HYDRAULIC RELATIONSHIPS BETWEEN SWIMMING FISH AND WATER FLOWING IN CULVERTS

C.E. Behlke, Professor Emeritus
University of Alaska-Fairbanks
Box 82230
Fairbanks, AK 99708

ABSTRACT

Proper passage of fish through culverts is an important element of design of highways, railroads and pipelines for the North. Though several investigators have attempted to define swimming capabilities of fish, it does not appear that the swimming requirements for fish to pass through culverts and other fish passage structures have been properly analyzed and defined.

This paper defines and analyzes the forces which fish are confronted with in entering and passing through barrels of sloping culverts flowing full and as open channels. The perched culvert receives particular analytical attention. Power and total energy requirements are also defined and developed for fish passing through barrels of culverts. The methods of analysis are general and can be utilized for analyses of fish passage swimming requirements through other types of fish passage facilities.

Field observations of Arctic grayling, by the writer and others at Poplar Grove Creek, Alaska, are used as examples of potential validity of conclusions, suggested by the analytical developments of the paper.

INTRODUCTION

A substantial body of research has been directed toward identifying the hydraulic drag forces which must be overcome by a swimming fish for it to progress through fish passage structures. Past studies and design specifications for fish passage devices have generally focussed on profile drag -- drag due to pressure and frictional effects as the fish moves through water -- as being the only force which the fish must overcome to progress through fish passage structures. It must be recognized explicitly, however, that fish passage facilities are utilized almost always to pass fish both horizontally and vertically beyond some obstruction and must, therefore, be designed to assist fish in moving upward while simultaneously moving horizontally. Fish passage facility designers and researchers are all well aware of the fact that they personally have greater difficulty walking uphill at a given speed than when walking on the level at the same speed. The fact that fish swim through water does not magically make them immune to the exact same problems which humans face in moving to higher potential energy levels (moving uphill). This topic was addressed in a much earlier paper (Ziemer and Behlke, 1966) which has, apparently, gone unnoticed until recently (Orsborn and Powers, 1985) -- unfortunately then to be misunderstood. Because of the obvious importance of often ignored forces acting on swimming fish as they negotiate fish passage structures, a detailed discussion of some important elements of the hydraulics of culverts as they relate to fish passage is warranted. To negotiate a culvert successfully, migrating fish must first enter the culvert; second, pass through the barrel of the culvert; and third, exit the culvert. Because of space limitations only the first two of these will be discussed.

Force Acting on Swimming Fish

Figure 1 is a free-body-diagram of a fish swimming at constant velocity, $\overline{V}_f$, in a water body which has a horizontal surface and current velocity, $\overline{V}_w$, equal to zero. This figure depicts the various fundamental forces which affect the fish's motion including the propulsive force, $\overline{P}$, which a fish must generate to move at velocity, V_{fw}, through the water (overbars indicate vector quantities).

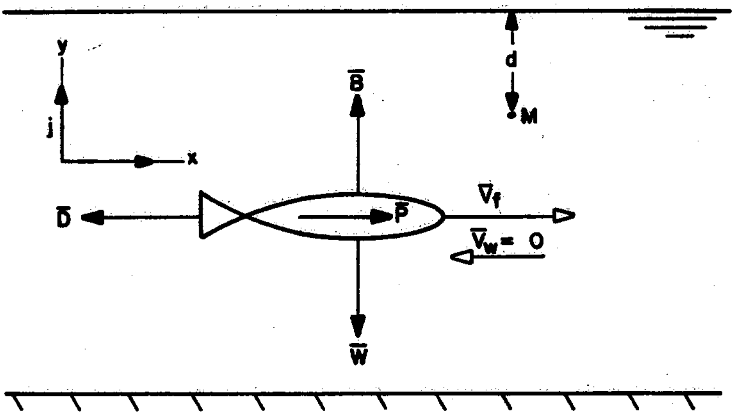


Figure 1. Free-body-diagram of fish swimming in static water.

Here $\overline{D}$ is the profile drag force resulting from all frictional effects between the surrounding water and the fish (Force), $|\overline{D}| = D$; $\overline{P}$ is the propulsive force generated by the fish to overcome other forces (in this situation only the drag force, $\overline{D}$) (F), $|\overline{P}| = P$; $\overline{B}$ is the buoyant force created on the fish by the surrounding water (F), $|\overline{B}| = B$; $\overline{W}$ is the fish's weight (F), $|\overline{W}| = W$; $\overline{V}_f$ is the velocity of the fish with respect to a fixed reference system (L/T), $|\overline{V}_f| = V_f$; $\overline{V}_w$ is the velocity of the water with respect to a fixed reference system (in this case $\overline{V}_w = 0$) (L/T), $|\overline{V}_w| = V_w$; $\overline{V}_{fw}$ is the velocity of the fish with respect to the water (in this case $\overline{V}_{fw} = \overline{V}_f$) (L/T); x and y define the fixed coordinate system for any figure, x being positive in the direction of the fish's motion in this and subsequent figures; i and j are unit vectors in the x and y directions, respectively. Each of the forces, velocities and unit vectors is a vector quantity having both magnitude and direction (unit vectors have magnitudes of unity).

Since the fish depicted in Figure 1 moves with constant velocity (i.e., no acceleration), the four forces shown are in equilibrium. Much of the following discussion will be focussed on the buoyant force, $\overline{B}$, and the fish's weight, $\overline{W}$, since these forces appear to be virtually unrecognized and poorly understood by fish passage structure designers, yet, under some conditions, may strongly influence the success or failure of a fish's successfully negotiating a fish passage facility. For the situation shown, the water pressure, p, at any point, M, in the fluid is

$$p = \gamma(d) \qquad \text{----Eq. 1}$$

where p is the pressure at a given point in the water, (F/L^2); d is the depth, measured normal to the water surface downward to the point M, (L); and γ is the specific weight of water, (F/L^3).

Fundamental laws of fluid mechanics state that at any point in the fluid the buoyant force/unit volume of fluid displaced is equal but opposite in direction to the vector gradient of the pressure, $\overline{\nabla}p$. That is

$$\overline{b} = -\overline{\nabla}p \qquad \text{----Eq. 2}$$

where $\overline{b}$ is the buoyant force per unit volume of fluid displaced at point M in the water, (F/L^3); and $\overline{\nabla}$ is the two-dimensional vector operator $\partial/\partial x\ i + \partial/\partial y\ j$.

For the conditions of Figure 1, the magnitude of ∇p equals the specific weight, γ, of the fluid. Thus, the buoyant force, $\overline{B}$, acting on the fish indicated in Figure 1 is

$$\overline{B} = (-\overline{\nabla}p)(\mathcal{V}) \qquad \text{----Eq. 3}$$

$$= (\gamma)(\mathcal{V})(j) \qquad \text{----Eq. 4}$$

where $\mathcal{V}$ is the volume of the fish's body (L^3).

It is important to recognize that Equation 2 is general and applies to static or dynamic fluid situations. If $\overline{\nabla}p$ is not variable in the flow field which the fish swims through, Equation 3 also has general applicability. Equation 4, however, applies only to the situation where the fluid surface is horizontal and streamlines in the fluid are straight, parallel and horizontal. Because Equation 3 is general, subject to the qualification above, it may be applied to situations where water surfaces slope and where streamline curvature and divergence is small compared to the size of the design fish. Since the fish is a very "streamlined" object, it is assumed for this discussion that the virtual mass correction of the water which it displaces can be ignored.

If the fish's specific weight is assumed to be the same as that of water (i.e., $\gamma_f = \gamma$), the buoyant force, $\overline{B}$, and the weight, $\overline{W}$, of the fish of Figure 1 are equal and opposite in direction and cancel each other. Thus, the propulsive force, $\overline{P}$, generated by the fish just equals the profile drag force, $\overline{D}$. Unfortunately, artificial fish passage facilities do not normally involve static water conditions -- water flows through the facility, water surfaces slope, and streamlines are not straight and parallel. Hence, in dynamic situations the buoyant force is best determined by surface integration of the pressure over the surface of the fish's body or by means of Equation 3, where applicable, but Equation 4 often cannot be used for design of fish passage facilities. This latter point frequently is overlooked with the result that both the buoyant force and the weight of the fish are incorrectly neglected in determinations of what types of flow situations fish can successfully negotiate. Since it is desirable to know ∇p for the determination of the magnitude and direction of the fish's buoyant force, it is important to consider how pressure responds to depth for some other rather typical situations found in flow through fish passage facilities, including culverts, under some flow conditions.

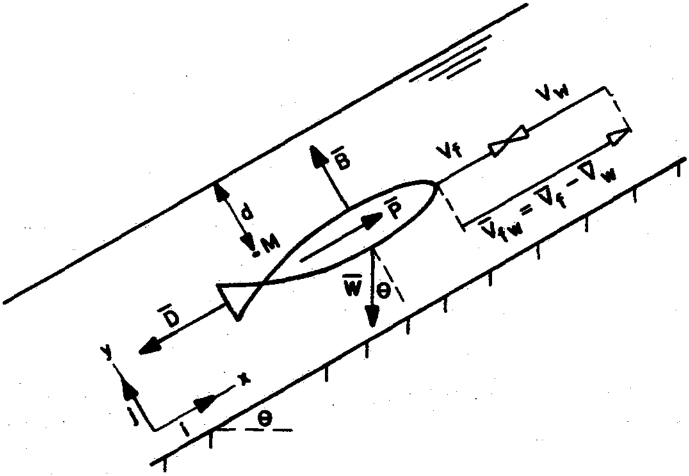


Figure 2. Definition sketch of fish swimming in sloping open channel.

The situation depicted in Figure 2 is one-dimensional, uniform, steady flow down an open channel (possibly a culvert flowing partially full) inclined to the horizontal at an angle θ.

Open Channel hydraulics texts (Henderson, 1955) show the pressure, p, at a point, M, in the moving fluid of Figure 2 to be,

$$p = d(\gamma)(\cos\theta) \qquad \text{----Eq. 5}$$

and it follows that,

$$-\overline{\nabla}p = \gamma(\cos\theta)(j) \qquad \text{----Eq. 6}$$

From Equation 3, which is applicable under the conditions of Figure 2, the buoyant force, $\overline{B}$, acting on a fish swimming in a sloping channel, is normal to the water surface and sloped at the angle θ to the vertical in the +y direction. $\overline{B}$'s magnitude is also less than it was in the example of Figure 1, now being

$$\overline{B} = \gamma(\forall)(\cos\theta)j \qquad \text{----Eq. 7}$$

If the fish swims up this channel at constant depth, the forces which its propulsive force must overcome and its propulsive force are as depicted in Figure 2.

The drag force, $\overline{D}$, due to frictional effects remains a function of the fish's velocity with respect to the water, $\overline{V}_{fw}$, so the magnitude of this force would be the same as in the condition of Figure 1, if the velocity of the fish with respect to the water, $\overline{V}_{fw}$, is the same. The weight of the fish, $\overline{W}$, remains the same in magnitude and direction as in Figure 1. The y component of the fish's weight normal to the direction of the fish's motion is now Wcosθ. If the specific weight of the fish is assumed the same as that of the water (no aeration of the water), this too becomes $\gamma(\forall)\cos\theta$ which is precisely the same as the new buoyant force but is opposite in direction. So, as in Figure 1, the buoyant force and the component of the fish's weight in the direction parallel to $-\overline{\nabla}p$ cancel each other. However, the fish's weight now also has a down-slope component normal to the direction of $\overline{\nabla}p$. The fish's propulsive force, $\overline{P}$, must now overcome the hydraulic profile drag force, $\overline{D}$, due to its motion through the water (which occurs regardless of slope of channel or direction of the fish's progressive motion through the water), but additionally, it must overcome the down-slope component of the fish's weight, Wsinθ. In equation form the propulsive force becomes

$$\overline{P} = W(\sin\theta)i + Di \qquad \text{----Eq.8}$$

Clearly, if the slope of the channel shown in Figure 2 is small, sinθ is small, and the additional propulsive force necessary to overcome this weight component may be negligible. It should be noted that $\overline{B}$ and $\overline{W}$ do not depend on the fish's motion through the channel where uniform, steady flow exists, so if the fish is moving up or down the channel or "resting" in it, these forces still exist, and they must be resisted by an appropriate propulsive response. This point has considerable importance when the fish's energy requirements are considered later.

The next step is to consider a fish swimming through an underdesigned, horizontal culvert which is flowing full and with a considerable headwater as conceptually indicated in Figure 3.

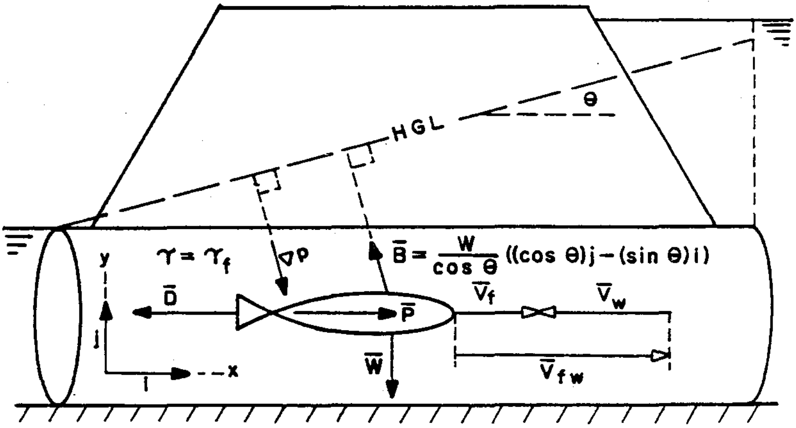


Figure 3. Forces acting on fish swimming in horizontal culvert.

The hydraulic grade line, HGL, has its usual engineering meaning (the locus of the hydraulic piezometric head in the conduit). It slopes at an angle θ with the horizontal. The gradient of the pressure, $\bar{\nabla}p$, in the culvert body, where the fish is located, is normal to the HGL as indicated in Figure 3. Thus, the fish's buoyant force also acts normal to the HGL. The magnitude of the vertical component of the buoyant force is $B\cos\theta$, and that of the horizontal component of the buoyant force is $-B\sin\theta$. It is a simple exercise to show that the magnitude of $-\bar{\nabla}p$ is $\gamma/\cos\theta$ everywhere in the culvert. So, if $\gamma_f = \gamma$, the vertical component of $\bar{B}$ cancels $\bar{W}$, but $\bar{B}$ also has a horizontal component which adds to profile drag, as indicated in the free-body-diagram of the fish in Figure 3 (if significant air entrainment occurs, $\gamma_f > \gamma$).

If the fish does not accelerate in the facility, the propulsive force, $\bar{P}$, is

$$\bar{P} = -(-(\bar{\nabla}p)(\forall) + \bar{W} + \bar{D}) \qquad \text{----Eq. 9}$$

$$= (\gamma/\cos\theta)(\forall)(\sin\theta i - \cos\theta j) - \gamma\forall j + Di \qquad \text{----Eq. 10}$$

and if $\gamma_f = \gamma$, Equation 10 becomes

$$\overline{P} = W\tan\theta i + \overline{D} \qquad \text{----Eq. 11}$$

What this really means is, as in the case of the sloping open channel, the fish's propulsive force, $\overline{P}$, must do more than just overcome the frictional effects of the current. It must also counteract the downstream component of the buoyant force (the first term in the right side of Equation 11). Thus, the fish must generate an additional horizontal propulsive force, to overcome the negative x component of the buoyant force beyond that necessary to realize the same $\overline{V}_{fw}$ as of Figure 1. The first term in the right side of Equation 11 is often not recognized in design of fish passage facilities, including culverts with substantial headwater and flowing full. If the angle θ is small such that $\tan\theta$ is approximately zero, this term can, of course, be ignored safely.

Figure 4 indicates the general case of fish attempting to negotiate a closed conduit sloping at an angle ϕ and with the HGL sloping at an angle θ, both with respect to the horizontal.

Assuming all velocities are constant, Figure 4 shows the forces which the fish's propulsive force, $\overline{P}$, must overcome, thus

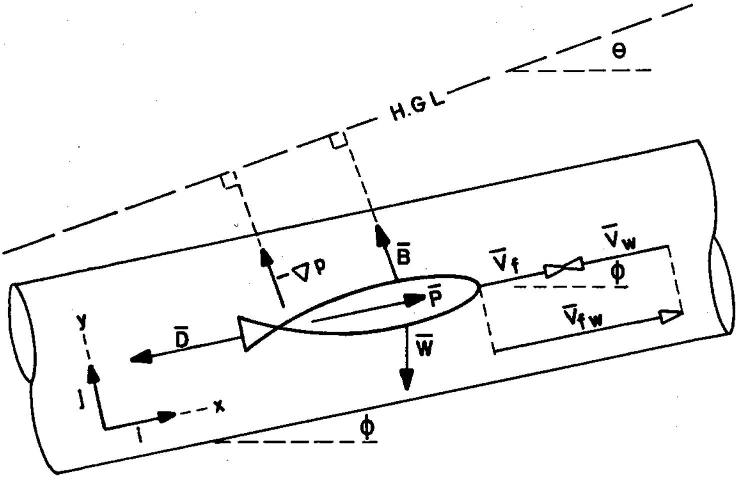


Figure 4. Forces acting on fish swimming in sloping conduit with headwater.

$$\overline{P} = (D+W\sin\phi+B\sin(\theta-\phi))i + (W\cos\phi-B\cos(\theta-\phi))j \quad \text{----Eq. 12}$$

It is an elementary exercise of fluid mechanics to show that

$$\nabla p = \gamma\cos\theta/\cos(\theta-\phi)$$

and, again, if $\gamma_f = \gamma$,

$$\begin{aligned} B &= -\nabla p(\Psi) = W(\gamma\cos\phi)/\gamma(\cos(\theta-\phi) \\ &= W\cos\phi/\cos(\theta-\phi) \quad \text{----Eq. 13} \end{aligned}$$

So, Equation 12 becomes

$$\begin{aligned} \overline{P} &= (D+W\sin\phi+W\cos\phi\sin(\theta-\phi)/\cos(\theta-\phi))i \\ &\quad + (W\cos\phi-W(\cos\phi)(\cos(\theta-\phi)/\cos(\theta-\phi))j \\ &= (D+W(\sin\phi+\cos\phi(\sin(\theta-\phi)/\cos(\theta-\phi))i \\ &= (D+W(\sin\phi+\cos\phi(\tan(\theta-\phi)))i \quad \text{----Eq. 14} \end{aligned}$$

In all cases the y components of $\overline{W}$ and $\overline{B}$ cancel. It is clear from the Equation 2, however, that, except in the cases of $\theta = \phi$ or ϕ = zero, both $\overline{B}$ and $\overline{W}$ have components which add to the drag force which $\overline{P}$ must overcome. The free-body-diagram described by Equation 14 is also shown in Figure 4.

WEIGHT AND BUOYANCY IMPLICATIONS FOR FISH ENTERING PERCHED CULVERTS

If an essentially hydrostatic pressure distribution (i.e., pressure increases linearly with depth from the water surface at a rate of γ) exists in the flow in a perched culvert several normal depths, d_o, upstream from its outlet, water flowing in the culvert must accelerate as it approaches the culvert's free-outfall. The convective acceleration in the vicinity of the outfall is a result of the change from hydrostatic pressure distribution to smaller or zero pressures in the jet of water as it exits the end of the culvert. The writer is not aware of analytical or experimental studies of the pressure in the vicinity of free-outfalls in circular open channels. However, Rouse (1938) and Henderson (1966) give information on the pressure distribution in the vicinity of free-outfalls in two-dimensional channels. Recognizing that rectangular and circular channels do not have precisely the same pressure distributions in the vicinity of the free-outfall, two-dimensional analogies may be utilized to advantage along the vertical center-plane of a circular culvert to give some insights into the magnitude and direction of the pressure gradient vector, $\overline{\nabla p}$, in the vicinity of the culvert outfall lip.

Rouse's (1938) graphical descriptions of the pressure distribution for frictionless, potential flow in the vicinity of a free-outfall is essentially reproduced in Figure 5.

The lines of constant pressure of Figure 5 describe a pressure potential field. Thus, the gradient of the pressure, $\overline{\nabla}p$, at any point in the field is directed normal to the line of constant pressure passing through that point and is directed in the direction of increasing pressure. The magnitude of $\overline{\nabla}p$ is inversely proportional to the spacing of the lines of constant pressure. Of course, $-\overline{\nabla}p$, which defines the buoyant force per unit of volume of fluid at any point in the flow field, and is the point of interest here, has the same magnitude as $\overline{\nabla}p$ but is opposite in direction. Without defining the scale, Figure 6 indicates the direction and relative magnitudes of $-\overline{\nabla}p$ at several locations of the pressure field of Figure 5.

Water following close to the invert at point b, experiences a large pressure gradient as it approaches the culvert lip. Here $-\overline{\nabla}p$ is directed downstream parallel to the channel invert and has a large magnitude. Just above the channel lip at point a, $-\overline{\nabla}p$ remains large, and it has a large downward component. In the center of the jet just above the channel lip at point c, the magnitude of $-\overline{\nabla}p$ is small, and it contains only a weak vertical component to assist in partially cancelling the fish's weight force. Only toward the top of the jet does $-\overline{\nabla}p$ develop a considerable upward component. Observations by the writer at Poplar Grove Creek, in May 1986, indicated that Arctic grayling (Thymallus arcticus) attempting to enter a partially perched culvert struggled in the vicinity of the downstream lip, but most of those which attempted to enter the culvert were able to negotiate the pressure gradient and approximately 2.13 m/sec velocities which existed at that point on a given day. Very few were observed leaping out of the water in their attempts to enter the culvert.

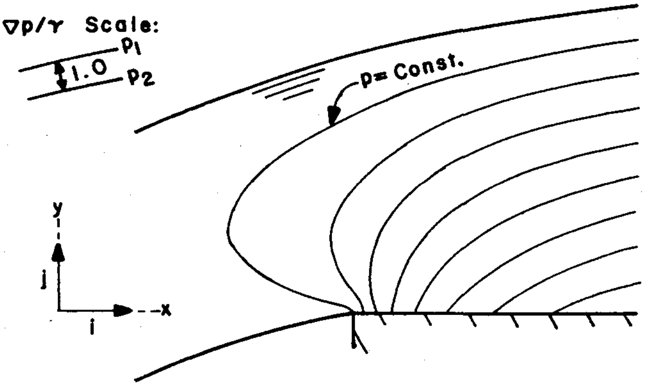


Figure 5. Pressure distribution in the vicinity of a two-dimensional free-outfall (After Rouse, 1938). (Scale by writer.)

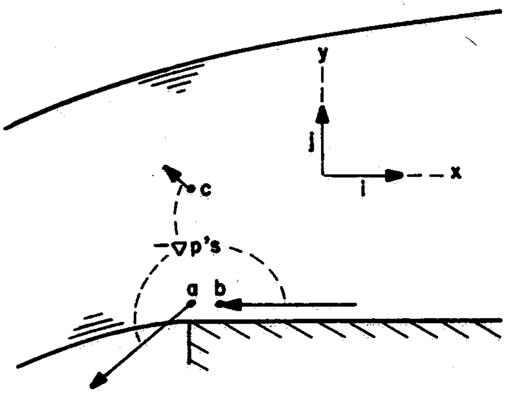
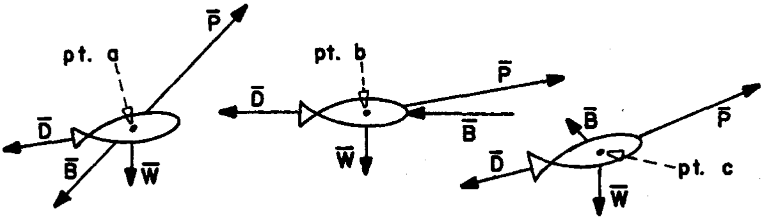


Figure 6. Description of $-\overline{\nabla}p$ at selected locations in the flow field of Figure 5.

Travis (1986) reports that in May 1985 when outlet velocities were in the order of 2.7 m/sec and pressure gradients at the culvert lip were undoubtedly stronger than in May of 1986, he observed a great many grayling leaping out of the water in their attempts to enter the top surface of the jet at the end of the culvert. Many fish which attempted to enter the culvert in this fashion were successful in doing so. Thus, it appears at some point the adverse effects of weight, buoyant force, and profile drag appear to force grayling to change their tactics from a bottom entry into the culvert to that of leaping and entering where pressure gradients are less severe. Travis (1986) observed grayling leaping as high as 1.5 m from the surrounding water surface in attempting to enter the upper surface of the jet at the end of the culvert. In the absence of possible assistance from local water currents, a fish must leave the surface of the surrounding pool with a minimum velocity of 5.46 m/sec in order to leap to that elevation. Travis also reports grayling leaping such that they covered a horizontal distance of up to 6.1 m from the point at which they left the pool surface to the point which they reentered the water. In the absence of assisting currents, this requires a minimum initial leaping velocity of 7.74 m/sec. Clearly, these are extremely high velocities for grayling (perhaps higher than have been previously reported). Such leaping velocities are high enough to indicate that these grayling may have entered the culvert at the invert lip if they were faced only with profile drag forces created by velocities of 2.7 m/sec. Thus, it appears strong pressure gradients in the vicinity of the lip may well have been the determining factor in grayling's abandoning their attempts to enter the lip of the culvert in favor of leaping to enter the less severe pressure gradients in the water at the upper surface of the water jet.

A true picture of pressure gradient forces which the fish experiences at various locations along its upstream path would require an integration of pressure over the fish's body surface as it passes through this complex pressure gradient flow field. If the fish is small in relation to the scale of the flow field, values of $-\bar{\nabla}p$, calculated for the unoccupied flow field at various points of interest in the flow field, can be multiplied by the fish's body volume to yield an approximate, but probably reasonable, buoyant force, $\bar{B}$ at those locations. Conceptual results thus obtained would be most accurate for fingerlings and least accurate for adult salmon.

The relative magnitudes and directions for $-\bar{\nabla}p$ indicated in Figure 6 are best appreciated when they are converted (approximately) to buoyant force, B, and are applied to the free-body-diagrams of fish occupying specific locations in the flow field. This is shown in Figure 7 for points a, b and c of Figure 6.



7–a 7–b 7–c

Figure 7. Free-body-diagrams for fish at points a, b and c of Figure 6.

Here $\bar{B}$ is assumed to be the product of the fish's volume and $-\bar{\nabla}p$, evaluated in the flow field of Figure 6 at the location corresponding approximately to that of the centroid of the fish's body. It is clear from these free-body-diagrams that $\bar{B}$ has taken on an unusual character for the locations shown. It and the fish's weight combine to act like an anvil which the fish must carry until it reaches the upstream zone of relatively straight and parallel streamlines. The zone of greatest difficulty is in the immediate vicinity of the outfall lip where the profile drag is also greatest. Adversity eases simultaneously with progress upstream away from this critical point.

A fish approaching the invert of a perched culvert may be neutrally buoyant as it swims upward toward the surface of the pool below the culvert outlet ($\bar{B} = -\bar{W}$). As it leaps clear of the pool, it suddenly loses its buoyant force and is accelerated downward by its uncancelled (by any buoyant force) weight. Then, as it enters the jet emitting from the culvert, it is met by a hostile buoyant force. Figures 7a, 7b and 7c indicate the degree of hostility of the buoyant force which depends on what part of the water jet the fish enters. Though the culvert on Poplar Grove Creek has an adverse (upward) invert slope at its outlet, the writer's (1986) visual observations of grayling entering at the invert lip of the culvert clearly indicated the fish were swimming very strenuously and in a much different mode (high frequency, low amplitude back and forth articulation of the body) from that of only one lip-depth distance upstream from the culvert lip. It was observed that fish clearing this obstacle were seldom subsequently swept out of the culvert. It is difficult to separate degree of distress relieved as the fish move a short distance upstream in the lessening and redirecting pressure gradient from that of the reducing drag force, $\overline{D}$, as the fish simultaneously moves into slower water velocities. However, an attempt can be made to evaluate approximately, the drag force, $\overline{D}$, and the buoyant force, $\overline{B}$. As an example of computations (admittedly approximate because of the qualitative aspects of one of the critical reference (Rouse, 1938)) to determine the relative values of the two forces just mentioned, a .25 m fish will be assumed entering a perched, two-dimensional culvert (point a of Figure 6) of very small slope and transporting water such that the hydraulic critical depth, d_c, is .4 m. It will also be assumed that the flow farther upstream is at normal depth, d_o, and that $d_o > d_c$. Henderson (1955) shows the lip depth, d_L, to be 0.715 d_c. From the assumed critical depth, the critical velocity, V_c, can readily be calculated to be 1.98 m/sec. Thus, the average water velocity at the culvert mouth, V_L, is $V_L = 1.98/.715 = 2.77$ m/sec. The standard profile, fluid drag equation for rigid bodies is

$$D = C_D \rho A V^2/2 \qquad \text{----Eq. 15}$$

where C_D is the drag coefficient which depends on the geometry of the body for which the drag is being calculated and its Reynolds number, (dimensionless); ρ is the mass density of the water, (M/L^3); A is the frontal projected area of the body, (L^2); V is the velocity of the body relative to the surrounding fluid. In this case, $V = V_{fw}$.

The writer's earlier analysis (Ziemer and Behlke, 1966) of J.R. Brett's data from young, swimming Sockeye salmon (Brett, 1963) yielded an equation for the value of the profile drag coefficient which will be used here for grayling and which is as follows.

$$C_d = \frac{3.3}{(N_R)^{.417}} \qquad \text{----Eq. 16}$$

where $N_R = V_{fw} L\rho/\mu$ = the fish's Reynolds number, and L is the fish's fork length. In order to apply this profile drag coefficient to Equation 15, it is necessary to substitute L^2 for A. This is essentially a scale change which is also reflected in the value of the profile drag coefficient calculated by Equation 16. The fish's fork length is used both in the Reynolds number and the profile drag equation because that is the length dimension which those in fisheries tend to use in characterizing fish. (The reader is cautioned not to use this drag coefficient in Equation 15.) The appropriate profile drag equation for fish then becomes

$$D = C_d(\rho)L^2V_{fw}^2/2 \qquad \text{----Eq. 17}$$

Accepting Equation 16 for grayling and for Reynolds numbers which are approximately an order of magnitude greater than those for the swimming fish of Brett's experiments, the profile drag of the example is calculated as follows. For water at 10°C the Reynolds number for the example .25 m fish, swimming through the water at a velocity arbitrarily selected to be 5% greater than the culvert lip water velocity (that is, $V_{fw} = 1.05(2.77)$ m/sec = 2.90 m/sec), the profile drag coefficient (Equation 16) becomes $C_d = 0.0133$, and the profile drag from Equation 16 becomes

$$\begin{aligned} D &= .0133(1000)(0.25)^2(2.90)^2/2 \\ &= 3.5\ \text{N}. \qquad \text{----Eq. 18} \end{aligned}$$

The writer's cursory analysis of 1986 data collected for arctic grayling at Poplar Grove Creek reveals the following approximate, average relationship between weight and length for a wide range of fish which were sampled.

$$\begin{aligned} W &= (9\text{x}10^{-3})L^3 \ \text{(grams)} \qquad \text{----Eq. 19} \\ &= 88\ L^3 \ \text{(Newtons)} \qquad \text{----Eq. 19} \end{aligned}$$

Thus the example .25 fish (if a Poplar Grove Creek grayling) would weigh 1.38 N, and, if its specific weight is assumed the same as for water, its volume would be $1.38/\gamma = .00014\ \text{m}^3$. Scaling, approximately, the pressure pattern indicated by (Rouse, 1938) and recalling that the lip water depth is .715 d_c, the value of $-\overline{\nabla}p$ close to the culvert lip at one-half of a body height (here assumed to be L/10) above the culvert lip, approximately where the fish's centroid would pass if it hugged the invert of the culvert (point a of Figure 6), is approximately, $1.7\gamma\ \text{N/m}^2\text{/m}$, directed downward, and against the fish's progress, at an approximately 45 degree angle. From Equation the fish's buoyant force, $\overline{B}$, would be

$$\begin{aligned} \overline{B} &= (-\overline{\nabla}p)(\forall) \qquad &\text{----Eq. 3} \\ &= 1.7(\gamma)(\forall)(-.707i-.707j) \qquad &\text{----Eq. 20} \\ &= 1.7(9810)(.00014)(-.707i-.707j) \qquad &\text{----Eq. 21} \\ &= 2.33(-.707i-.707j)\text{N}. \qquad &\text{----Eq. 22} \end{aligned}$$

Or, the magnitude of $\overline{B}$ is 2.33 N. This is almost the magnitude of the approximate drag force, $\overline{D}$, calculated in Equation 18. Though the calculations here of both $\overline{B}$ and $\overline{D}$ are certainly not precise because of the approximate use of a small diagram of the Rouse reference, the writer's approximate evaluation of the surface integral of pressure over the fish's body, neglect of virtual mass complications, and some disagreement among investigators regarding the proper determination of values used in Equation 17, it is clear that, for a fish, entrance to the invert of a perched culvert probably is not an easy matter even if the invert is perched only a short distance above the surface of the downstream pool.

Since, for the example just considered, the upstream uniform depth for the discharge defined by the given critical depth was assumed greater than d_c, and the culvert was functioning in the upstream barrel at its normal depth of flow, d_o, the water surface profile passed through d_c as it approached the culvert lip. This was a situation where some distance upstream the flow was at normal depth, hydraulically subcritical, and followed an M-2 curve from the initial point of contraction to approximately critical depth. At approximately that point the flow became rapidly varied and began to lose its hydrostatic pressure distribution.

The writer is not aware of analytical or laboratory experiments which detail the pressure distribution in flow at a free-outfall lip for supercritical approaching flow in the barrel of the culvert. However, a nonhydrostatic pressure gradient must exist in such flow as it approaches the culvert lip. Thus, supercritical flow in the culvert as the flow approaches the perched invert lip must exhibit some of the same fundamental characteristics as approaching subcritical flow (i.e., $-\overline{\nabla}p$ has a strong downstream component along the invert near the downstram culvert lip). The buoyant force in that vicinity, therefore, consists of a downstream component as well as a downward, vertical component. Thus, the same fundamental problem is created as resulted from subcritical flow in the upstream approaching section, though the magnitude and direction of $\overline{\nabla}p$ would be somewhat different from that in the subcritical approaching flow situation.

A fundamental problem, therefore, is that the inducement of large additional, adverse, horizontal and vertical forces (beyond those which the fish would face if only subject to profile drag) in the vicinity of the lip of perched culverts may impede or block successful fish passage. These forces can be avoided completely if the culvert is submerged so that the water surface in the receiving pool matches the elevation of the water surface in the culvert at the outlet. If the slope of the approaching water surface profile, on leaving the culvert and entering the pool, does not increase, the pressure gradient vector and, therefore, the buoyant force, would change very little from its direction and magnitude in the culvert barrel to that in the pool. $\overline{B}$ would not exhibit the problems illustrated in Figure 7. Between this situation and that of a highly perched culvert, for which the receiving pool's backwater does not affect the flow as it passes the outlet lip of the culvert, there lie all degrees of backwater effects which partially reduce the rapid fluid accelerations presented by a highly perched culvert. Though the term "perched culvert" usually means that the culvert invert at the outlet lies above the elevation of the pool, it should be clear that any onfiguration, where the surface of the receiving pool is at a lower elevation than that of the water surface profile in the culvert near the outlet, is really a perched situation by a greater or lesser degree. Since the flow geometry at the culvert outlet depends on many parameters of flow, culvert geometry and receiving water elevation, it is doubtful that a design rule-of-thumb suggesting that the downstream pool be kept at or above a small, specific, fixed, minimum elevation above the lip elevation can be generally effective for all culvert diameters and all flows. Possible changes of culvert outlet geometry and of the degree of downstream backwater required to reduce significantly pressure gradients in the vicinity of the culvert invert's downstream lip are, therefore, potential areas of future hydraulic laboratory and field fish passage investigations.

Because of Newton's second law (F=ma), pressure gradients exist to greater or lesser degrees in almost any situation where water accelerates (speeding up, slowing down, changing direction of flow), depending on the magnitude of the acceleration. Since pressure gradients solely create "buoyant" forces on submerged objects, any zone of water acceleration (in addition to steeply sloping open channels functioning at normal depth or pipeflow with steeply sloping hydraulic grade lines) may be problem zones for fish because of the possible adverse effect on the buoyant force, $\bar{B}$, in that zone.

ENERGY AND POWER EXPENDITURE REQUIREMENTS OF SWIMMING FISH IN CULVERTS

Thus far the discussion has addressed only the forces which swimming fish must deal with to progress into and through simple fish passage structures, specifically culverts. However, a fish must overcome these forces for some distance and for some time period if it is to move through the structure. This leads to energy and power requirements necessary for the fish to expend in its journey through a fish passage facility. This discussion will be limited to what energy and power the fish must expend rather than how much it is capable of expending. The latter has been the topic of much already published material and will not be repeated here. The discussion will deal only with fish passage through the barrel of a culvert and will consider the sloping open channel and the full flowing pipe or culvert (i.e., those situations previously discussed in the context of forces).

Figure 7 indicates a fish swimming in a sloping open channel under conditions of uniform, steady flow. Here L_c is the length of the channel, and the other symbols are as before. As the fish swims upstream or remains stationary in the channel it must swim and release energy continuously. The instantaneous time rate at which it releases energy is the power, Pwr, which it is generating at that instant. Pwr is defined as follows:

$$Pwr = P\ V_{fw} \qquad \text{----Eq. 23}$$

Ignoring entrance and exit energy requirements, the total energy, TE, released by the fish in negotiating the structure is

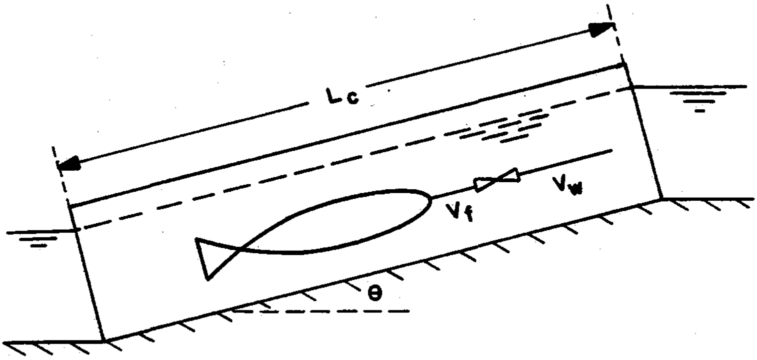


Figure 8. Definition sketch of fish ascending sloping open channel.

$$TE = \int_{o}^{t_p} (Pwr)\ dt \qquad \text{----Eq. 24}$$

where t_p is the fish's passage time through the structure.

For conceptual purposes and to simplify the discussion, it will be assumed the fish's velocity, V_f, the water velocity, V_w, and the velocity of the fish with respect to the water, V_{fw}, are each constant as the fish swims through the structure. If these velocities are constant, the time of passage through the culvert is L_c/V_f, so

$$TE = Pwr(L_c/V_f) \qquad \text{----Eq. 25}$$
$$= P(V_{fw})(L_c/V_f)$$
$$= P\ L_c(V_{fw}/V_f) \qquad \text{----Eq. 26}$$

Since $V_{fw} = V_f + V_w$, TE can be expressed as

$$TE = P\ L_c(1+(V_w/V_f)) \qquad \text{----Eq. 27}$$

Substituting the magnitude of P from Equation 8 for identical conditions, Equation 27 becomes

$$TE = L_c(W(\sin\theta)+D)\ \ (1+(V_w/V_f) \qquad \text{----Eq. 28}$$

The profile drag force, D, was defined by Equation 17. For a given fish, water temperature, and small changes in V_{fw}, the fish's Reynolds number remains relatively constant, so all the terms in Equation 17 except V_{fw} are constants and can be replaced by a constant, K, where $K = C_d(\rho)(L^2)/2$. Making these substitutions for D, Equation 26 becomes

$$TE = L_c(W(\sin\theta)+K\ V_{fw}^{\ 2})(V_{fw}/V_f) \qquad \text{----Eq. 29}$$

It is assumed the example fish swims upstream in a heavily baffled open channel (culvert) which is set at a steep 20% slope and the flow is both uniform and steady (i.e., constant velocity and depth everywhere in the channel). The channel is baffled so that the water velocity of flow, V_w, where the fish swims is 1.5 m/sec. K of Equation 33 becomes .55 N-sec^2/m^2, and W = 1.38 N from the previous example. If the fish just enters the channel but decides to remain at a fixed location in the channel, $V_{fw} = V_w = 1.5$ m/sec, and Equation 33 becomes

$$\begin{aligned} Pwr &= 1.38(.2)(1.5) + .55(1.5)^3 \\ &= 0.41 + 1.86 = 2.27 \text{ Joules/sec} \end{aligned} \qquad \text{----Eq. 34}$$

If the same fish progresses up the channel with $V_f = 0.3$ m/sec, Equation 33 becomes

$$\begin{aligned} Pwr &= 1.38(.2)1.8 + .55(1.8)^3 \\ &= 0.50 + 3.2 = 3.7 \text{ J/sec} \end{aligned} \qquad \text{----Eq. 35}$$

Thus, a 63% greater power expenditure results from only a 20% increase in V_{fw}. In this very steep structure, the example shows the importance of the first term in Equation 33 which results from the fish's weight not being completely cancelled by its buoyant force. More important, however, this example also clearly shows that the fish minimizes its power expenditure if it progresses up the channel very slowly.

Experience at Poplar Grove Creek by Travis (1986) indicates that, under the high water velocity conditions existing in May 1985, large grayling moved up the culvert at a mean, average speed, V_f, of only .028 m/sec. The fastest, average V_f recorded for a single fish was only .052 m/sec. Passage times through the Poplar Grove Creek highway culvert for two grayling, instrumented with transponders, were observed by the writer and others in May 1986. One grayling passed through the culvert, $L_c = 33.5$ m, in 5 minutes and the other passed through in 12 minutes. This yields V_f's of .113 and .046 m/sec. V_w varied within the culvert because of changes in vertical alignment of the culvert which was functioning as an open channel. However, culvert average V_w was in the order of .92 m/sec, so it appears V_{fw} was only slightly greater than V_w. These observations seem to lend some credibility to the concept that, when faced with a structure of unknown extent, a grayling adopts a strategy of minimal power expenditure consistent with upstream progress and its ability to generate that minimum power.

How the fish sense Pwr changing with an increase in its V_f, for constant V_w, can be determined for any value of V_f. Substituting $V_f + V_w$ for V_{fw} in Equation 33

$$Pwr = W\sin\theta(V_f+V_w) + K(V_f+V_w)^3, \qquad \text{----Eq. 36}$$

and Equation 28 becomes

$$TE = L_c(W(\sin\theta)+K\ V_{fw}^{2})(1+(V_w/V_f)) \qquad \text{----Eq. 30}$$

Each of the velocity terms of Equations 26 through 30 appears as a result of the fact that the fish swims in a channel and cannot walk on a ramp having the same length and slope as the channel discussed. That is, Equation 30 for a walker would reduce to

$$TE = L_c\ W\sin\theta \qquad \text{----Eq. 31}$$

Clearly, fish are not efficient "climbers".

For the fish to progress up the channel, V_{fw} must be greater than V_w. If it is only slightly greater than V_w, V_f is only slightly greater than zero. Reference to Equation 29 shows that for such a condition, and a specific V_w, $K(V_{fw}^{2})$ is minimized, but, since V_f would be very small, the ratio V_{fw}/V_f becomes very large, and Equation 26 shows that TE increases greatly as V_{fw} approaches V_w, (i.e., as V_f approaches zero). Conversely, as V_f increases, TE decreases. So, for fish of equal size, faster moving fish expend less total energy in moving through the structure than do slower fish, though reference to Equation 23 shows that the concurrent power requirements of faster moving fish are greater. Thus, more aggressive fish may consume less energy than do slower fish in passing through a culvert -- the "rich" get richer.

As observed by Ziemer and Behlke (1966), the fish negotiating a long structure does not know about TE (Equation 30) until the structure has been successfully negotiated (i.e., after the fact). (From its past experience with natural or artificial structures, the fish is, no doubt, well aware of the possible effects of Equation 30 as it enters the culvert, but it doesn't know what value of L_c to substitute into the equation until it reaches the upstream end of the culvert.) Thus, TE cannot enter into the fish's strategy for negotiating the culvert. Immediately on entering the channel, however, the fish senses the power requirement, Pwr, for it to move ahead.

Substituting D from Equation 17 into Equation 8, and substituting the scalar form of Equation 8 into Equation 23, Pwr becomes

$$Pwr = (W(\sin\theta)+C_d(\rho)(L^{2})(V_{fw}^{2}/2))V_{fw} \qquad \text{----Eq. 32}$$

Or, introducing K, as before, and rearranging slightly

$$Pwr = W(\sin\theta)(V_{fw}) + K\ V_{fw}^{3} \qquad \text{----Eq. 33}$$

An example using previous constants for a .25 m fish, will, perhaps, reveal the strategy which fish adopt in negotiating an obstacle of unknown extent, such as a culvert.

and differentiating with respect to V_f, yields the rate of change of Pwr with respect to a change in V_f. That is,

$$d(Pwr)/dV_f = W\sin\theta + 3K(V_f+V_w)^2 \qquad \text{----Eq. 37}$$

Equation 37 shows $d(Pwr)/dV_f$ to be positive for all positive values of V_f and for all realistic values of θ from zero to $\pi/2$, thus showing that power consumption always increases with increasing V_f.

For a full flowing pipe of length L_c, Equations 23 through 26 apply. Multiplying the scalar magnitude of P, taken from Equation 14, by V_{fw} yields Pwr for the general case of full flowing pipes or culverts and for open channels experiencing uniform, steady flow. That is,

$$\begin{aligned} Pwr &= PV_{fw} \\ &= (D+W(\sin\phi+(\cos\phi)(\tan(\theta-\phi)))V_{fw} \end{aligned} \qquad \text{----Eq. 38}$$

Substituting $K\,V_{fw}^{\;2}$, as before, for D,

$$Pwr = (K\,V_{fw}^{\;2} + W(\sin\phi+(\cos\phi)(\tan(\theta-\phi)))V_{fw} \qquad \text{----Eq. 39}$$

For an open channel operating under conditions of uniform, steady flow, $\theta = \phi$, and Equation 39 reduces to Equation 33. For a horizontal conduit flowing full, $\phi = 0$, and Equation 39 reduces to

$$Pwr = K\,V_{fw}^{\;3} + (W\tan\theta)V_{fw} \qquad \text{----Eq. 40}$$

If the HGL drops by an amount, H, from upstream to downstream ends of the conduit, $\tan\theta = H/L_c$. For the open channel, $\sin\theta = H/L_c$, so Equations 33 and 40 yield identical values for Pwr. Thus, the power necessary for the fish to generate in order to move at a specific speed is the same in the horizontal enclosed conduit as it is in the open channel if L_c, V_{fw} and H are the same in both structures. It can be shown (albeit, with some difficulty) that Pwr is the same for all values of ϕ if L_c, V_{fw} and H remain the same.

CONCLUSIONS

The inclusion of the fish's buoyant force and weight in analyses of the forces which fish must swim against, the power which they must generate, and the total energy which they consume in negotiating fish passage structures is potentially important in the analysis of open channels or pipe type fish passage structures where the hydraulic grade line slopes more than about 10% or anywhere that significant water pressure gradients exist.

Buoyant forces and weight are very important, previously ignored, forces acting on fish attempting to enter perched culverts. Culverts installed with receiving pool surface elevations near those of the water surface emerging from culverts eliminate problems associated with these two forces.

Only if the slope of an open channel culvert becomes appreciable do the fish's buoyant force and its weight become a problem for the fish passing through the body of the barrel of the culvert. It appears that culverts having appropriate entrance and exit conditions could be placed at much greater slopes than is present practice if sufficient, dependable roughness can be introduced to keep the fish's profile drag, power and total energy expenditures at acceptable levels, though what those acceptable levels are is not presently well defined.

The introduction of the fish's buoyant force, its weight, and the relationship between pressure gradients in the water and the fish's buoyant force provides a powerful new tool which, in addition to the long used profile drag, makes possible a much more complete picture of the obstacles which fish face in attempting to negotiate any fish passage structure, not just culverts. The principles introduced may make possible the generation of entirely new concepts in economic fish passage facilities.

The paper has set forth the hydraulic conditions which fish must be capable of overcoming to pass through a fish passage structure. Work should be pursued to identify fishes' abilities to cope simultaneously with buoyant, weight, and profile drag forces and with their limits of power and total energy expenditures while passing through various structures. This work should begin with analyses of past information, published by various investigators, in the context of the principles introduced.

ACKNOWLEDGMENTS

Partial funding for this study was provided by the U.S. Department of Transportation, Federal Highway Administration Grant HPR-86-5. The writer also wishes to thank Dr. John Roberson of Washington State University and Mr. Robert McLean of the Alaska Department of Fish and Game for their draft reviews and helpful suggestions.

BIBLIOGRAPHY

Brett, J.R., 1963. The energy required for swimming by young Sockeye salmon with a comparison of the drag force. Transactions of the Royal Society of Canada, Vol. 1, Series IV, June 1963, p. 441-457.

Henderson, F.M., 1966. Open Channel Flow. MacMillan Publishing Co., Inc., New York, NY, 522 p.

Orsborn, J.F., Powers, P.D., 1985. Fishways--an assessment of their development and design. Part 3 of 4, Report to the Bonneville Power Administration, Project 82-14, 161 p.

Rouse, H., 1938. Fluid mechanics for hydraulic engineers, engineering societies monograms, 1st ed. McGraw-Hill Book Co., New York, NY, 415 p.

Travis, M.D., 1986. Fish passage through Poplar Grove Creek culvert. M.S. Thesis, University of Alaska-Fairbanks, Fairbanks, AK, 107 p.

Ziemer, G.L., Behlke, C.E., 1966. Analysis of salmon capabilities in steep fish ladders. Proceedings of 2nd Annual American Water Resources Conference, 1966, pp. 328-339.

COLD REGIONS
ENVIRONMENTAL ENGINEERING

Proceedings of the Second International Conference

Edmonton, Alberta
23 - 24 March 1987

Edited by
D.W. Smith
T. Tilsworth

Sponsored by

Department of Civil Engineering
University of Alberta
Edmonton, Alberta

Environmental Quality Engineering
University of Alaska
Fairbanks, Alaska

Co-Sponsored by

Canadian Society for Civil Engineering
Cold Regions Engineering Division
Environmental Engineering Division

American Society of Civil Engineers
Alaska Section
Technical Council on Cold Regions
Engineering

Alaska Water Management
Association