	1.24800	0.05548
1.09300	0.10725	1.14500	0.08617	1.19700	0.06908	1.24900	0.05524
1.09400	0.10680	1.14600	0.08580	1.19800	0.06878	1.25000	0.05500
1.09500	0.10635	1.14700	0.08544	1.19900	0.06849	1.25100	0.05477
1.09600	0.10591	1.14800	0.08508	1.20000	0.06820	1.25200	0.05453
1.09700	0.10546	1.14900	0.08472	1.20100	0.06790	1.25300	0.05430
1.09800	0.10502	1.15000	0.08436	1.20200	0.06761	1.25400	0.05406
1.09900	0.10458	1.15100	0.08401	1.20300	0.06733	1.25500	0.05383
1.10000	0.10415	1.15200	0.08365	1.20400	0.06704	1.25600	0.05360
1.10100	0.10371	1.15300	0.08330	1.20500	0.06675	1.25700	0.05336
1.10200	0.10327	1.15400	0.08294	1.20600	0.06647	1.25800	0.05313
1.10300	0.10284	1.15500	0.08259	1.20700	0.06618	1.25900	0.05290
1.10400	0.10241	1.15600	0.08224	1.20800	0.06590	1.26000	0.05267
1.10500	0.10198	1.15700	0.08190	1.20900	0.06562	1.26100	0.05245
1.10600	0.10156	1.15800	0.08155	1.21000	0.06534	1.26200	0.05222
1.10700	0.10113	1.15900	0.08120	1.21100	0.06506	1.26300	0.05199
1.10800	0.10071	1.16000	0.08086	1.21200	0.06478	1.26400	0.05177
1.10900	0.10028	1.16100	0.08052	1.21300	0.06450	1.26500	0.05155
1.11000	0.09986	1.16200	0.08018	1.21400	0.06423	1.26600	0.05132
1.11100	0.09944	1.16300	0.07984	1.21500	0.06395	1.26700	0.05110
1.11200	0.09903	1.16400	0.07950	1.21600	0.06368	1.26800	0.05088
1.11300	0.09861	1.16500	0.07916	1.21700	0.06341	1.26900	0.05066
1.11400	0.09820	1.16600	0.07883	1.21800	0.06313	1.27000	0.05044
1.11500	0.09779	1.16700	0.07849	1.21900	0.06286	1.27100	0.05022
1.11600	0.09738	1.16800	0.07816	1.22000	0.06259	1.27200	0.05000
1.11700	0.09697	1.16900	0.07783	1.22100	0.06233	1.27300	0.04979
1.11800	0.09656	1.17000	0.07750	1.22200	0.06206	1.27400	0.04957
1.11900	0.09615	1.17100	0.07717	1.22300	0.06179	1.27500	0.04935
1.12000	0.09575	1.17200	0.07684	1.22400	0.06153	1.27600	0.04914
1.12100	0.09535	1.17300	0.07652	1.22500	0.06126	1.27700	0.04893
1.12200	0.09495	1.17400	0.07619	1.22600	0.06100	1.27800	0.04871
1.12300	0.09455	1.17500	0.07587	1.22700	0.06074	1.27900	0.04850
1.12400	0.09415	1.17600	0.07555	1.22800	0.06048	1.28000	0.04829
1.12500	0.09376	1.17700	0.07523	1.22900	0.06022	1.28100	0.04808
1.12600	0.09336	1.17800	0.07491	1.23000	0.05996	1.28200	0.04787
1.12700	0.09297	1.17900	0.07459	1.23100	0.05970	1.28300	0.04767
1.12800	0.09258	1.18000	0.07427	1.23200	0.05945	1.28400	0.04746
1.12900	0.09219	1.18100	0.07396	1.23300	0.05919	1.28500	0.04725
1.13000	0.09180	1.18200	0.07364	1.23400	0.05894	1.28600	0.04705
1.13100	0.09141	1.18300	0.07333	1.23500	0.05868	1.28700	0.04684
1.13200	0.09103	1.18400	0.07302	1.23600	0.05843	1.28800	0.04664
1.13300	0.09065	1.18500	0.07271	1.23700	0.05818	1.28900	0.04644

TABLE 4.—*Values of* $\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}\,du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$
1. 29000	0. 04623	1. 34200	0. 03680	1. 39400	0. 02920	1. 44600	0. 02311
1. 29100	0. 04603	1. 34300	0. 03664	1. 39500	0. 02907	1. 44700	0. 02300
1. 29200	0. 04583	1. 34400	0. 03647	1. 39600	0. 02894	1. 44800	0. 02290
1. 29300	0. 04563	1. 34500	0. 03631	1. 39700	0. 02882	1. 44900	0. 02280
1. 29400	0. 04543	1. 34600	0. 03615	1. 39800	0. 02869	1. 45000	0. 02269
1. 29500	0. 04524	1. 34700	0. 03599	1. 39900	0. 02856	1. 45100	0. 02259
1. 29600	0. 04504	1. 34800	0. 03584	1. 40000	0. 02843	1. 45200	0. 02249
1. 29700	0. 04484	1. 34900	0. 03568	1. 40100	0. 02830	1. 45300	0. 02239
1. 29800	0. 04465	1. 35000	0. 03552	1. 40200	0. 02818	1. 45400	0. 02228
1. 29900	0. 04445	1. 35100	0. 03536	1. 40300	0. 02805	1. 45500	0. 02218
1. 30000	0. 04426	1. 35200	0. 03521	1. 40400	0. 02793	1. 45600	0. 02208
1. 30100	0. 04406	1. 35300	0. 03505	1. 40500	0. 02780	1. 45700	0. 02198
1. 30200	0. 04387	1. 35400	0. 03490	1. 40600	0. 02768	1. 45800	0. 02188
1. 30300	0. 04368	1. 35500	0. 03474	1. 40700	0. 02755	1. 45900	0. 02178
1. 30400	0. 04349	1. 35600	0. 03459	1. 40800	0. 02743	1. 46000	0. 02168
1. 30500	0. 04330	1. 35700	0. 03444	1. 40900	0. 02731	1. 46100	0. 02159
1. 30600	0. 04311	1. 35800	0. 03428	1. 41000	0. 02718	1. 46200	0. 02149
1. 30700	0. 04292	1. 35900	0. 03413	1. 41100	0. 02706	1. 46300	0. 02139
1. 30800	0. 04274	1. 36000	0. 03398	1. 41200	0. 02694	1. 46400	0. 02129
1. 30900	0. 04255	1. 36100	0. 03383	1. 41300	0. 02682	1. 46500	0. 02120
1. 31000	0. 04236	1. 36200	0. 03368	1. 41400	0. 02670	1. 46600	0. 02110
1. 31100	0. 04218	1. 36300	0. 03353	1. 41500	0. 02658	1. 46700	0. 02100
1. 31200	0. 04199	1. 36400	0. 03338	1. 41600	0. 02646	1. 46800	0. 02091
1. 31300	0. 04181	1. 36500	0. 03323	1. 41700	0. 02634	1. 46900	0. 02081
1. 31400	0. 04163	1. 36600	0. 03309	1. 41800	0. 02622	1. 47000	0. 02072
1. 31500	0. 04144	1. 36700	0. 03294	1. 41900	0. 02611	1. 47100	0. 02062
1. 31600	0. 04126	1. 36800	0. 03279	1. 42000	0. 02599	1. 47200	0. 02053
1. 31700	0. 04108	1. 36900	0. 03265	1. 42100	0. 02587	1. 47300	0. 02044
1. 31800	0. 04090	1. 37000	0. 03250	1. 42200	0. 02575	1. 47400	0. 02034
1. 31900	0. 04072	1. 37100	0. 03236	1. 42300	0. 02564	1. 47500	0. 02025
1. 32000	0. 04054	1. 37200	0. 03222	1. 42400	0. 02552	1. 47600	0. 02016
1. 32100	0. 04037	1. 37300	0. 03207	1. 42500	0. 02541	1. 47700	0. 02007
1. 32200	0. 04019	1. 37400	0. 03193	1. 42600	0. 02529	1. 47800	0. 01998
1. 32300	0. 04001	1. 37500	0. 03179	1. 42700	0. 02518	1. 47900	0. 01988
1. 32400	0. 03984	1. 37600	0. 03165	1. 42800	0. 02507	1. 48000	0. 01979
1. 32500	0. 03966	1. 37700	0. 03151	1. 42900	0. 02495	1. 48100	0. 01970
1. 32600	0. 03949	1. 37800	0. 03137	1. 43000	0. 02484	1. 48200	0. 01961
1. 32700	0. 03931	1. 37900	0. 03123	1. 43100	0. 02473	1. 48300	0. 01952
1. 32800	0. 03914	1. 38000	0. 03109	1. 43200	0. 02462	1. 48400	0. 01943
1. 32900	0. 03897	1. 38100	0. 03095	1. 43300	0. 02451	1. 48500	0. 01935
1. 33000	0. 03880	1. 38200	0. 03081	1. 43400	0. 02440	1. 48600	0. 01926
1. 33100	0. 03863	1. 38300	0. 03068	1. 43500	0. 02429	1. 48700	0. 01917
1. 33200	0. 03846	1. 38400	0. 03054	1. 43600	0. 02418	1. 48800	0. 01908
1. 33300	0. 03829	1. 38500	0. 03040	1. 43700	0. 02407	1. 48900	0. 01899
1. 33400	0. 03812	1. 38600	0. 03027	1. 43800	0. 02396	1. 49000	0. 01891
1. 33500	0. 03795	1. 38700	0. 03013	1. 43900	0. 02385	1. 49100	0. 01882
1. 33600	0. 03779	1. 38800	0. 03000	1. 44000	0. 02374	1. 49200	0. 01873
1. 33700	0. 03762	1. 38900	0. 02986	1. 44100	0. 02364	1. 49300	0. 01865
1. 33800	0. 03745	1. 39000	0. 02973	1. 44200	0. 02353	1. 49400	0. 01856
1. 33900	0. 03729	1. 39100	0. 02960	1. 44300	0. 02342	1. 49500	0. 01848
1. 34000	0. 03713	1. 39200	0. 02947	1. 44400	0. 02332	1. 49600	0. 01839
1. 34100	0. 03696	1. 39300	0. 02934	1. 44500	0. 02321	1. 49700	0. 01831

TABLE 4.—*Values of* $\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$
1.49800	0.01823	1.55000	0.01433	1.60200	0.01123	1.65400	0.00877
1.49900	0.01814	1.55100	0.01426	1.60300	0.01117	1.65500	0.00872
1.50000	0.01806	1.55200	0.01420	1.60400	0.01112	1.65600	0.00868
1.50100	0.01798	1.55300	0.01413	1.60500	0.01107	1.65700	0.00864
1.50200	0.01789	1.55400	0.01406	1.60600	0.01102	1.65800	0.00860
1.50300	0.01781	1.55500	0.01400	1.60700	0.01096	1.65900	0.00856
1.50400	0.01773	1.55600	0.01393	1.60800	0.01091	1.66000	0.00852
1.50500	0.01765	1.55700	0.01387	1.60900	0.01086	1.66100	0.00848
1.50600	0.01757	1.55800	0.01380	1.61000	0.01081	1.66200	0.00844
1.50700	0.01749	1.55900	0.01374	1.61100	0.01076	1.66300	0.00839
1.50800	0.01741	1.56000	0.01368	1.61200	0.01071	1.66400	0.00835
1.50900	0.01733	1.56100	0.01361	1.61300	0.01066	1.66500	0.00831
1.51000	0.01725	1.56200	0.01355	1.61400	0.01061	1.66600	0.00827
1.51100	0.01717	1.56300	0.01348	1.61500	0.01056	1.66700	0.00823
1.51200	0.01709	1.56400	0.01342	1.61600	0.01051	1.66800	0.00820
1.51300	0.01701	1.56500	0.01336	1.61700	0.01046	1.66900	0.00816
1.51400	0.01693	1.56600	0.01330	1.61800	0.01041	1.67000	0.00812
1.51500	0.01685	1.56700	0.01323	1.61900	0.01036	1.67100	0.00808
1.51600	0.01678	1.56800	0.01317	1.62000	0.01031	1.67200	0.00804
1.51700	0.01670	1.56900	0.01311	1.62100	0.01026	1.67300	0.00800
1.51800	0.01662	1.57000	0.01305	1.62200	0.01021	1.67400	0.00796
1.51900	0.01654	1.57100	0.01299	1.62300	0.01016	1.67500	0.00792
1.52000	0.01647	1.57200	0.01293	1.62400	0.01011	1.67600	0.00789
1.52100	0.01639	1.57300	0.01287	1.62500	0.01007	1.67700	0.00785
1.52200	0.01632	1.57400	0.01281	1.62600	0.01002	1.67800	0.00781
1.52300	0.01624	1.57500	0.01275	1.62700	0.00997	1.67900	0.00777
1.52400	0.01617	1.57600	0.01269	1.62800	0.00992	1.68000	0.00773
1.52500	0.01609	1.57700	0.01263	1.62900	0.00988	1.68100	0.00770
1.52600	0.01602	1.57800	0.01257	1.63000	0.00983	1.68200	0.00766
1.52700	0.01594	1.57900	0.01251	1.63100	0.00978	1.68300	0.00762
1.52800	0.01587	1.58000	0.01245	1.63200	0.00974	1.68400	0.00759
1.52900	0.01580	1.58100	0.01239	1.63300	0.00969	1.68500	0.00755
1.53000	0.01572	1.58200	0.01234	1.63400	0.00964	1.68600	0.00751
1.53100	0.01565	1.58300	0.01228	1.63500	0.00960	1.68700	0.00748
1.53200	0.01558	1.58400	0.01222	1.63600	0.00955	1.68800	0.00744
1.53300	0.01551	1.58500	0.01216	1.63700	0.00951	1.68900	0.00741
1.53400	0.01543	1.58600	0.01211	1.63800	0.00946	1.69000	0.00737
1.53500	0.01536	1.58700	0.01205	1.63900	0.00942	1.69100	0.00733
1.53600	0.01529	1.58800	0.01199	1.64000	0.00937	1.69200	0.00730
1.53700	0.01522	1.58900	0.01194	1.64100	0.00933	1.69300	0.00726
1.53800	0.01515	1.59000	0.01188	1.64200	0.00928	1.69400	0.00723
1.53900	0.01508	1.59100	0.01182	1.64300	0.00924	1.69500	0.00719
1.54000	0.01501	1.59200	0.01177	1.64400	0.00919	1.69600	0.00716
1.54100	0.01494	1.59300	0.01171	1.64500	0.00915	1.69700	0.00712
1.54200	0.01487	1.59400	0.01166	1.64600	0.00911	1.69800	0.00709
1.54300	0.01480	1.59500	0.01160	1.64700	0.00906	1.69900	0.00706
1.54400	0.01473	1.59600	0.01155	1.64800	0.00902	1.70000	0.00702
1.54500	0.01467	1.59700	0.01149	1.64900	0.00898	1.70100	0.00699
1.54600	0.01460	1.59800	0.01144	1.65000	0.00893	1.70200	0.00695
1.54700	0.01453	1.59900	0.01139	1.65100	0.00889	1.70300	0.00692
1.54800	0.01446	1.60000	0.01133	1.65200	0.00885	1.70400	0.00689
1.54900	0.01440	1.60100	0.01128	1.65300	0.00881	1.70500	0.00685

TABLE 4.—*Values of* $\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$
1.70600	0.00682	1.75900	0.00526	1.81200	0.00404	1.86500	0.00309
1.70700	0.00679	1.76000	0.00523	1.81300	0.00402	1.86600	0.00308
1.70800	0.00675	1.76100	0.00521	1.81400	0.00400	1.86700	0.00306
1.70900	0.00672	1.76200	0.00518	1.81500	0.00398	1.86800	0.00305
1.71000	0.00669	1.76300	0.00516	1.81600	0.00396	1.86900	0.00303
1.71100	0.00666	1.76400	0.00513	1.81700	0.00394	1.87000	0.00302
1.71200	0.00662	1.76500	0.00511	1.81800	0.00392	1.87100	0.00300
1.71300	0.00659	1.76600	0.00508	1.81900	0.00390	1.87200	0.00299
1.71400	0.00656	1.76700	0.00506	1.82000	0.00388	1.87300	0.00297
1.71500	0.00653	1.76800	0.00503	1.82100	0.00386	1.87400	0.00296
1.71600	0.00650	1.76900	0.00501	1.82200	0.00384	1.87500	0.00294
1.71700	0.00646	1.77000	0.00498	1.82300	0.00382	1.87600	0.00293
1.71800	0.00643	1.77100	0.00496	1.82400	0.00381	1.87700	0.00291
1.71900	0.00640	1.77200	0.00493	1.82500	0.00379	1.87800	0.00290
1.72000	0.00637	1.77300	0.00491	1.82600	0.00377	1.87900	0.00288
1.72100	0.00634	1.77400	0.00488	1.82700	0.00375	1.88000	0.00287
1.72200	0.00631	1.77500	0.00486	1.82800	0.00373	1.88100	0.00285
1.72300	0.00628	1.77600	0.00484	1.82900	0.00371	1.88200	0.00284
1.72400	0.00625	1.77700	0.00481	1.83000	0.00369	1.88300	0.00282
1.72500	0.00622	1.77800	0.00479	1.83100	0.00367	1.88400	0.00281
1.72600	0.00619	1.77900	0.00476	1.83200	0.00366	1.88500	0.00279
1.72700	0.00616	1.78000	0.00474	1.83300	0.00364	1.88600	0.00278
1.72800	0.00613	1.78100	0.00472	1.83400	0.00362	1.88700	0.00277
1.72900	0.00610	1.78200	0.00469	1.83500	0.00360	1.88800	0.00275
1.73000	0.00607	1.78300	0.00467	1.83600	0.00358	1.88900	0.00274
1.73100	0.00604	1.78400	0.00465	1.83700	0.00356	1.89000	0.00272
1.73200	0.00601	1.78500	0.00462	1.83800	0.00355	1.89100	0.00271
1.73300	0.00598	1.78600	0.00460	1.83900	0.00353	1.89200	0.00270
1.73400	0.00595	1.78700	0.00458	1.84000	0.00351	1.89300	0.00268
1.73500	0.00592	1.78800	0.00456	1.84100	0.00349	1.89400	0.00267
1.73600	0.00589	1.78900	0.00453	1.84200	0.00348	1.89500	0.00265
1.73700	0.00586	1.79000	0.00451	1.84300	0.00346	1.89600	0.00264
1.73800	0.00583	1.79100	0.00449	1.84400	0.00344	1.89700	0.00263
1.73900	0.00580	1.79200	0.00447	1.84500	0.00342	1.89800	0.00261
1.74000	0.00578	1.79300	0.00444	1.84600	0.00341	1.89900	0.00260
1.74100	0.00575	1.79400	0.00442	1.84700	0.00339	1.90000	0.00259
1.74200	0.00572	1.79500	0.00440	1.84800	0.00337	1.90100	0.00257
1.74300	0.00569	1.79600	0.00438	1.84900	0.00336	1.90200	0.00256
1.74400	0.00566	1.79700	0.00436	1.85000	0.00334	1.90300	0.00255
1.74500	0.00564	1.79800	0.00433	1.85100	0.00332	1.90400	0.00253
1.74600	0.00561	1.79900	0.00431	1.85200	0.00330	1.90500	0.00252
1.74700	0.00558	1.80000	0.00429	1.85300	0.00329	1.90600	0.00251
1.74800	0.00555	1.80100	0.00427	1.85400	0.00327	1.90700	0.00250
1.74900	0.00553	1.80200	0.00425	1.85500	0.00325	1.90800	0.00248
1.75000	0.00550	1.80300	0.00423	1.85600	0.00324	1.90900	0.00247
1.75100	0.00547	1.80400	0.00421	1.85700	0.00322	1.91000	0.00246
1.75200	0.00545	1.80500	0.00419	1.85800	0.00321	1.91100	0.00244
1.75300	0.00542	1.80600	0.00417	1.85900	0.00319	1.91200	0.00243
1.75400	0.00539	1.80700	0.00414	1.86000	0.00317	1.91300	0.00242
1.75500	0.00537	1.80800	0.00412	1.86100	0.00316	1.91400	0.00241
1.75600	0.00534	1.80900	0.00410	1.86200	0.00314	1.91500	0.00240
1.75700	0.00531	1.81000	0.00408	1.86300	0.00313	1.91600	0.00238
1.75800	0.00529	1.81100	0.00406	1.86400	0.00311	1.91700	0.00237

TABLE 4.—*Values of* $\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$
1. 91800	0. 00236	1. 97100	0. 00179	2. 02400	0. 00135	2. 07700	0. 00102
1. 91900	0. 00235	1. 97200	0. 00178	2. 02500	0. 00135	2. 07800	0. 00101
1. 92000	0. 00233	1. 97300	0. 00177	2. 02600	0. 00134	2. 07900	0. 00101
1. 92100	0. 00232	1. 97400	0. 00176	2. 02700	0. 00133	2. 08000	0. 00100
1. 92200	0. 00231	1. 97500	0. 00175	2. 02800	0. 00133	2. 08100	0. 00100
1. 92300	0. 00230	1. 97600	0. 00174	2. 02900	0. 00132	2. 08200	0. 00099
1. 92400	0. 00229	1. 97700	0. 00173	2. 03000	0. 00131	2. 08300	0. 00099
1. 92500	0. 00227	1. 97800	0. 00173	2. 03100	0. 00130	2. 08400	0. 00098
1. 92600	0. 00226	1. 97900	0. 00172	2. 03200	0. 00130	2. 08500	0. 00098
1. 92700	0. 00225	1. 98000	0. 00171	2. 03300	0. 00129	2. 08600	0. 00097
1. 92800	0. 00224	1. 98100	0. 00170	2. 03400	0. 00128	2. 08700	0. 00097
1. 92900	0. 00223	1. 98200	0. 00169	2. 03500	0. 00128	2. 08800	0. 00096
1. 93000	0. 00222	1. 98300	0. 00168	2. 03600	0. 00127	2. 08900	0. 00095
1. 93100	0. 00220	1. 98400	0. 00167	2. 03700	0. 00126	2. 09000	0. 00095
1. 93200	0. 00219	1. 98500	0. 00166	2. 03800	0. 00126	2. 09100	0. 00094
1. 93300	0. 00218	1. 98600	0. 00165	2. 03900	0. 00125	2. 09200	0. 00094
1. 93400	0. 00217	1. 98700	0. 00165	2. 04000	0. 00124	2. 09300	0. 00093
1. 93500	0. 00216	1. 98800	0. 00164	2. 04100	0. 00124	2. 09400	0. 00093
1. 93600	0. 00215	1. 98900	0. 00163	2. 04200	0. 00123	2. 09500	0. 00092
1. 93700	0. 00214	1. 99000	0. 00162	2. 04300	0. 00122	2. 09600	0. 00092
1. 93800	0. 00213	1. 99100	0. 00161	2. 04400	0. 00122	2. 09700	0. 00091
1. 93900	0. 00212	1. 99200	0. 00160	2. 04500	0. 00121	2. 09800	0. 00091
1. 94000	0. 00210	1. 99300	0. 00159	2. 04600	0. 00120	2. 09900	0. 00090
1. 94100	0. 00209	1. 99400	0. 00159	2. 04700	0. 00120	2. 10000	0. 00090
1. 94200	0. 00208	1. 99500	0. 00158	2. 04800	0. 00119	2. 10100	0. 00089
1. 94300	0. 00207	1. 99600	0. 00157	2. 04900	0. 00118	2. 10200	0. 00089
1. 94400	0. 00206	1. 99700	0. 00156	2. 05000	0. 00118	2. 10300	0. 00089
1. 94500	0. 00205	1. 99800	0. 00155	2. 05100	0. 00117	2. 10400	0. 00088
1. 94600	0. 00204	1. 99900	0. 00155	2. 05200	0. 00117	2. 10500	0. 00088
1. 94700	0. 00203	2. 00000	0. 00154	2. 05300	0. 00116	2. 10600	0. 00087
1. 94800	0. 00202	2. 00100	0. 00153	2. 05400	0. 00115	2. 10700	0. 00087
1. 94900	0. 00201	2. 00200	0. 00152	2. 05500	0. 00115	2. 10800	0. 00086
1. 95000	0. 00200	2. 00300	0. 00151	2. 05600	0. 00114	2. 10900	0. 00086
1. 95100	0. 00199	2. 00400	0. 00150	2. 05700	0. 00113	2. 11000	0. 00085
1. 95200	0. 00198	2. 00500	0. 00150	2. 05800	0. 00113	2. 11100	0. 00085
1. 95300	0. 00197	2. 00600	0. 00149	2. 05900	0. 00112	2. 11200	0. 00084
1. 95400	0. 00196	2. 00700	0. 00148	2. 06000	0. 00112	2. 11300	0. 00084
1. 95500	0. 00195	2. 00800	0. 00147	2. 06100	0. 00111	2. 11400	0. 00083
1. 95600	0. 00194	2. 00900	0. 00147	2. 06200	0. 00110	2. 11500	0. 00083
1. 95700	0. 00193	2. 01000	0. 00146	2. 06300	0. 00110	2. 11600	0. 00082
1. 95800	0. 00192	2. 01100	0. 00145	2. 06400	0. 00109	2. 11700	0. 00082
1. 95900	0. 00191	2. 01200	0. 00144	2. 06500	0. 00109	2. 11800	0. 00082
1. 96000	0. 00190	2. 01300	0. 00143	2. 06600	0. 00108	2. 11900	0. 00081
1. 96100	0. 00189	2. 01400	0. 00143	2. 06700	0. 00108	2. 12000	0. 00081
1. 96200	0. 00188	2. 01500	0. 00142	2. 06800	0. 00107	2. 12100	0. 00080
1. 96300	0. 00187	2. 01600	0. 00141	2. 06900	0. 00106	2. 12200	0. 00080
1. 96400	0. 00186	2. 01700	0. 00140	2. 07000	0. 00106	2. 12300	0. 00079
1. 96500	0. 00185	2. 01800	0. 00140	2. 07100	0. 00105	2. 12400	0. 00079
1. 96600	0. 00184	2. 01900	0. 00139	2. 07200	0. 00105	2. 12500	0. 00078
1. 96700	0. 00183	2. 02000	0. 00138	2. 07300	0. 00104	2. 12600	0. 00078
1. 96800	0. 00182	2. 02100	0. 00138	2. 07400	0. 00104	2. 12700	0. 00078
1. 96900	0. 00181	2. 02200	0. 00137	2. 07500	0. 00103	2. 12800	0. 00077
1. 97000	0. 00180	2. 02300	0. 00136	2. 07600	0. 00102	2. 12900	0. 00077

TABLE 4.—*Values of* $\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$
2.13000	0.00076	2.18300	0.00057	2.23600	0.00042	2.28900	0.00031
2.13100	0.00076	2.18400	0.00057	2.23700	0.00042	2.29000	0.00031
2.13200	0.00076	2.18500	0.00056	2.23800	0.00042	2.29100	0.00031
2.13300	0.00075	2.18600	0.00056	2.23900	0.00042	2.29200	0.00031
2.13400	0.00075	2.18700	0.00056	2.24000	0.00041	2.29300	0.00031
2.13500	0.00074	2.18800	0.00055	2.24100	0.00041	2.29400	0.00030
2.13600	0.00074	2.18900	0.00055	2.24200	0.00041	2.29500	0.00030
2.13700	0.00073	2.19000	0.00055	2.24300	0.00041	2.29600	0.00030
2.13800	0.00073	2.19100	0.00055	2.24400	0.00040	2.29700	0.00030
2.13900	0.00073	2.19200	0.00054	2.24500	0.00040	2.29800	0.00030
2.14000	0.00072	2.19300	0.00054	2.24600	0.00040	2.29900	0.00030
2.14100	0.00072	2.19400	0.00054	2.24700	0.00040	2.30000	0.00029
2.14200	0.00072	2.19500	0.00053	2.24800	0.00040	2.30100	0.00029
2.14300	0.00071	2.19600	0.00053	2.24900	0.00039	2.30200	0.00029
2.14400	0.00071	2.19700	0.00053	2.25000	0.00039	2.30300	0.00029
2.14500	0.00070	2.19800	0.00052	2.25100	0.00039	2.30400	0.00029
2.14600	0.00070	2.19900	0.00052	2.25200	0.00039	2.30500	0.00029
2.14700	0.00070	2.20000	0.00052	2.25300	0.00038	2.30600	0.00028
2.14800	0.00069	2.20100	0.00052	2.25400	0.00038	2.30700	0.00028
2.14900	0.00069	2.20200	0.00051	2.25500	0.00038	2.30800	0.00028
2.15000	0.00068	2.20300	0.00051	2.25600	0.00038	2.30900	0.00028
2.15100	0.00068	2.20400	0.00051	2.25700	0.00038	2.31000	0.00028
2.15200	0.00068	2.20500	0.00050	2.25800	0.00037	2.31100	0.00028
2.15300	0.00067	2.20600	0.00050	2.25900	0.00037	2.31200	0.00027
2.15400	0.00067	2.20700	0.00050	2.26000	0.00037	2.31300	0.00027
2.15500	0.00067	2.20800	0.00050	2.26100	0.00037	2.31400	0.00027
2.15600	0.00066	2.20900	0.00049	2.26200	0.00037	2.31500	0.00027
2.15700	0.00066	2.21000	0.00049	2.26300	0.00036	2.31600	0.00027
2.15800	0.00065	2.21100	0.00049	2.26400	0.00036	2.31700	0.00027
2.15900	0.00065	2.21200	0.00048	2.26500	0.00036	2.31800	0.00026
2.16000	0.00065	2.21300	0.00048	2.26600	0.00036	2.31900	0.00026
2.16100	0.00064	2.21400	0.00048	2.26700	0.00036	2.32000	0.00026
2.16200	0.00064	2.21500	0.00048	2.26800	0.00035	2.32100	0.00026
2.16300	0.00064	2.21600	0.00047	2.26900	0.00035	2.32200	0.00026
2.16400	0.00063	2.21700	0.00047	2.27000	0.00035	2.32300	0.00026
2.16500	0.00063	2.21800	0.00047	2.27100	0.00035	2.32400	0.00026
2.16600	0.00063	2.21900	0.00047	2.27200	0.00035	2.32500	0.00025
2.16700	0.00062	2.22000	0.00046	2.27300	0.00034	2.32600	0.00025
2.16800	0.00062	2.22100	0.00046	2.27400	0.00034	2.32700	0.00025
2.16900	0.00062	2.22200	0.00046	2.27500	0.00034	2.32800	0.00025
2.17000	0.00061	2.22300	0.00046	2.27600	0.00034	2.32900	0.00025
2.17100	0.00061	2.22400	0.00045	2.27700	0.00034	2.33000	0.00025
2.17200	0.00061	2.22500	0.00045	2.27800	0.00033	2.33100	0.00025
2.17300	0.00060	2.22600	0.00045	2.27900	0.00033	2.33200	0.00024
2.17400	0.00060	2.22700	0.00045	2.28000	0.00033	2.33300	0.00024
2.17500	0.00060	2.22800	0.00044	2.28100	0.00033	2.33400	0.00024
2.17600	0.00059	2.22900	0.00044	2.28200	0.00033	2.33500	0.00024
2.17700	0.00059	2.23000	0.00044	2.28300	0.00032	2.33600	0.00024
2.17800	0.00059	2.23100	0.00044	2.28400	0.00032	2.33700	0.00024
2.17900	0.00058	2.23200	0.00043	2.28500	0.00032	2.33800	0.00024
2.18000	0.00058	2.23300	0.00043	2.28600	0.00032	2.33900	0.00023
2.18100	0.00058	2.23400	0.00043	2.28700	0.00032	2.34000	0.00023
2.18200	0.00057	2.23500	0.00043	2.28800	0.00031	2.34100	0.00023

TABLE 4.—*Values of* $\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$
2.34200	0.00023	2.39500	0.00017	2.44800	0.00012	2.50100	0.00009
2.34300	0.00023	2.39600	0.00017	2.44900	0.00012	2.50200	0.00009
2.34400	0.00023	2.39700	0.00017	2.45000	0.00012	2.50300	0.00009
2.34500	0.00023	2.39800	0.00017	2.45100	0.00012	2.50400	0.00009
2.34600	0.00023	2.39900	0.00016	2.45200	0.00012	2.50500	0.00009
2.34700	0.00022	2.40000	0.00016	2.45300	0.00012	2.50600	0.00009
2.34800	0.00022	2.40100	0.00016	2.45400	0.00012	2.50700	0.00009
2.34900	0.00022	2.40200	0.00016	2.45500	0.00012	2.50800	0.00009
2.35000	0.00022	2.40300	0.00016	2.45600	0.00012	2.50900	0.00008
2.35100	0.00022	2.40400	0.00016	2.45700	0.00012	2.51000	0.00008
2.35200	0.00022	2.40500	0.00016	2.45800	0.00012	2.51100	0.00008
2.35300	0.00022	2.40600	0.00016	2.45900	0.00011	2.51200	0.00008
2.35400	0.00021	2.40700	0.00016	2.46000	0.00011	2.51300	0.00008
2.35500	0.00021	2.40800	0.00016	2.46100	0.00011	2.51400	0.00008
2.35600	0.00021	2.40900	0.00016	2.46200	0.00011	2.51500	0.00008
2.35700	0.00021	2.41000	0.00015	2.46300	0.00011	2.51600	0.00008
2.35800	0.00021	2.41100	0.00015	2.46400	0.00011	2.51700	0.00008
2.35900	0.00021	2.41200	0.00015	2.46500	0.00011	2.51800	0.00008
2.36000	0.00021	2.41300	0.00015	2.46600	0.00011	2.51900	0.00008
2.36100	0.00021	2.41400	0.00015	2.46700	0.00011	2.52000	0.00008
2.36200	0.00020	2.41500	0.00015	2.46800	0.00011	2.52100	0.00008
2.36300	0.00020	2.41600	0.00015	2.46900	0.00011	2.52200	0.00008
2.36400	0.00020	2.41700	0.00015	2.47000	0.00011	2.52300	0.00008
2.36500	0.00020	2.41800	0.00015	2.47100	0.00011	2.52400	0.00008
2.36600	0.00020	2.41900	0.00015	2.47200	0.00011	2.52500	0.00008
2.36700	0.00020	2.42000	0.00015	2.47300	0.00011	2.52600	0.00008
2.36800	0.00020	2.42100	0.00014	2.47400	0.00010	2.52700	0.00008
2.36900	0.00020	2.42200	0.00014	2.47500	0.00010	2.52800	0.00008
2.37000	0.00020	2.42300	0.00014	2.47600	0.00010	2.52900	0.00007
2.37100	0.00019	2.42400	0.00014	2.47700	0.00010	2.53000	0.00007
2.37200	0.00019	2.42500	0.00014	2.47800	0.00010	2.53100	0.00007
2.37300	0.00019	2.42600	0.00014	2.47900	0.00010	2.53200	0.00007
2.37400	0.00019	2.42700	0.00014	2.48000	0.00010	2.53300	0.00007
2.37500	0.00019	2.42800	0.00014	2.48100	0.00010	2.53400	0.00007
2.37600	0.00019	2.42900	0.00014	2.48200	0.00010	2.53500	0.00007
2.37700	0.00019	2.43000	0.00014	2.48300	0.00010	2.53600	0.00007
2.37800	0.00019	2.43100	0.00014	2.48400	0.00010	2.53700	0.00007
2.37900	0.00019	2.43200	0.00014	2.48500	0.00010	2.53800	0.00007
2.38000	0.00018	2.43300	0.00013	2.48600	0.00010	2.53900	0.00007
2.38100	0.00018	2.43400	0.00013	2.48700	0.00010	2.54000	0.00007
2.38200	0.00018	2.43500	0.00013	2.48800	0.00010	2.54100	0.00007
2.38300	0.00018	2.43600	0.00013	2.48900	0.00010	2.54200	0.00007
2.38400	0.00018	2.43700	0.00013	2.49000	0.00010	2.54300	0.00007
2.38500	0.00018	2.43800	0.00013	2.49100	0.00009	2.54400	0.00007
2.38600	0.00018	2.43900	0.00013	2.49200	0.00009	2.54500	0.00007
2.38700	0.00018	2.44000	0.00013	2.49300	0.00009	2.54600	0.00007
2.38800	0.00018	2.44100	0.00013	2.49400	0.00009	2.54700	0.00007
2.38900	0.00017	2.44200	0.00013	2.49500	0.00009	2.54800	0.00007
2.39000	0.00017	2.44300	0.00013	2.49600	0.00009	2.54900	0.00007
2.39100	0.00017	2.44400	0.00013	2.49700	0.00009	2.55000	0.00007
2.39200	0.00017	2.44500	0.00013	2.49800	0.00009	2.55100	0.00007
2.39300	0.00017	2.44600	0.00012	2.49900	0.00009	2.55200	0.00006
2.39400	0.00017	2.44700	0.00012	2.50000	0.00009	2.55300	0.00006

TABLE 4.—*Values of* $\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}\,du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$
2. 55400	0. 00006	2. 60700	0. 00005	2. 66000	0. 00003	2. 71300	0. 00002
2. 55500	0. 00006	2. 60800	0. 00005	2. 66100	0. 00003	2. 71400	0. 00002
2. 55600	0. 00006	2. 60900	0. 00005	2. 66200	0. 00003	2. 71500	0. 00002
2. 55700	0. 00006	2. 61000	0. 00004	2. 66300	0. 00003	2. 71600	0. 00002
2. 55800	0. 00006	2. 61100	0. 00004	2. 66400	0. 00003	2. 71700	0. 00002
2. 55900	0. 00006	2. 61200	0. 00004	2. 66500	0. 00003	2. 71800	0. 00002
2. 56000	0. 00006	2. 61300	0. 00004	2. 66600	0. 00003	2. 71900	0. 00002
2. 56100	0. 00006	2. 61400	0. 00004	2. 66700	0. 00003	2. 72000	0. 00002
2. 56200	0. 00006	2. 61500	0. 00004	2. 66800	0. 00003	2. 72100	0. 00002
2. 56300	0. 00006	2. 61600	0. 00004	2. 66900	0. 00003	2. 72200	0. 00002
2. 56400	0. 00006	2. 61700	0. 00004	2. 67000	0. 00003	2. 72300	0. 00002
2. 56500	0. 00006	2. 61800	0. 00004	2. 67100	0. 00003	2. 72400	0. 00002
2. 56600	0. 00006	2. 61900	0. 00004	2. 67200	0. 00003	2. 72500	0. 00002
2. 56700	0. 00006	2. 62000	0. 00004	2. 67300	0. 00003	2. 72600	0. 00002
2. 56800	0. 00006	2. 62100	0. 00004	2. 67400	0. 00003	2. 72700	0. 00002
2. 56900	0. 00006	2. 62200	0. 00004	2. 67500	0. 00003	2. 72800	0. 00002
2. 57000	0. 00006	2. 62300	0. 00004	2. 67600	0. 00003	2. 72900	0. 00002
2. 57100	0. 00006	2. 62400	0. 00004	2. 67700	0. 00003	2. 73000	0. 00002
2. 57200	0. 00006	2. 62500	0. 00004	2. 67800	0. 00003	2. 73100	0. 00002
2. 57300	0. 00006	2. 62600	0. 00004	2. 67900	0. 00003	2. 73200	0. 00002
2. 57400	0. 00006	2. 62700	0. 00004	2. 68000	0. 00003	2. 73300	0. 00002
2. 57500	0. 00006	2. 62800	0. 00004	2. 68100	0. 00003	2. 73400	0. 00002
2. 57600	0. 00006	2. 62900	0. 00004	2. 68200	0. 00003	2. 73500	0. 00002
2. 57700	0. 00006	2. 63000	0. 00004	2. 68300	0. 00003	2. 73600	0. 00002
2. 57800	0. 00006	2. 63100	0. 00004	2. 68400	0. 00003	2. 73700	0. 00002
2. 57900	0. 00005	2. 63200	0. 00004	2. 68500	0. 00003	2. 73800	0. 00002
2. 58000	0. 00005	2. 63300	0. 00004	2. 68600	0. 00003	2. 73900	0. 00002
2. 58100	0. 00005	2. 63400	0. 00004	2. 68700	0. 00003	2. 74000	0. 00002
2. 58200	0. 00005	2. 63500	0. 00004	2. 68800	0. 00003	2. 74100	0. 00002
2. 58300	0. 00005	2. 63600	0. 00004	2. 68900	0. 00003	2. 74200	0. 00002
2. 58400	0. 00005	2. 63700	0. 00004	2. 69000	0. 00003	2. 74300	0. 00002
2. 58500	0. 00005	2. 63800	0. 00004	2. 69100	0. 00003	2. 74400	0. 00002
2. 58600	0. 00005	2. 63900	0. 00004	2. 69200	0. 00003	2. 74500	0. 00002
2. 58700	0. 00005	2. 64000	0. 00004	2. 69300	0. 00003	2. 74600	0. 00002
2. 58800	0. 00005	2. 64100	0. 00004	2. 69400	0. 00003	2. 74700	0. 00002
2. 58900	0. 00005	2. 64200	0. 00004	2. 69500	0. 00003	2. 74800	0. 00002
2. 59000	0. 00005	2. 64300	0. 00004	2. 69600	0. 00003	2. 74900	0. 00002
2. 59100	0. 00005	2. 64400	0. 00004	2. 69700	0. 00003	2. 75000	0. 00002
2. 59200	0. 00005	2. 64500	0. 00004	2. 69800	0. 00003	2. 75100	0. 00002
2. 59300	0. 00005	2. 64600	0. 00004	2. 69900	0. 00003	2. 75200	0. 00002
2. 59400	0. 00005	2. 64700	0. 00004	2. 70000	0. 00003	2. 75300	0. 00002
2. 59500	0. 00005	2. 64800	0. 00004	2. 70100	0. 00002	2. 75400	0. 00002
2. 59600	0. 00005	2. 64900	0. 00004	2. 70200	0. 00002	2. 75500	0. 00002
2. 59700	0. 00005	2. 65000	0. 00003	2. 70300	0. 00002	2. 75600	0. 00002
2. 59800	0. 00005	2. 65100	0. 00003	2. 70400	0. 00002	2. 75700	0. 00002
2. 59900	0. 00005	2. 65200	0. 00003	2. 70500	0. 00002	2. 75800	0. 00002
2. 60000	0. 00005	2. 65300	0. 00003	2. 70600	0. 00002	2. 75900	0. 00002
2. 60100	0. 00005	2. 65400	0. 00003	2. 70700	0. 00002	2. 76000	0. 00002
2. 60200	0. 00005	2. 65500	0. 00003	2. 70800	0. 00002	2. 76100	0. 00002
2. 60300	0. 00005	2. 65600	0. 00003	2. 70900	0. 00002	2. 76200	0. 00002
2. 60400	0. 00005	2. 65700	0. 00003	2. 71000	0. 00002	2. 76300	0. 00002
2. 60500	0. 00005	2. 65800	0. 00003	2. 71100	0. 00002	2. 76400	0. 00002
2. 60600	0. 00005	2. 65900	0. 00003	2. 71200	0. 00002	2. 76500	0. 00002

TABLE 4.—*Values of* $\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}\,du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$
2. 76600	0. 00002	2. 81900	0. 00001	2. 87200	0. 00001	2. 92500	0. 00001
2. 76700	0. 00002	2. 82000	0. 00001	2. 87300	0. 00001	2. 92600	0. 00001
2. 76800	0. 00002	2. 82100	0. 00001	2. 87400	0. 00001	2. 92700	0. 00001
2. 76900	0. 00002	2. 82200	0. 00001	2. 87500	0. 00001	2. 92800	0. 00001
2. 77000	0. 00002	2. 82300	0. 00001	2. 87600	0. 00001	2. 92900	0. 00001
2. 77100	0. 00002	2. 82400	0. 00001	2. 87700	0. 00001	2. 93000	0. 00001
2. 77200	0. 00002	2. 82500	0. 00001	2. 87800	0. 00001	2. 93100	0. 00001
2. 77300	0. 00002	2. 82600	0. 00001	2. 87900	0. 00001	2. 93200	0. 00000
2. 77400	0. 00002	2. 82700	0. 00001	2. 88000	0. 00001	2. 93300	0. 00000
2. 77500	0. 00002	2. 82800	0. 00001	2. 88100	0. 00001	2. 93400	0. 00000
2. 77600	0. 00002	2. 82900	0. 00001	2. 88200	0. 00001	2. 93500	0. 00000
2. 77700	0. 00001	2. 83000	0. 00001	2. 88300	0. 00001	2. 93600	0. 00000
2. 77800	0. 00001	2. 83100	0. 00001	2. 88400	0. 00001	2. 93700	0. 00000
2. 77900	0. 00001	2. 83200	0. 00001	2. 88500	0. 00001	2. 93800	0. 00000
2. 78000	0. 00001	2. 83300	0. 00001	2. 88600	0. 00001	2. 93900	0. 00000
2. 78100	0. 00001	2. 83400	0. 00001	2. 88700	0. 00001	2. 94000	0. 00000
2. 78200	0. 00001	2. 83500	0. 00001	2. 88800	0. 00001	2. 94100	0. 00000
2. 78300	0. 00001	2. 83600	0. 00001	2. 88900	0. 00001	2. 94200	0. 00000
2. 78400	0. 00001	2. 83700	0. 00001	2. 89000	0. 00001	2. 94300	0. 00000
2. 78500	0. 00001	2. 83800	0. 00001	2. 89100	0. 00001	2. 94400	0. 00000
2. 78600	0. 00001	2. 83900	0. 00001	2. 89200	0. 00001	2. 94500	0. 00000
2. 78700	0. 00001	2. 84000	0. 00001	2. 89300	0. 00001	2. 94600	0. 00000
2. 78800	0. 00001	2. 84100	0. 00001	2. 89400	0. 00001	2. 94700	0. 00000
2. 78900	0. 00001	2. 84200	0. 00001	2. 89500	0. 00001	2. 94800	0. 00000
2. 79000	0. 00001	2. 84300	0. 00001	2. 89600	0. 00001	2. 94900	0. 00000
2. 79100	0. 00001	2. 84400	0. 00001	2. 89700	0. 00001	2. 95000	0. 00000
2. 79200	0. 00001	2. 84500	0. 00001	2. 89800	0. 00001	2. 95100	0. 00000
2. 79300	0. 00001	2. 84600	0. 00001	2. 89900	0. 00001	2. 95200	0. 00000
2. 79400	0. 00001	2. 84700	0. 00001	2. 90000	0. 00001	2. 95300	0. 00000
2. 79500	0. 00001	2. 84800	0. 00001	2. 90100	0. 00001	2. 95400	0. 00000
2. 79600	0. 00001	2. 84900	0. 00001	2. 90200	0. 00001	2. 95500	0. 00000
2. 79700	0. 00001	2. 85000	0. 00001	2. 90300	0. 00001	2. 95600	0. 00000
2. 79800	0. 00001	2. 85100	0. 00001	2. 90400	0. 00001	2. 95700	0. 00000
2. 79900	0. 00001	2. 85200	0. 00001	2. 90500	0. 00001	2. 95800	0. 00000
2. 80000	0. 00001	2. 85300	0. 00001	2. 90600	0. 00001	2. 95900	0. 00000
2. 80100	0. 00001	2. 85400	0. 00001	2. 90700	0. 00001	2. 96000	0. 00000
2. 80200	0. 00001	2. 85500	0. 00001	2. 90800	0. 00001	2. 96100	0. 00000
2. 80300	0. 00001	2. 85600	0. 00001	2. 90900	0. 00001	2. 96200	0. 00000
2. 80400	0. 00001	2. 85700	0. 00001	2. 91000	0. 00001	2. 96300	0. 00000
2. 80500	0. 00001	2. 85800	0. 00001	2. 91100	0. 00001	2. 96400	0. 00000
2. 80600	0. 00001	2. 85900	0. 00001	2. 91200	0. 00001	2. 96500	0. 00000
2. 80700	0. 00001	2. 86000	0. 00001	2. 91300	0. 00001	2. 96600	0. 00000
2. 80800	0. 00001	2. 86100	0. 00001	2. 91400	0. 00001	2. 96700	0. 00000
2. 80900	0. 00001	2. 86200	0. 00001	2. 91500	0. 00001	2. 96800	0. 00000
2. 81000	0. 00001	2. 86300	0. 00001	2. 91600	0. 00001	2. 96900	0. 00000
2. 81100	0. 00001	2. 86400	0. 00001	2. 91700	0. 00001	2. 97000	0. 00000
2. 81200	0. 00001	2. 86500	0. 00001	2. 91800	0. 00001	2. 97100	0. 00000
2. 81300	0. 00001	2. 86600	0. 00001	2. 91900	0. 00001	2. 97200	0. 00000
2. 81400	0. 00001	2. 86700	0. 00001	2. 92000	0. 00001	2. 97300	0. 00000
2. 81500	0. 00001	2. 86800	0. 00001	2. 92100	0. 00001	2. 97400	0. 00000
2. 81600	0. 00001	2. 86900	0. 00001	2. 92200	0. 00001	2. 97500	0. 00000
2. 81700	0. 00001	2. 87000	0. 00001	2. 92300	0. 00001	2. 97600	0. 00000
2. 81800	0. 00001	2. 87100	0. 00001	2. 92400	0. 00001	2. 97700	0. 00000

TABLE 4.—*Values of* $\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$ *for given values of the parameter* $\frac{r}{\sqrt{4\alpha t}}$—Continued

$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$	$\frac{r}{\sqrt{4\alpha t}}$	$\sqrt{\pi}\int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}du$
2. 97800	0. 00000	2. 98400	0. 00000	2. 99000	0. 00000	2. 99600	0. 00000
2. 97900	0. 00000	2. 98500	0. 00000	2. 99100	0. 00000	2. 99700	0. 00000
2. 98000	0. 00000	2. 98600	0. 00000	2. 99200	0. 00000	2. 99800	0. 00000
2. 98100	0. 00000	2. 98700	0. 00000	2. 99300	0. 00000	2. 99900	0. 00000
2. 98200	0. 00000	2. 98800	0. 00000	2. 99400	0. 00000	3. 00000	0. 00000
2. 98300	0. 00000	2. 98900	0. 00000	2. 99500	0. 00000		

bank storage and return will contribute to the storage capacity of the reservoir. The bank storage capacity can be estimated by idealizing the rise and fall of the reservoir surface as sinusoidal with a period T.

Fluctuation of reservoir level. If the instantaneous departure of the reservoir surface from the mean level is h_r and the maximum departure from the mean H_r, then it is assumed that:

$$h_r=H_r \sin \omega t \qquad (62)$$

where

$$\omega=\frac{2\pi}{T}. \qquad (63)$$

A solution of Equation (3) which reduces to the above variation when $x=0$ is:

$$h_r=H_r e^{-x\sqrt{\frac{\omega}{2\alpha}}} \sin\left(\omega t-x\sqrt{\frac{\omega}{2\alpha}}\right). \qquad (64)$$

The total volume of water stored, per unit length of bank, as the reservoir is filled and then returned as the reservoir is drained is:

$$S=\frac{2KDH_r}{\sqrt{\omega\alpha}}. \qquad (65)$$

The total amount stored would be obtained by multiplying this quantity by the perimeter of the reservoir.

Since the cycles of filling and emptying of reservoirs are generally irregular, Formula (65) can only serve as a means of estimating the increase of reservoir capacity provided by the bank storage. If the reservoir has only recently been placed in service or the operation is irregular, the contribution of bank storage can be approximated by a stepwise application of Formula (61) to increments of reservoir rise or fall corresponding to suitably chosen uniform increments of time. The total flow would be obtained as a summation of the flows originating in the individual increments. In the case of a reservoir recently placed in service, the flows will generally be moving outward to build up the ground-water mound which will ultimately be created by the surface reservoir.

A few words of explanation concerning the nature of Formula (65) may be helpful.

This formula represents an ultimate stable regimen which would prevail after all of the starting transients had died out. A solution containing the starting transients would be quite complicated and would not add much in the way of useful results. It will be worthwhile to remember that Formulas (64) and (65) become applicable only after the reservoir has been through several cycles of filling and emptying.

Example

A reservoir is to be constructed in a valley filled to considerable depths with alluvial and wind-blown sands. The visible storage, with a depth of 80 feet, will be 25,000 acre-feet. The transmissibility of the material adjacent to the reservoir is 0.220 ft² per sec. The yield on drawdown is $V=0.15$. The reservoir will have a shoreline, when filled, of about 75,000 feet. A yearly fluctuation of 40 feet is expected. At the mean water surface, corresponding to a 60-foot depth, the shoreline will be about 56,000 feet. Estimate the rate of flow into the banks after the reservoir has been filled 1 year, the total bank storage at the end of 1 year and the effective increase in capacity due to the bank storage and return.

$$\alpha=\frac{KD}{V}=\frac{0.220}{0.15}=1.47 \text{ feet}^2/\text{second.}$$

The average depth of water applied to the banks is 80/2=40 feet. Then $H=40$ feet. One year is 31,536,000 seconds.

From Formula (60) the flow into the banks is:

$$-F_0=\frac{HKD}{\sqrt{\pi\alpha t}}=\frac{(40)(0.220)}{\sqrt{\pi(1.47)(31{,}536{,}000)}}$$

or

$$-F_0=\frac{8.80}{12{,}070}$$

cubic-feet-per-second per foot of bank. The total outflow is obtained by multiplying this figure by the length of the bank. Then the total flow into the banks at the end of a year would be estimated as:

$$\frac{(8.80)(75{,}000)}{12{,}070}=55$$

cubic feet per second. The total flow into the banks in the first year would be, by Formula (61),

$$-R=HV\sqrt{\frac{4\alpha t}{\pi}}=(40)(0.15)\sqrt{\frac{(4)(1.47)31{,}536{,}000}{\pi}}$$

$$=(6.0)(7{,}680)=46{,}000$$

cubic feet per foot of bank. Then the total bank storage accumulated in the first year is estimated to be

$$\frac{(46{,}000)(75{,}000)}{43{,}560}=79{,}200 \text{ acre-feet.}$$

The increase in effective capacity of the reservoir due to bank storage would be, by Formula (65),

$$S=\frac{2KDH_r}{\sqrt{\omega\alpha}}.$$

The amplitude of the yearly fluctuation is $H_r=$ 40/2=20. Also with a period of 1 year, or 31,536,000 seconds,

$$\omega=\frac{2\pi}{T}=\frac{6.2832}{31{,}536{,}000}=(0.1992)(10)^{-6}=(1/\text{sec})$$

$$\sqrt{\omega\alpha}=\sqrt{(0.293)(10)^{-6}}=0.000541 \text{ (ft/sec)}$$

then

$$S=\frac{(2)(0.220)(20)}{0.000541}=16{,}270$$

cubic feet per foot of bank. The total capacity increase would be, for the 56,000 feet of bank at the mean level,

$$\frac{(16{,}270)(56{,}000)}{43{,}560}=20{,}900 \text{ acre-feet.}$$

The conditions chosen for this example represent an unusually permeable bank material. They are such conditions as might be found in a sand-dune area and the seepage losses and reservoir capacity increases are correspondingly high. In such an area there would be a leakage around the dam and the volume of the ground-water mound would be limited to the volume it could attain under ultimate steady state conditions. An electric analogy tray is a useful device for finding the ultimate size and shape of the ground-water mound.

Use of Images

The usefulness of the formulas for ascertaining the drawdown that can be attributed to well pumping in indefinitely extended aquifers may be extended by the use of images. Suppose, for example, that a well, drawing water from an unconfined aquifer, is located near a stream and it is decided to compute the pattern of drawdown around the well at some given time after pumping begins. If the stream were absent there would be a drawdown along the line where the near bank of the stream is located, but so long as the stream is there, no drawdown can occur along this line. This fact can be accounted for by assuming that the stream is absent and introducing an image well at the point where the pumped well would be imaged by the line representing the location of the near bank of the stream. Since the condition maintained by the stream is one of zero drawdown, the image well is a recharge well which is supplied with the same flow as is being taken from the pumped well. Since both wells are at the same distance from the line representing the near bank of the stream and produce drawdowns of equal magnitude but of opposite sign, the net result will be to keep the drawdown along this line zero. Then the mathematical procedure for computing the drawdowns around the pumped well will be to assume the aquifer is infinite in extent to assume that the stream is absent, to introduce the image well, and to compute the drawdowns from the two wells.

Variations of this expedient can be used to account for an impermeable boundary, for the case of a well in a corner between a stream and a tributary which comes in at a right angle, for the case of a well between two streams, and for other similar cases.

In some cases, a succession of imagings will be needed to meet the appropriate boundary conditions. This process often leads to an infinite series of terms, but the series generally converges rapidly.

As an example of the use of this process it will be used to estimate the depletion of a stream due to pumping a well at the rate Q at the distance x_1 from the stream. It is assumed that the well draws water from alluvial sediments and that the stream runs over the surface of these sediments and is in contact with the ground water in them. The course of the stream will be idealized as a straight line.

In an infinitely extended aquifer, as postulated in the development of Formula (9), a drawdown would develop along the course of the stream. The stream, however, will maintain the water-table elevation along its course. The condition of no drawdown along the course of the stream can be met by introducing a recharge well of strength Q located at the distance x_1 from the stream at a point directly opposite the pumped well.

The flow of ground water across the stream boundary, due to pumping the well, can be found by introducing a rectangular coordinate system with the coordinate y measured along the course of the stream and the coordinate x measured toward the well from the point $y=0$. In this system, the radius, drawn from the well, is:

$$r=\sqrt{x^2+y^2}.$$

The gradient across the stream is found by differentiating Expression 9 with respect to x. The flow per unit length along the axis of y is found by multiplying this gradient by the transmissivity KD. And the total flow q across the y axis is found by integrating this unit flow along the axis of y. This procedure is described in detail in Reference (10). The result, including the effect of the image well, is:

$$\frac{q_e}{Q}=\left[1-\int_0^{\frac{x_1}{\sqrt{4at}}} e^{-u^2}du\right]. \qquad (66)$$

The integral which appears here is the tabulated "Probability Integral."

Analogs

The treatment of ground-water movement has proceeded along analytical lines in the preceding sections of this monograph. Analogs, however, provide an alternative approach, which becomes particularly valuable when the conditions to be dealt with are complex. Analog procedures are basically experimental in nature. If time and funds permitted, it would be possible, in many cases, to develop needed information by direct experimentation in the field. Such an approach is seldom permissible, however, because of the large expenditures which would be needed to support such investigations. Experimentation becomes feasible if the field phenomena can be replaced by analogous phenomena of such nature that it becomes possible to bring the problem into the laboratory.

Electrical phenomena provide such a possibility. They are, in addition, easily controlled and measured and also are capable of tremendous speeds of operation. Analogous quantities in the prototype and its electrical representation are the following:

Analogous quantities

Hydraulic prototype	*Electric analog*
Flow of ground water	Current
Water-table elevation	Voltage
Transmissivity	Conductance
Storage	Capacitance

The procedure for establishing correlations among the prototype and analog quantities is explained in detail in Reference (12) but it is essentially the following:

a. Write a system of equations expressing the prototype relationships
b. Write a similar set of equations for the analog
c. Write a set of correlation equations expressing the prototype quantities in terms of the analog quantities.

The last set of equations will contain a set of constants. Several trials will generally be necessary before a satisfactory relationship can be worked out, but these constants will fix the size of the analog components and its speed of operation.

Highly developed electrical and experimental and analytical skills are needed if economical and expeditious analog procedures are to be realized. A few wrong choices can bog the whole operation down in a morass of unnecessary construction and operational difficulties. A combination of analog and analytical procedures is often effective.

Comparisons of Observed and Computed Quantities

The formulas presented herein have been developed on bases which include certain simplifying assumptions. These simplifications were introduced to make the cases analytically tractable but they do impose some limitations on the range of validity. It will be worthwhile, therefore, to compare some computed and observed quantities to learn how important these limitations are. Comparisons are made for a pumped well operating under water-table conditions, for a leaky aquifer case, and for the return flows originating in an irrigated area. Simplifying assumptions often must be used in engineering work and it is generally recognized that formulas derived on such a basis must be used with a knowledge of the limitations these simplifications impose. The comparisons shown here indicate that these formulas, when so used, can be expected to serve as well as those having a longer history of use in the practice of engineering.

Pumping test in an unconfined aquifer. This test was made on a gravel-packed well installed to a depth of 90 feet. The well casing was 8 inches in diameter. The pump column was set at 88 feet. The well penetrated the entire thickness of the aquifer which extended from 24 to 90 feet. The depth of water was 60 feet, leaving 6 feet of unsaturated material at the top of the aquifer. The pump was operated continuously for 5 days with a flow of 150 gallons per minute. Depth to water was measured in the pumped well and in six observation wells installed to a depth of 45 feet. These were perforated from 20 feet down and sealed to 20 feet. Based upon the performance of the well, the aquifer properties were determined to be:

$K=0.0020$ ft per sec
$D=60$ (feet)
$V=0.154$ (dimensionless)
$=0.78$ ft^2 per sec.

The theory for such cases, as described in the development leading to Formula (9), indicates that it should be possible to draw a type curve, as shown by the solid line on Figure 14, and that all drawdown observations at all radii and at all times should fall on this curve if they are plotted in terms of the dimensionless parameters

$$r/\sqrt{4\alpha t} \text{ and } s\Big/\left(\frac{Q}{2\pi KD}\right).$$

The test of the validity of the theoretical development is here made to depend upon conformity with the observed pattern of drawdowns. If the observations do plot on the type curve, then the pattern derived from the theoretical consideration is acceptable, otherwise not. A list of observed drawdowns is shown in the following table:

TABLE 5.—*Observed drawdowns due to pumping from an unconfined aquifer*

Well	Distance from pumped well (feet)	Drawdown (feet)							
		July 16 9:15-1/2	July 16 9:35	July 16 22:30	July 17 12:00	July 18 12:00	July 19 12:00	July 20 12:30	July 21 8:30
N-2-35	100N	0	0.10	0.50	0.67	0.78	0.86	0.91	0.97
N-4-48	200N	0	0.03	0.24	0.35	0.48	0.59	0.60	0.67
N-6-50	400N	0	0.00	0.09	0.15	0.24	0.29	0.32	0.39
W-2-50	101W	0	0.10	0.47	0.62	0.75	0.81	0.85	0.93
W-4-50	201W	0	0.04	0.24	.0.36	0.47	0.53	0.56	0.64
W-6-48	403W	0	0.00	0.08	0.15	0.22	0.28	0.30	0.36

The following computations are based upon these observations:

Values of $\frac{r}{\sqrt{4\alpha t}}$

TABLE 5.—*Observed drawdowns due to pumping from an unconfined aquifer*—Continued

Well	Radius (feet)	July 16 9:15-1/2 0 sec	July 16 9:35 1,170 sec	July 16 22:30 47,670	July 17 12:00 96,270	July 18 12:00 182,670	July 19 12:00 269,070	July 20 12:30 357,270	July 21 8:30 429,270
N-2-35	100	∞	1.65	0.259	0.182	0.132	0.109	0.095	0.086
N-4-48	200	∞	3.30	0.519	0.365	0.265	0.218	0.189	0.173
N-6-50	400	∞	6.62	1.037	0.730	0.530	0.436	0.379	0.346
W-2-50	101	∞	1.67	0.262	0.184	0.134	0.110	0.096	0.087
W-4-50	201	∞	3.33	0.521	0.367	0.266	0.219	0.190	0.174
W-6-48	403	∞	6.67	1.045	0.735	0.534	0.440	0.382	0.348

$\alpha = 0.78$ ft² per sec $4\alpha = 3.12$ ft² per sec

A type curve and the corresponding values obtained from the pumping test are shown on Figure 14.

A scrutiny of Figure 14 will show that, while the observations exhibit some scatter, they do follow substantially the theoretical type curve. It is concluded that the validity of the theory is confirmed by this test.

A leaky roof aquifer case. The test described here was made in an area of previous glaciation. As the glaciers receded, they first deposited a bed of sand and later covered the sand with a thick deposit of glacial till. The permeability of the till is very low compared to that of the sand. In the area of the test, the water table is in the till. When a well casing is installed and perforated through the thickness of the sand bed, removal of water by a pump will reduce the pressure at the base of the till and induce a vertical movement of water in the till. The situation is amenable to treatment by an analysis of the type described under the heading, "Aquifer with a Semipermeable Upper Confining Bed."

The present case differs fundamentally from the

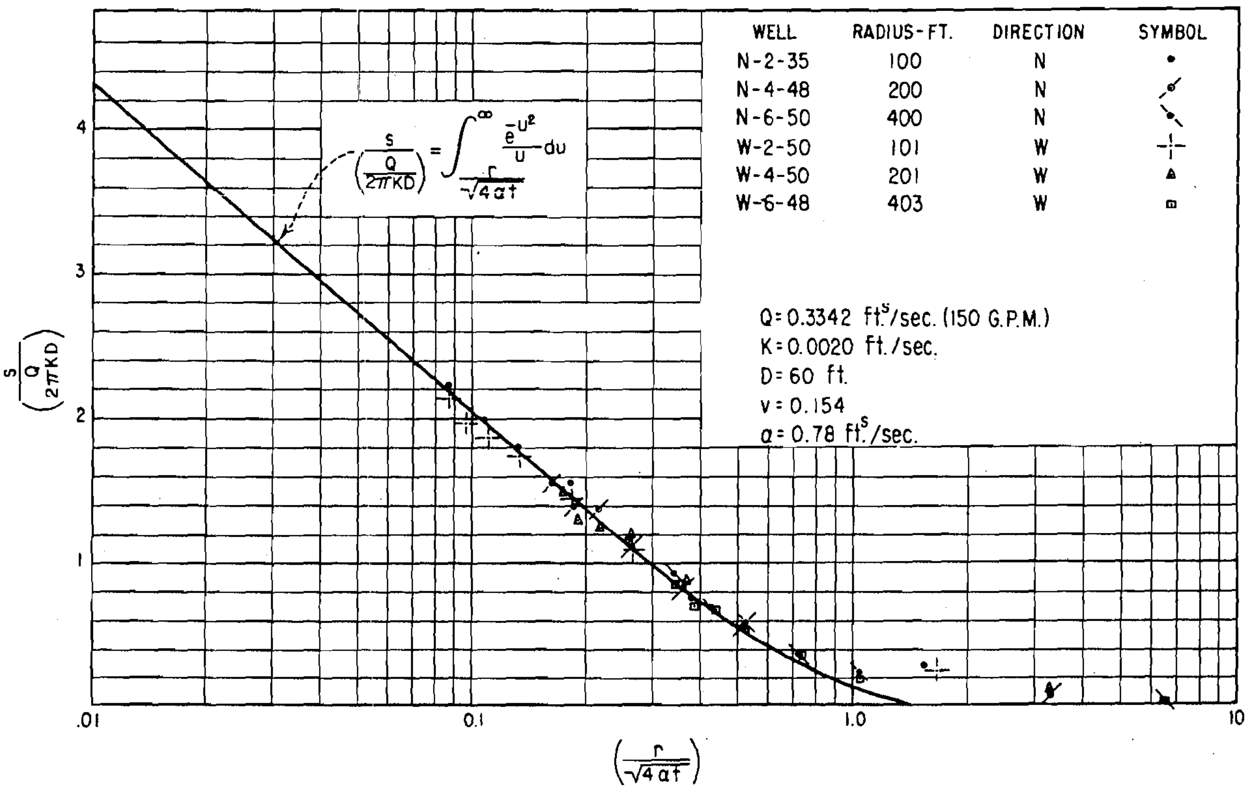


FIGURE 14.—*Comparison of observed and computed performance. Well drawing water from an unconfined aquifer.*

one previously described since the drawdowns, represented in this instance by a pressure reduction, ultimately reach a steady state where the leakage from the upper confining bed will supply the flow from the well. In the unconfined case no ultimate steady state is ever reached. An attempt to apply the analysis for an unconfined aquifer does not produce correlation, as is indicated on Figure 15, even though the transmissibility of the aquifer and its specific yield may be well defined. If the analysis appropirate for an aquifer with a semipermeable confining bed is applied, a good correlation is obtained, as is shown on Figure 16.

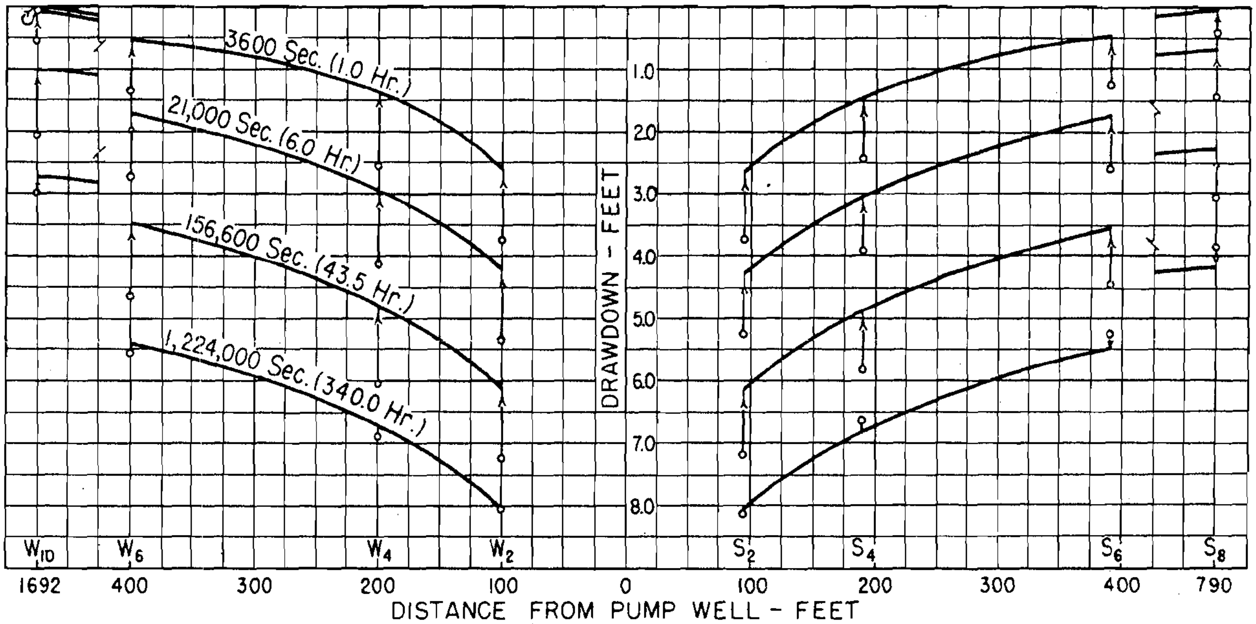


FIGURE 15.—*Drawdown curves as computed from Equation 9 with physical constants K and V based on measured drawdowns at maximum time shown. Circled points represent corresponding measured drawdowns. Observation wells are designated* S_2, W_2, *etc.*

Values of $\frac{s}{\left(\frac{Q}{2\pi KD}\right)}$

Well	July 16 9:15½	July 16 9:35	July 16 22:30	July 17 12:00	July 18 12:00	July 19 12:00	July 20 12:30	July 21 8:30
N-2-35	0	0.23	1.15	1.54	1.79	1.97	2.08	2.22
N-4-48	0	0.07	0.55	0.80	1.10	1.35	1.37	1.54
N-6-50	0	0.00	0.21	0.34	0.55	0.66	0.73	0.89
W-2-50	0	0.23	1.08	1.42	1.72	1.86	1.95	2.13
W-4-50	0	0.09	0.55	0.82	1.08	1.21	1.28	1.47
W-6-48	0	0.00	0.18	0.34	0.50	0.64	0.69	0.82

$$2\pi KD = (6.2832)\ (0.122) = 0.766$$

$$Q = 0.3342 \text{ ft}^3 \text{ per sec} \qquad \frac{Q}{(2\pi KD)} = 0.4363.$$

$$\left(\frac{1}{0.4363}\right) = 2.2916$$

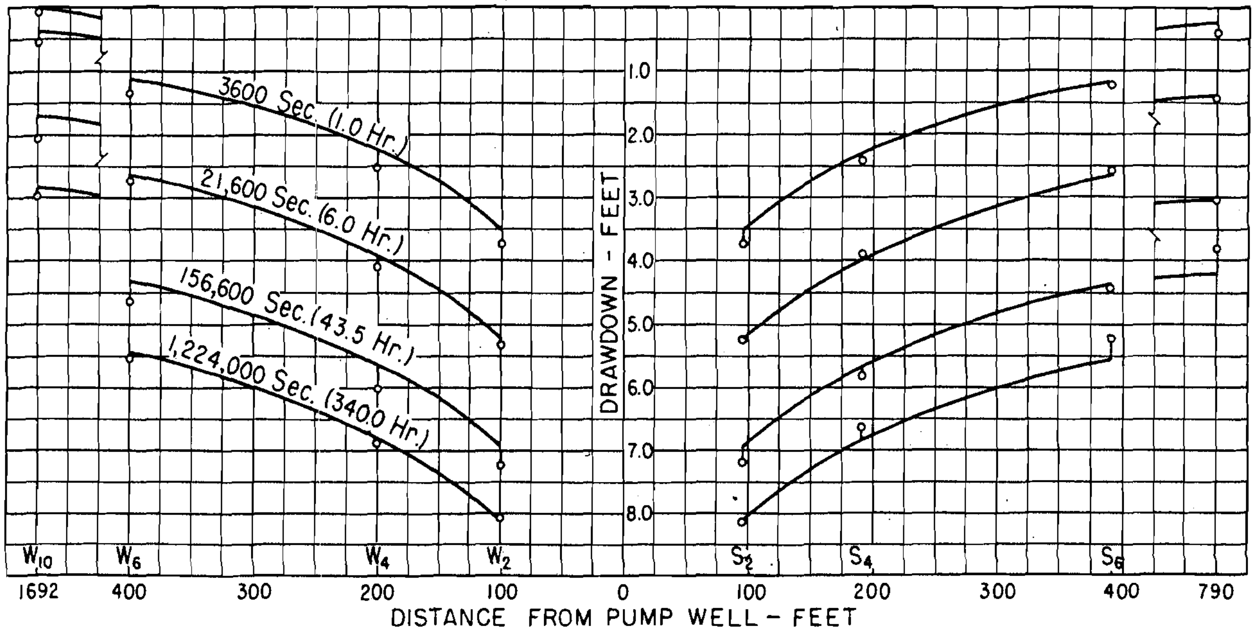


FIGURE 16.—*Drawdown curves as computed by use of the idealization of Figure 4 which includes the effect of leakage through a bed of permeability* p *overlying the aquifer. Circled points represent corresponding measured drawdown observations. Wells are designated* S_2, W_2, *etc.*

It can be concluded in this case also that the validity of the analyses is confirmed by the test.

Comparisons of Observed and Computed Return Flows

A comparison of observed and computed return flows in the Mesilla Valley in New Mexico is given in detail in Bureau Technical Memorandum No. 660 (Hurley, Reference 15).

This study covers a 12-year period terminating in 1945. No significant amount of pumping was being done at this time. The Mesilla Valley is an irrigated area served by the Rio Grande Project, covering a long narrow strip bordering the Rio Grande in southern New Mexico. It is about 50 miles in length and 5 miles wide. The gross area is about 180.7 square miles and is served by about 152 miles of drains. From these figures an average drain spacing of 1.19 miles was obtained. The surrounding area is desert which contributes little or no inflow. Diversions are made at the Leasburg and Mesilla Diversion Dams. Project operation was quite consistent and without major construction changes during the period of the study. The irrigated area in the valley is about 70,000 acres.

Records for diversions and return flows are available for the period. The analysis was based upon the use of the chart of Figure 8. The computations were based upon a time interval of 1 month. The results are shown on the following six figures. (Figures 17, 18, 19, 20, 21, and 22.)

A close correlation is obtained for 10 of the 12 years of the study period. The author states that "The differences in 1935 and 1945 can be explained by unusual situations during these years." He concludes that "Considering the quantity of water dealt with and all the variables and assumptions, the results obtained are remarkably good. This seems to indicate that the theory and the the method are valid, at least in the range of accuracy anticipated."

Acknowledgments

The content of this monograph has benefited from a review by Mr. William T. Moody.

Tables 1, 2 and 4 were prepared for this monograph using electronic digital computed programs developed by the Engineering Applications Section, Automatic Data Processing Branch of the Bureau of Reclamation. Messrs. W. T. Moody and Q. L. Florey contributed computations used as a basis for construction of Figures 3 and 5.

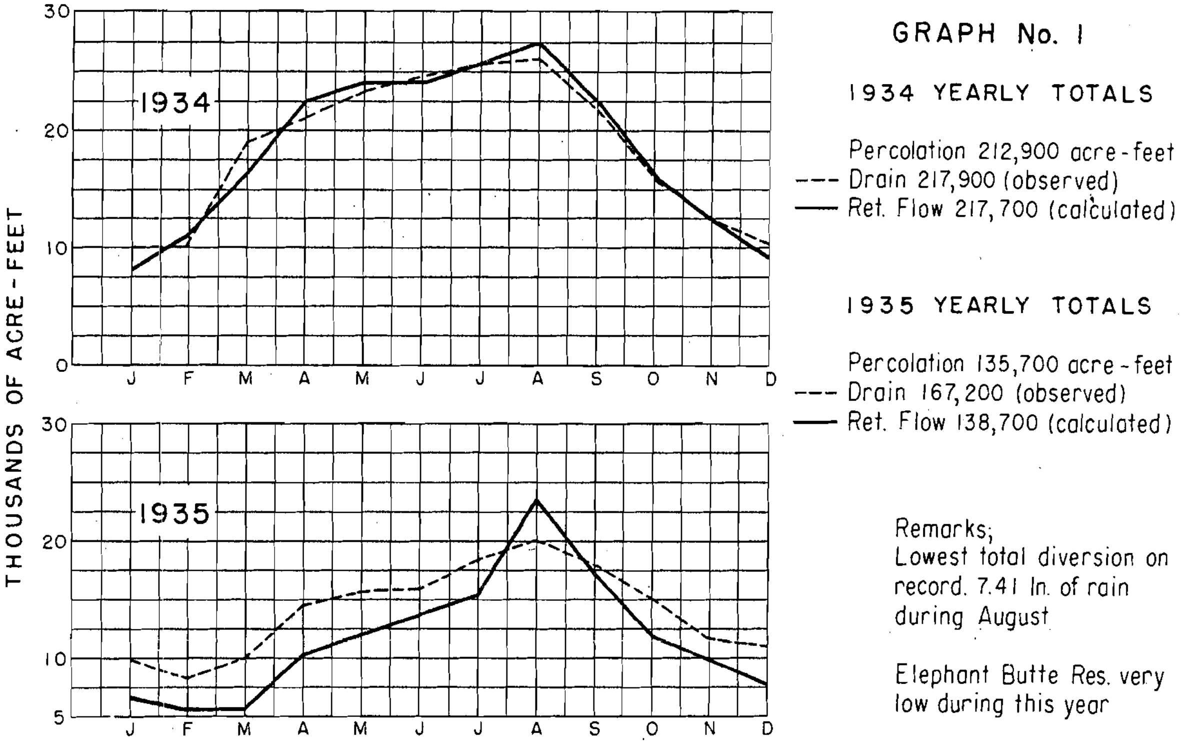


FIGURE 17.—*Comparison of observed and computed drain flows.*

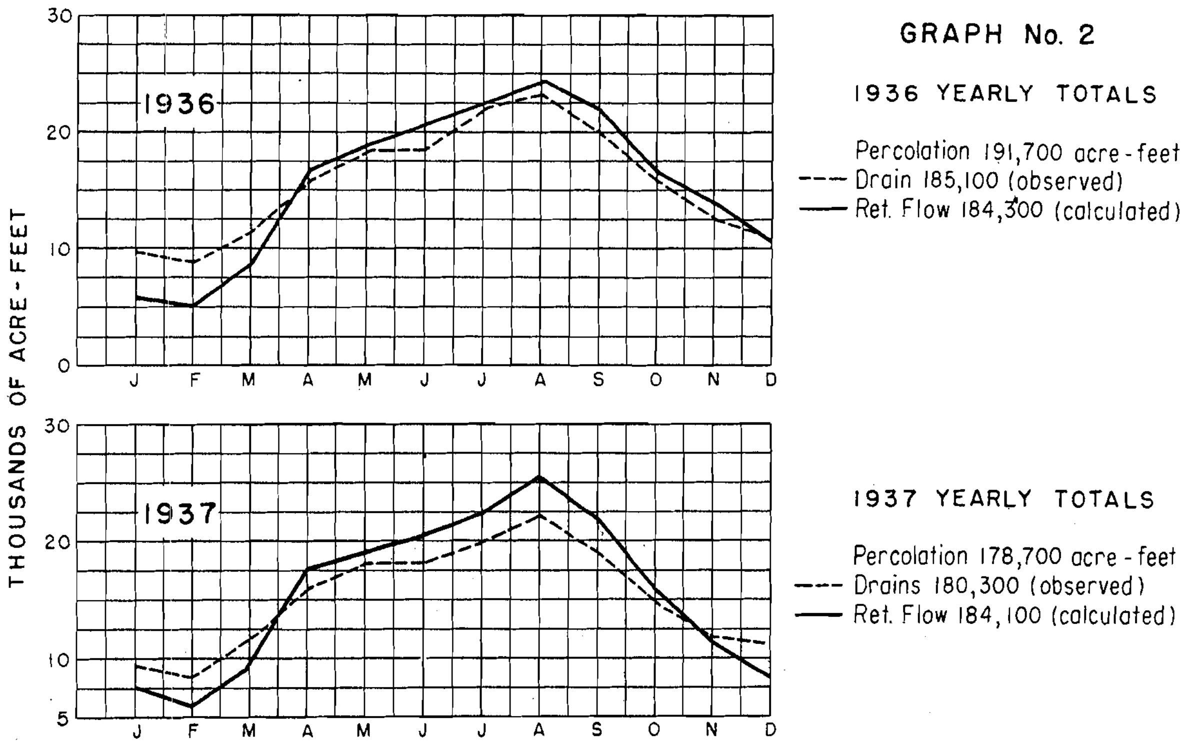


FIGURE 18.—*Comparison of observed and computed drain flows.*

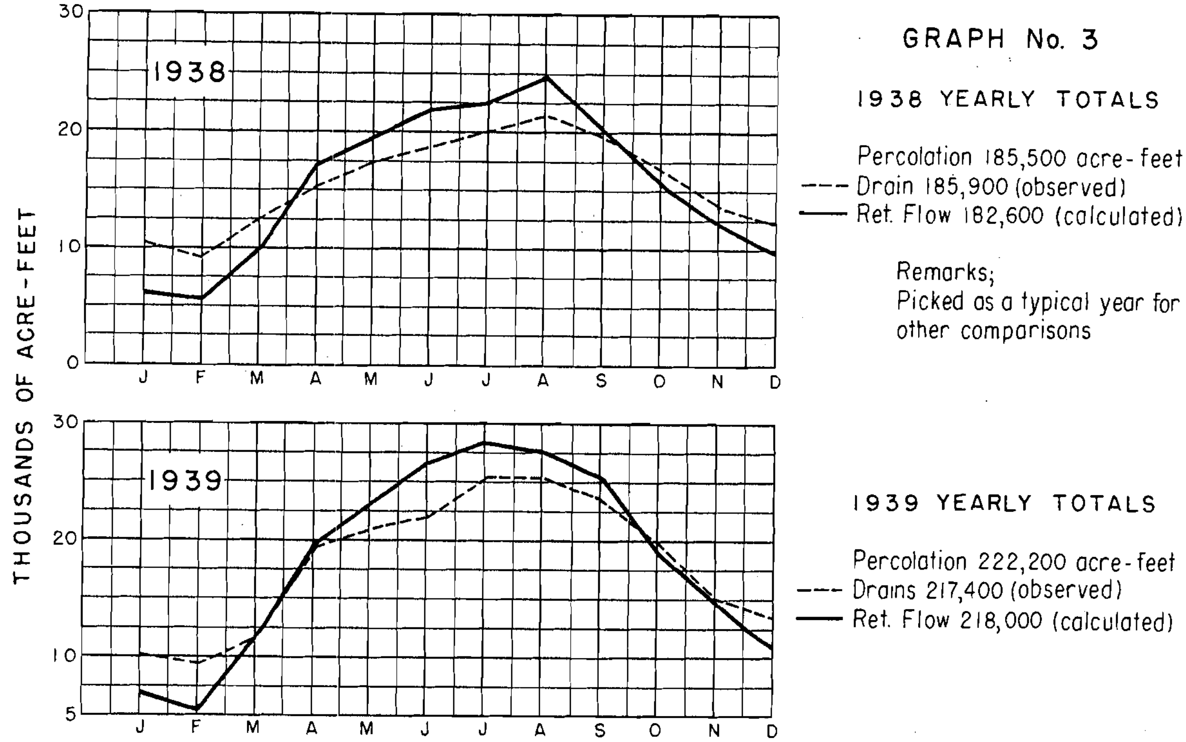


FIGURE 19.—*Comparison of observed and computed drain flows.*

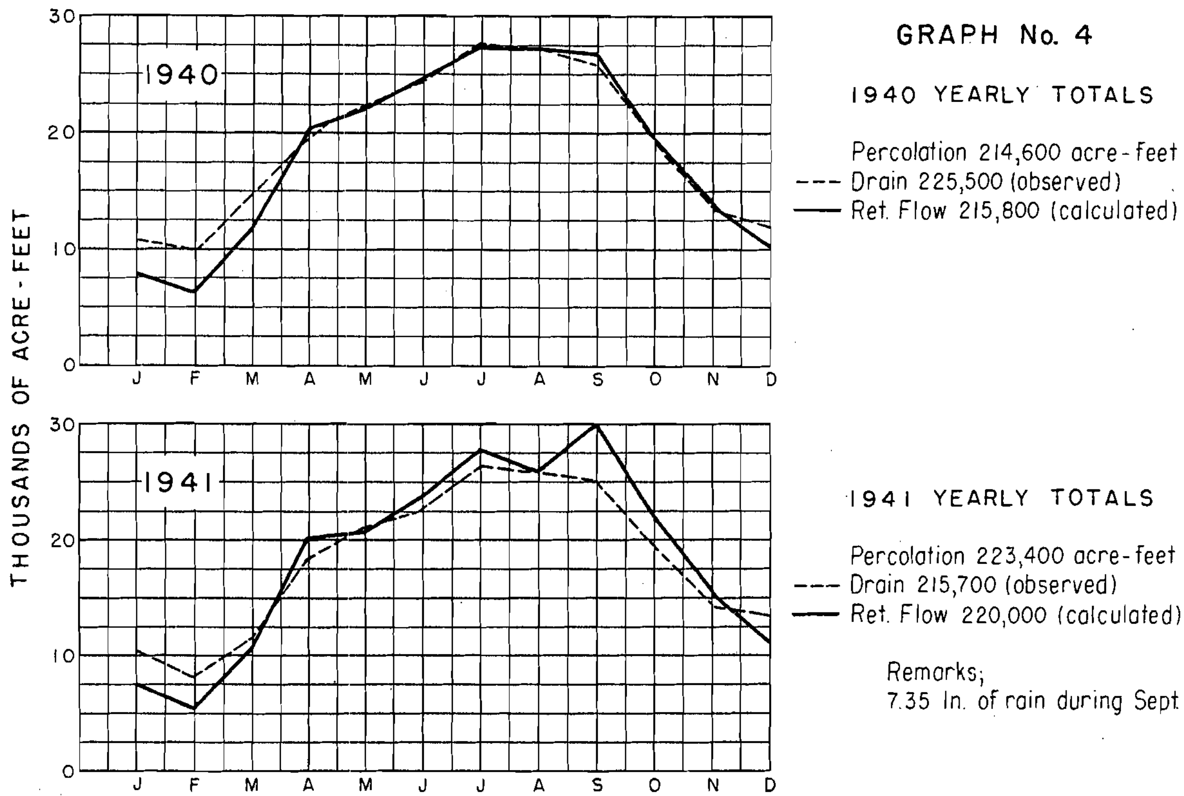


FIGURE 20.—*Comparison of observed and computed drain flows.*

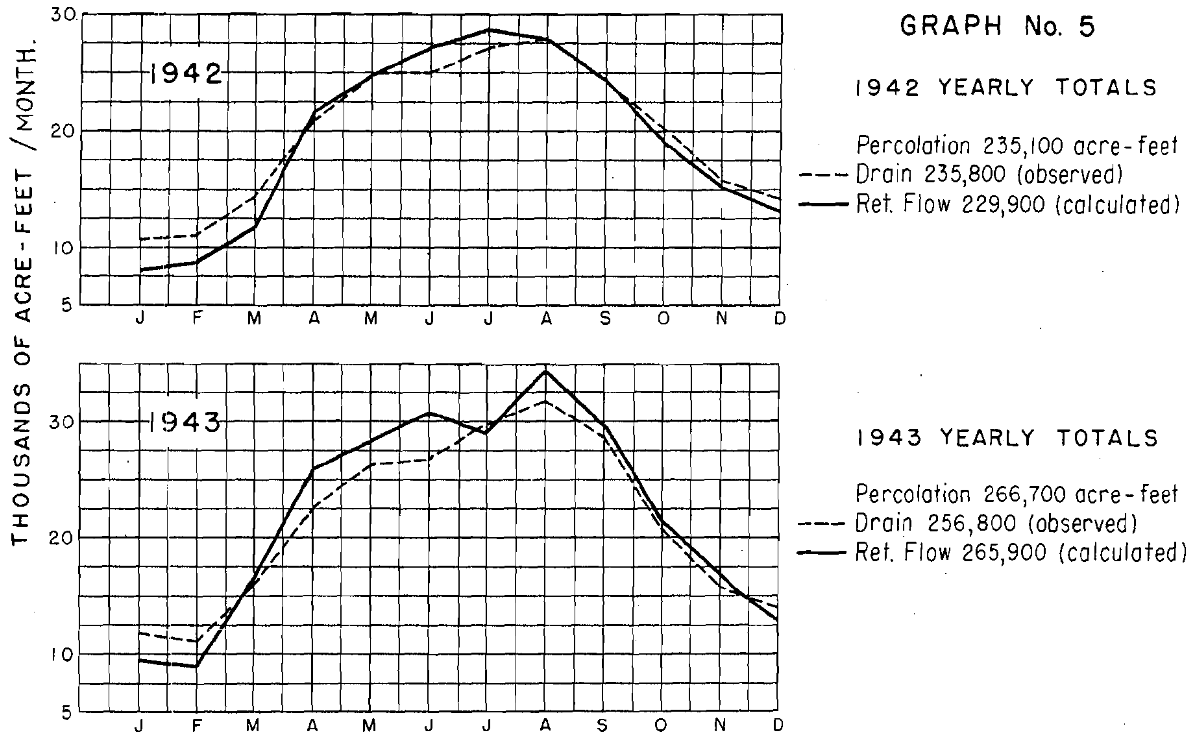


FIGURE 21.—*Comparison of observed and computed drain flows.*

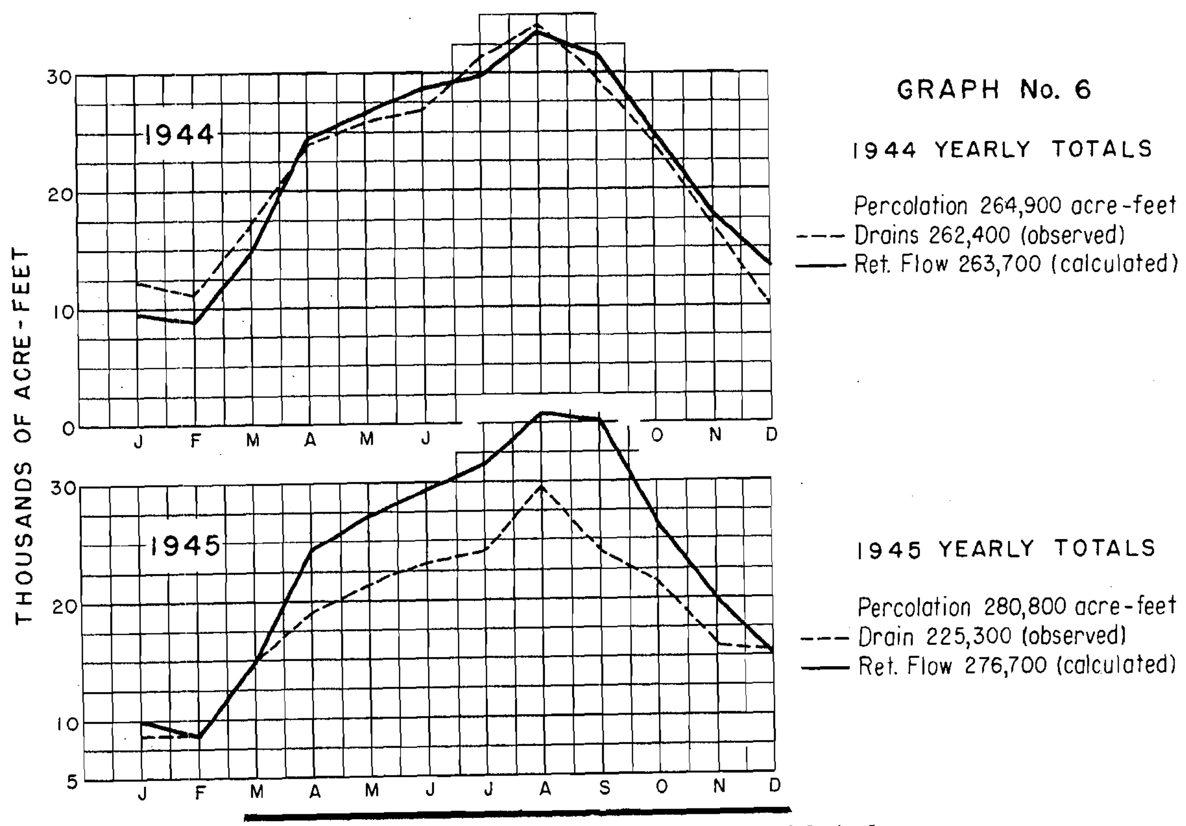


FIGURE 22.—*Comparison of observed and computed drain flows.*

List of References

1. Boreli, M., "Free Surface Flow Toward Partially Penetrating Walls," *Transactions of the American Geophysical Union*, 1955, Vol. 36, pp. 664–672 (contains some flow nets constructed by relaxation procedures which show the seepage surface above the water level in the well).
2. Boulton, N. S., "The Drawdown of the Water Table Under Nonsteady Conditions Near a Pumped Well in an Unconfined Formation," Paper 5979, *Proceedings of the Institution of Civil Engineers*, Vol. 3, 1954, pp. 564–579, incl.
3. Boussinesq, J., "Recherches, Theoretique sur l'ecoulement des nappes d'eau infiltrees dans le sol et sur le debit des sources," *Journal Mathematique Pures et Appliques*, 1904, Vol. 10, Fifth Series.
4. British Association Mathematical Tables VI, Bessel Functions, Part 1, British Association for the Advancement of Science, Cambridge University Press, 1937.
5. Brooks, R. H., "Unsteady Flow of Ground Water Into Drain Tile," *Journal of the Irrigation and Drainage Division, American Society of Civil Engineers*, Vol. 87, No. IR 2, June 1961, Part 1.
6. Bureau of Reclamation, "Cooling of Concrete Dams," Boulder Canyon Project Final Reports, 1949, Bulletin 3, Part VII.
7. Crank, J., *The Mathematics of Diffusion*, Oxford, 1957.
8. Dumm, Lee D., "Drain Spacing Formula," *Agricultural Engineering*, October 1954.
9. Dumm, Lee D., "Validity and Use of the Transient Flow Concept in Subsurface Drainage," a paper presented at the 1960 Winter Meeting of the American Society of Agricultural Engineers, Memphis, Tenn., December 4–7, 1960.
10. Glover, R. E. and Balmer, G. G., "River Depletion Resulting From Pumping a Well Near a River." *Transactions of the American Geophysical Union*, Vol. 35, No. 3, June 1954.
11. Glover, R. E. and Bittinger, M. W., "Drawdown Due to Pumping From an Unconfined Aquifer," Paper 2594, *Journal of the Irrigation and Drainage Division, American Society of Civil Engineers*, September 1960, Vol. 86, No. IR 3, Part 1, pp. 63–70, incl.
12. Glover, R. E., Hebert, D. J., and Daum, C. R., "Electric Analogies and Electronic Computers—A Symposium—Application to an Hydraulic Problem," Paper 2569, *Transactions of the American Society of Civil Engineers*, Vol. 118, 1953.
13. Gray, A. and Matthews, G. B., "Treatise on Bessel Functions," MacMillan and Co., London, 1922.
14. Haushild, William and Kruse, Gordon, "Unsteady Flow of Ground Water Into a Surface Reservoir," Paper 2551, *Journal of the Hydraulics Division, American Society of Civil Engineers*, July 1960, pp. 13–20.
15. Hurley, Patrick A., "Predicting Return Flow From Irrigation," Bureau of Reclamation, Denver, Colo., August 1961, Technical Memorandum No. 660.
16. Ingersoll, L. R., Zobel, O. J., and Ingersoll, A. C., "Heat Conduction," McGraw-Hill Book Co., Inc., New York, 1948. (A tabulation of the function is given in Appendix F.)
17. Jacob, C. E., "Radial Flow in a Leaky Artesian Aquifer," *Transactions of the American Geophysical Union*, April 1946, Vol. 27, No. 11, pp. 198–205.
18. Jacob, C. E. and Lohman, S. W., "Nonsteady Flow to a Well of Constant Drawdown in an Extensive

Aquifer," *Transactions of the American Geophysical Union,* 1952, Vol. 33, pp. 559–569.

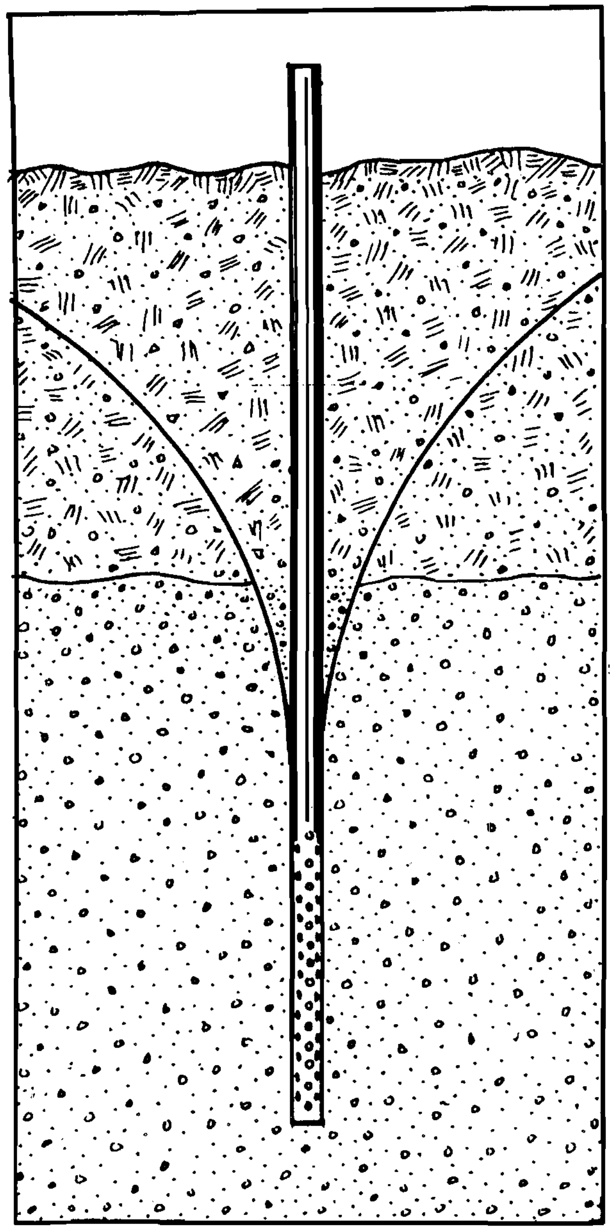
19. Jaeger, J. C., "Heat Flow in the Region Bounded Internally by a Circular Cylinder," *Proceedings of the Royal Society of Edinburgh,* 1942–43, Section A, Part III.
20. Jaeger, J. C. and Clarke, Martha, "A Short Table of

$$\int_0^\infty \frac{e^{-xu^2}}{J_0^2(u)+Y_0^2(u)} \frac{du}{u}",$$

Proceedings of the Royal Society of Edinburgh, 1942–43, Section A, Part III.
21. Jahnke, E. and Emde, F., "Tables of Functions With Formulas and Curves," Dover, 1945.
22. Kirkham, Don, "Exact Theory of Flow Into a Partially Penetrating Well," *Journal of Geophysical Research,* September 1959, Vol. 64, No. 9, pp. 1317–1327.
23. McLachlan, N. W., "Bessel Functions for Engineers," Oxford, 1934.
24. Muskat, M., "Flow of Homogeneous Fluids," McGraw-Hill Book Co., 1937.
25. "National Bureau of Standards, Tables of Sine, Cosine and Exponential Integrals", Superintendent of Documents, Washington, D.C., 1940, Vols. 1 and 2, Tables M.T. 5 and M.T. 6.
26. Pierce, B. O., "A Short Table of Integrals," Ginn and Co., 1929, Third Revised Edition.
27. Polubarinova-Kochina, P. YA., "The Problem of a System of Horizontal Drains Archiwum Meehaniki Stosowanej," 1955, Vol. 7, No. 3, pp. 287–300. (Yield of water to drains considered on a steady state basis.)
28. Polubarinova-Kochina, P. YA., " Theory of Ground Water Movement," 1952 (translated by R. J. DeWiest).
29. Theis, C. V., "The Relation Between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground Water Storage," *Transactions of the American Geophysical Union,* 1935, pp. 519–524.
30. Tuthill, L. H., Glover, R. E., Spencer, C. H., and Bierce, W. B., "Insulation for Protection of New Concrete in Winter," *Journal of the American Concrete Institute,* November 1951, Vol. 23, No. 3.
31. Watson, G. N., "Theory of Bessel Functions," Second Edition, Cambridge University Press, 1962.
32. Zangar, Carl N., "Theory and Problems of Water Percolation," U.S. Department of the Interior, Bureau of Reclamation, April 1953, Engineering Monograph No. 8.

A WATER RESOURCES TECHNICAL PUBLICATION

ENGINEERING MONOGRAPH No. 31



GROUND-WATER MOVEMENT

UNITED STATES DEPARTMENT
OF THE INTERIOR
BUREAU OF RECLAMATION